11-1 Permutations and Combinations

Essential Understanding You can use multiplication to quickly count the number of ways certain things can happen.

The **Fundamental Counting Principle** describes the method of using multiplication to count.

take note

Key Concept Fundamental Counting Principle

If event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in $m \cdot n$ ways.

Example 3 pants and 2 shirts give $3 \cdot 2 = 6$ possible outfits.

It is fairly easy to count the ways you can pick items from a short list. But, sometimes you have so many choices that counting the possibilities is impractical.

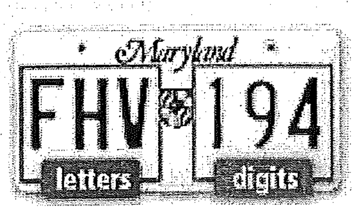
Example 1:

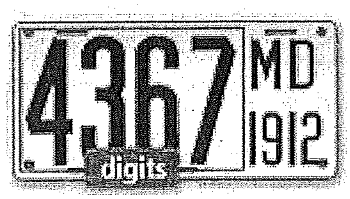
Michigan's standard license plate is a pattern of 3 letters, followed by 4 numbers. How many different possibilities are available?

____ x ____ x ____ x _____ x _____ x ____ x ____ =
letter letter letter digit digit digit digit

Problem 1 Using the Fundamental Counting Principle

Motor Vehicles The photos show Maryland license plates in 2004 and 1912. How many more 2004-style license plates were possible than 1912-style plates?





A **permutation** is an arrangement of items in a particular order.

Example 2:

How many different ways can you arrange the letters ABC? Make an organized list.

Permutations of an Entire Set

To find an arrangement of an ***entire set*** of objects ***with*** **no repetition**, you can use "*n* factorial":

$$n! = n*(n-1)*(n-2)*...*2*1$$

NOTE: The value of $0! = 1$.

Problem 2 Finding the Number of Permutations of *n* Items

In how many ways can you file 12 folders, one after another, in a drawer?

Permutations of a Portion of a Set

To find an arrangement of only a ***portion*** of a set of objects ***with no*** **repetition**, you can use:

$$_nP_r = \frac{n!}{(n-r)!}$$

n = number of different items in the set r = the number of items that are "taken" or used

Problem 3 Finding $_nP_r$

Track Ten students are in a race. First, second, and third places will win medals. In how many ways can 10 runners finish first, second, and third (no ties allowed)?

A selection in which order does not matter is a **combination**.

To find the number of **combinations** of an entire set of objects, you can use:

$$_nC_r = \frac{n!}{(n-r)!r!}$$

n = number of different items in the set r = the number of items that are "taken" or used

Problem 4 Finding $_nC_r$

What is $_{13}C_4$, the number of combinations of 13 items taken 4 at a time?

When determining whether to use a permutation or combination, you must decide whether order is important.

Problem 5 Identifying Whether Order Is Important

For each situation, determine whether you should use a permutation or combination. What is the answer to each question?

A A chemistry teacher divides his class into eight groups. Each group submits one drawing of the molecular structure of water. He will select four of the drawings to display. In how many different ways can he select the drawings?

B You will draw winners from a total of 25 tickets in a raffle. The first ticket wins $100. The second ticket wins $50. The third ticket wins $10. In how many different ways can you draw the three winning tickets?

Homework

9. You have five shirts and four pairs of pants. How many different ways can you arrange your shirts and pants into outfits?

See Problem 1.

10. To create an entry code for a push-button door lock, you need to first choose a letter and then, three single-digit numbers. How many different entry codes can you create?

11. The prom committee has four sites available for the banquet and three sites for the dance. How many arrangements are possible for the banquet and dance?

Evaluate each expression.

See Problem 2.

13. $10!$

15. $5!3!$

17. $5(4!)$

19. $\frac{15!}{10!5!}$

Evaluate each expression.

See Problem 3.

21. ${}_8P_1$

23. ${}_6P_3$

25. ${}_3P_2$

27. ${}_9P_6$

29. Scheduling Fifteen students ask to visit a college admissions counselor. Each scheduled visit includes one student. In how many ways can ten time slots be assigned?

Evaluate each expression. See Problem 4.

31. ${}_8C_5$

32. ${}_4C_4$

33. ${}_4C_3$

35. $3({}_5C_4)$

37. $\frac{{}_7C_4}{{}_9C_4}$

38. Awards There are eight swimmers in a competition where the top three swimmers advance. In how many ways can three swimmers advance?

