Algebra II

1-3 Algebraic Expressions

Content Standard
Reviews A.SSE.1.a Interpret parts of an expression, such as terms, factors, and coefficients.

Key

Objectives To evaluate algebraic expressions
To simplify algebraic expressions

Addition +	Subtraction −	Multiplication ×	Division ÷
Sum	difference	product	quotient
More than	Less than less	times	Divided by
Increased by	Fewer than		
total	Subtracted from		
Added to			

Problem 1 Modeling Words With an Algebraic Expression

What does "seven fewer than t" mean?
"Seven fewer than t" means your answer will be less than t.

Multiple Choice Which algebraic expression models the word phrase *seven fewer than a number t*?

(A) $t + 7$ (B) $-7t$ (C) $t - 7$ (D) $7 - t$

"Seven fewer than" suggests subtraction. Begin with the number t and subtract 7. This can be represented by the expression $t - 7$. The correct answer is C.

Got It? 1. Which algebraic expression models the word phrase *two times the sum of a and b*?

(F) $a + b$ (H) $2(a + b)$
(G) $2a + b$ (I) $a + 2b$

Write an algebraic expression that models each word phrase.

See Problem 1.

10. four more than a number b $b+4$ $4+b$
11. the product of 8 and the sum of a number x and 3 $8(x+3)$
12. the quotient of the difference between 5 and a number n, and 2 $\frac{5-n}{2}$

$3+5=5+3$
$1+2=2+1$
$3-5=5-3$

To **evaluate** an algebraic expression, substitute a number for each variable in the expression. Then simplify using the order of operations.

Problem 3 Evaluating Algebraic Expressions

What is the value of the expression for the given values of the variables?

(A) $7(d + 4) + 3b - 8$ for $a = -4$ and $b = 5$

$7(-4+4)+3(5)-8$
$0 + 15 - 8$
7

Evaluate each algebraic expression for $x = 3$ and $y = -2$.

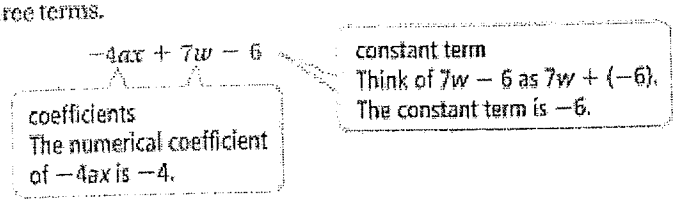
3. $2x - 3y$ 12

4. $5x + y$ 13

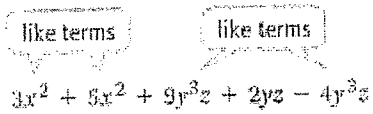
5. $y - x$ -5

6. $x + 4y$ -5

An expression that is a number, a variable, or the product of a number and one or more variables is a **term**. A **coefficient** is the numerical factor of a term. A **constant term** is a term with no variables. You can add terms to form longer expressions. The expression below has three terms.



Like terms have the same variables raised to the same powers.



You can simplify an algebraic expression that has like terms. You combine like terms using the properties of real numbers (Lesson 1-2). An expression and its simplified form are equivalent. Their values are equal for all values of their variables.

EXAMPLES:

Problem 5 **Simplifying Algebraic Expressions**

Combine like terms. What is a simpler form of each expression?

A $7x^2 + 3y^2 + 2y^2 - 4x^2$

$3x^2 + 5y^2$

B $-(3k + m) + 2(k - 4m)$

$-3K - m + 2K - 8m$

$-1K - 9m$

Got It? 5. Combine like terms. What is a simpler form of each expression?

a. $-4j^2 - 7k + 5j + j^2$

$-3j^2 + 5j - 7k$

b. $-(8a + 3b) + 10(2a - 5b)$

$-8a - 3b + 20a - 50b$

$12a - 53b$

Simplify by combining like terms. See Problem 5.

30. $5a - a$ $4a$

31. $5 + 10s - 8s$ $2s + 5$

32. $-5a - 4a + b$ $-9a + b$

33. $2a + 3b + 4a$ $6a + 3b$

34. $6r + 3s + 2s + 4r$ $10r + 5s$

35. $0.5x - |x$ $-.5x$

36. $7b - (3a - 8b)$

$-3a + 15b$

$7b - 3a + 8b$

$-3a + 15b$

37. $5 + (4g - 7)$

$4g - 2$

$5 + 4g - 7$

$4g - 2$

38. $-(3x - 4y) + x$

$-2x + 4y$

1-4 Solving Equations

Content Standards

A.CED.1 Create equations and inequalities in one variable and use them to solve problems.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Objectives To solve equations
To solve problems by writing equations

An **equation** is a statement that two expressions are equal. In this lesson you will use equations to model and solve problems.