For each situation, determine whether to use a permutation or a combination. Then solve the problem. See Problem 5.

39. How many different teams of 11 players can be chosen from a soccer team of 16?

40. Suppose you find seven equally useful articles related to the topic of your research paper. In how many ways can you choose five articles to read?

41. A salad bar offers eight choices of toppings for a salad. In how many ways can you choose four toppings?

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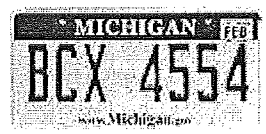
Example 1:



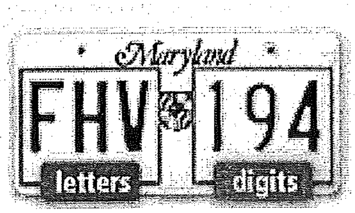
Michigan's standard license plate is a pattern of 3 letters, followed by 4 numbers. How many different possibilities are available?

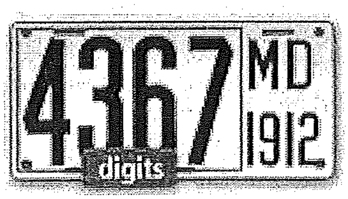
26 x 26 x 26 x 10 x 10 x 10 x 10 = 175,760,000

letter letter letter digit digit digit digit

Problem 1 Using the Fundamental Counting Principle

Motor Vehicles The photos show Maryland license plates in 2004 and 1912. How many more 2004-style license plates were possible than 1912-style plates?





$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$

$= 17576000$

$10 \cdot 10 \cdot 10 \cdot 10$

$= 10{,}000$

$17576000 - 10000$

$= 17{,}566{,}000$

A **permutation** is an arrangement of items in a particular order.

Example 2:

How many different ways can you arrange the letters ABC? Make an organized list.

ABC BAC CBA
ACB BCA CAB 6

Permutations of an Entire Set

To find an arrangement of an ***entire set*** of objects ***with*** **no repetition**, you can use "*n* factorial":

$$n! = n*(n-1)*(n-2)*...*2*1$$

$$6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$$

NOTE: The value of 0! = 1.

Problem 2 **Finding the Number of Permutations of *n* Items**

In how many ways can you file 12 folders, one after another, in a drawer?

$$12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\ldots\cdot1$$

$$12! = 479{,}001{,}600$$

Permutations of a Portion of a Set

To find an arrangement of only a ***portion*** of a set of objects ***with no*** **repetition**, you can use:

$${}_nP_r = \frac{n!}{(n-r)!}$$

n = number of different items in the set r = the number of items that are "taken" or used

Problem 3 **Finding $_nP_r$**

Track Ten students are in a race. First, second, and third places will win medals. In how many ways can 10 runners finish first, second, and third (no ties allowed)?

$${}_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10\cdot9\cdot8\cdot\cancel{7!}}{\cancel{7!}}$$

$$= 10\cdot9\cdot8$$

$$= 720$$

A selection in which order does not matter is a **combination**.

To find the number of **combinations** of an entire set of objects, you can use:

$$_nC_r = \frac{n!}{(n-r)!r!}$$

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Problem 4 Finding $_nC_r$

What is $_{13}C_4$, the number of combinations of 13 items taken 4 at a time?

$n \nearrow \quad \nwarrow r$

$$\frac{13!}{(13-4)!4!} = \frac{13!}{9!4!} = \frac{}{9\cdot8\cdot7\cdot6\cdot4\cdots1\cdot4\cdot3\cdot2\cdot1}$$

$= 715$

When determining whether to use a permutation or combination, you must decide whether order is important.

Problem 5 Identifying Whether Order Is Important

For each situation, determine whether you should use a permutation or combination. What is the answer to each question?

A A chemistry teacher divides his class into eight groups. Each group submits one drawing of the molecular structure of water. He will select four of the drawings to display. In how many different ways can he select the drawings?

Order Does NOT Matter!

$_nC_r = {_8C_4} = 70$

B You will draw winners from a total of 25 tickets in a raffle. The first ticket wins $100. The second ticket wins $50. The third ticket wins $10. In how many different ways can you draw the three winning tickets?

Order Matters!

$_nP_r = {_{25}P_3} = 13{,}800$

Homework

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