Solving an equation that contains a variable means finding all values of the variable that make the equation true. Such a value is a **solution of the equation.** To find a solution, isolate the variable on one side of the equation using *inverse operations.*

Inverse operations are operations that "undo" each other. Addition and subtraction have this inverse relationship, as do multiplication and division.

Problem 1 Solving a One-Step Equation

What is the solution of $x + 4 = -12$?

$-4 \quad -4$

$x = -16$

Solve each equation.

See Problem 1.

10. $h - 12 = 6$ — $h = 18$

11. $\frac{x}{3} = 27$ — $x = -81$

12. $4t = 48$ — $t = 12$

13. $22 + r = 36$ — $r = 14$

Problem 2 Solving a Multi-Step Equation

What is the solution of $-27 + 6y = 3(y - 3)$?

$-27 + 6y = 3y - 9$

$-3y \quad -3y$

$-27 + 3y = -9$

$+27 \quad +27$

$3y$

$\frac{3y}{3} = \frac{18}{3}$

$y = 6$

Got It? 2. What is the solution of $3(2x - 1) - 2(3x + 4) = 11x$?

$6x - 3 - 6x - 8 = 11x$

$0 - 11 = 11x$

$\frac{-11}{11} = \frac{11x}{11}$

$-1 = x$

$x = -1$

See Problem 2.

Solve each equation. Check your answer.

14. $7w + 2 = 3w + 94$

$w = 23$

15. $15 - g = 23 - 2g$

$g = 8$

16. $43 - 3d = d + 9$

$d = 17/2$

17. $5y + 1.8 = 4y - 3.2$

18. $6a - 5 = 4a + 2$

19. $7y + 4 = 3 - 2y$

$y = -5$

$a = \frac{7}{2}$

$y = -\frac{1}{9}$

20. $5c - 9 = 8 - 2c$

21. $4y - 8 - 2y + 5 = 0$

22. $6(n - 4) = 3n$

$c = \frac{17}{7}$

$y = \frac{3}{2}$

$n = 8$

23. $2 - 3(x + 4) = 8$

24. $5(2 - g) = 0$

25. $2(x + 4) = 8$

$x = -6$

$g = 2$

$x = 0$

An equation does not always have one solution. An equation has no solution if no value of the variable makes the equation true. An equation that is true for every value of the variable is an **identity**.

Problem 4 Equations With No Solution and Identities

Is the equation *always*, *sometimes*, or *never* true?

A $11 + 3x - 7 = 6x + 5 - 3x$

$3x + 4 = 3x + 5$

$-3x \quad -3x$

$4 = 5$

Never

No Solution

B $6x + 5 - 2x = 4 + 4x + 1$

$4x + 5 = 4x + 5$

$5 = 5$

Always

Infinite Solution

Got It? 4. Is the equation *always*, *sometimes*, or *never* true?

a. $7x + 6 - 4x = 12 + 3x - 8$

$3x + 6 = 3x + 4$

$6 = 4$

Never

No Solution

b. $2x + 3(x - 4) = 2(2x - 6) + x$

$2x + 3x - 12 = 4x - 12 + x$

$5x - 12 = 5x - 12$

Always Infinite Sol.

29. Sometimes

Determine whether the equation is *always*, *sometimes*, or *never* true. ◀ See Problem 4.

29. $5x + 3 - 2x = 7x + 3$

$3x + 3 = 7x + 3$
$-3x \quad -3x$
$3 = 4x + 3$

Sometimes

$0 = 4x$
$\frac{0}{4} = \frac{4x}{4}$
$0 = x$

30. $2(5x + 4) = 10x + 6$

never

31. $\frac{2}{3}x + 4 = 2x$

$3(\frac{2}{3}x) + 3(4) = 2x \cdot 3$
$2x + 12 = 6x$
$12 = 4x$
$3 = x$

Sometimes

32. $6x - 12 + 2x = 3 + 8x - 15$

always

A **literal equation** is an equation that uses at least two different letters as variables. You can solve a literal equation for any one of its variables by using the properties of equality. You solve for a variable "in terms of" the other variables.

$A = l \cdot w$
$\frac{A}{l} = \frac{l \cdot w}{l}$
$\frac{A}{l} = w$

Problem 5 Solving a Literal Equation

The equation $C = \frac{5}{9}(F - 32)$ relates temperatures in degrees Fahrenheit F and degrees Celsius C. What is F in terms of C?

$9 \cdot C = \frac{9}{9}\frac{5}{9}(F - 32)$
$\frac{9C}{5} = \frac{5}{5}(F - 32)$
$\frac{9}{5}C = F - 32$

$\frac{9}{5}C + 32 = F$

Solve each formula for the indicated variable. ◀ See Problem 5.

33. $A = \frac{1}{2}bh$, for h

$\frac{2A}{b} = \frac{bh}{b}$
$\frac{2A}{b} = h$

34. $s = \frac{1}{2}gt^2$, for g

$2s = gt^2$
$\frac{2s}{t^2} = g$

35. $V = lwh$, for w

$\frac{V}{lh} = w$

36. $I = prt$, for r

$\frac{I}{pt} = r$

Solve each formula for the indicated variable.

46. $R(r_1 + r_2) = r_1r_2$, for R

$\frac{R(r_1 + r_2)}{r_1 + r_2} = \frac{r_1r_2}{r_1 + r_2}$

$R = \frac{r_1r_2}{r_1 + r_2}$

47. $A = \frac{1}{2}h(b_1 + b_2)$, for b_2

$2A = h(b_1 + b_2)$
$\frac{2A}{h} - b_1 = b_2$
$b_2 = \frac{2A}{h} - b_1$

48. $S = 2\pi r^2 + 2\pi rh$, for h

$-2\pi r^2 \quad -2\pi r^2$
$\frac{S - 2\pi r^2}{2\pi r} = \frac{2\pi rh}{2\pi r}$
$h = \frac{S - 2\pi r^2}{2\pi r}$

1-5 Solving Inequalities

Vocabulary Builder

compound (noun) KAHM pound

Related Words: compound inequality, less than, greater than

Definition: A compound is made up of separate parts.

Example: A compound inequality is made up of two or more simple inequalities joined by *and* or *or*.

compound inequality
$x \leq 7$ or $x \geq 5$

Write an *inequality* for each verbal description.

7. x is at most 25

$x \leq 25$

8. x is not equal to 25

$x \neq 25$

9. x is greater than 25

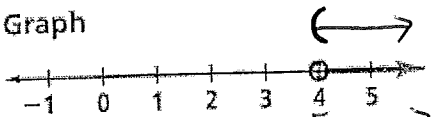
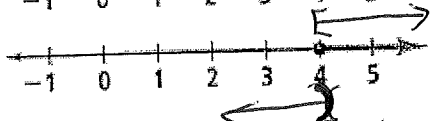
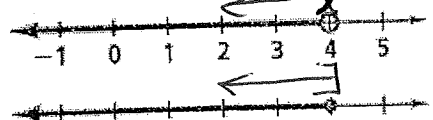
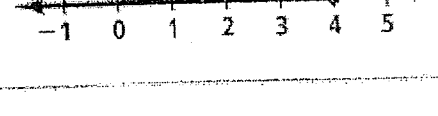
$x > 25$

10. x is less than 25

$x < 25$

TAKE NOTE

Key Concept Writing and Graphing Inequalities

Inequality	Word Sentence	Graph
$x > 4$	x is greater than 4.	
$x \geq 4$	x is greater than or equal to 4.	
$x < 4$	x is less than 4.	
$x \leq 4$	x is less than or equal to 4.	

Problem 1 Writing an Inequality From a Sentence

Got It? What inequality represents the sentence *The quotient of a number and 3 is no more than 15?*

$\frac{x}{3} < 15$

Write the inequality that represents the sentence.

10. The sum of a number and 5 is less than −7.

$x + 5 < -7$

11. The product of a number and 8 is at least 25.

$8x \geq 25$

Problem 2 Solving and Graphing an Inequality

Got It? What is the solution of $-2(x + 9) + 5 \geq 3$? Graph the solution.

$-2x - 18 + 5 \geq 3$

$-2x - 13 \geq 3$

$+13 \quad +13$

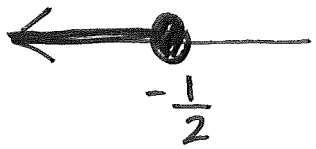
$\frac{-2x}{-2} \geq \frac{16}{-2}$

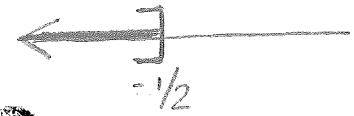
$x \leq -8$

Solve each inequality. Graph the solution.

14. $-12 \geq 24x$

$x \leq -\frac{1}{2}$





15. $-7k < 63$

$k > -9$

Problem 3 Using an Inequality

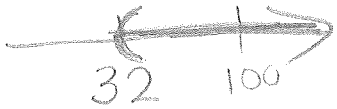
Got It? A digital music service offers two subscription plans. The first has a \$9 membership fee and charges \$1 per download. The second has a \$25 membership fee and charges \$.50 per download. How many songs must you download for the second plan to cost less than the first plan?

1st Plan $9 + 1d$ > 2nd Plan $25 + .50d$

\$109 \$75

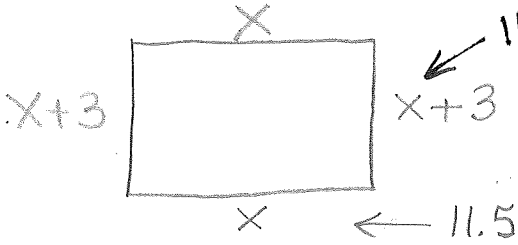
$.5d > 16$

$d > 32$



Solve each problem by writing an inequality.

24. The length of a picture frame is 3 in. greater than the width. The perimeter is less than 52 in. Describe the dimensions of the frame.



$11.5 + 3 = 14.5$

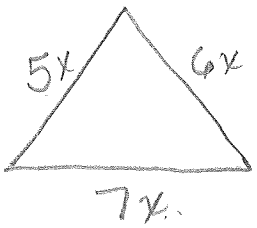
$x + x+3 + x+3 + x < 52$

$4x + 6 < 52$

$4x < 46$

$x < \frac{23}{2}$ or 11.5

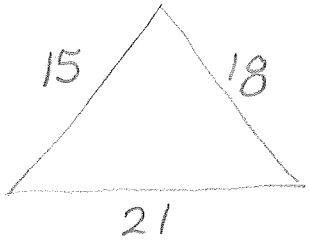
25. The lengths of the sides of a triangle are in the ratio 5 : 6 : 7. Describe the length of the longest side if the perimeter is less than 54 cm.



$5x + 6x + 7x < 54$

$18x < 54$

$x < 3$



Problem 4 No Solution or All Real Numbers as Solution

Got It? Is $4(2x - 3) < 8(x + 1)$ *always, sometimes,* or *never* true?

$8x - 12 < 8x + 8$

$-12 < 8$

always

Solve it

Infinite sol.

Is the inequality *always, sometimes,* or *never* true?

28. $9(x + 2) > 9(x - 3)$

$9x + 18 > 9x - 27$

$18 > -27$

always

29. $6x - 13 < 6(x - 2)$

always

Problem 5 Solving an *And* Inequality

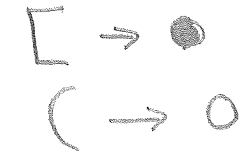
Got It? What is the solution of $8 \leq 3x - 1$ and $2x < 12$? Graph the solution.

$+1 \quad +1$

$\frac{9}{3} \leq \frac{3x}{3}$

$3 \leq x$

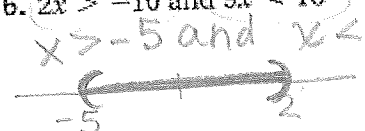
$x \geq 3$ and $x < 6$



Solve each compound inequality. Graph the solution.

36. $2x > -10$ and $9x < 18$

$x > -5$ and $x < 2$



$-5 < x < 2$

37. $3x \geq -12$ and $8x \leq 16$

$\frac{3x}{3} \geq \frac{-12}{3} \quad \frac{8x}{8} \leq \frac{16}{8}$

$-4 \leq x \leq 2$

$x \geq -4$ and $x \leq 2$

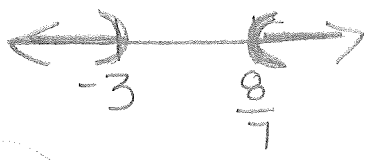
Problem 6 Solving an *Or* Inequality

Got It? What is the solution of $7w + 3 > 11$ or $4w - 1 < -13$? Graph the solution.

$-3 \quad -3 \qquad +1 \quad +1$

$\frac{7w}{7} > \frac{8}{7} \qquad \frac{4w}{4} < \frac{-12}{4}$

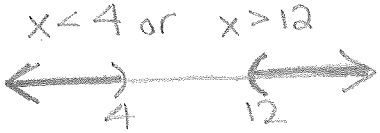
$w > 8/7$ or $w < -3$



Solve each compound inequality. Graph the solution.

40. $\frac{4x}{4} < \frac{16}{4}$ or $\frac{12x}{12} > \frac{144}{12}$

$x < 4$ or $x > 12$



41. $3x \geq 3$ or $9x < 54$

$x \geq 1$ all real numbers

$x < 6$