



Algebra 2 – Year 2

Lesson 11.4 – Conditional Probability
Notes & Examples

Name: _____
Date: _____ Hour: _____

Conditional Probability

The **conditional probability** of an event B is the probability that the event will occur given the knowledge that an event A has already occurred.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Examples

Problem 1 Finding Conditional Probability

Education The table shows students by gender at two- and four-year colleges, and graduate schools, in 2005. You pick a student at random.

A What is $P(\text{female} | \text{graduate school})$?

Student Genders

	Males (in thousands)	Females (in thousands)
Two-year colleges	1866	2462
Four-year colleges	4324	5517
Graduate schools	1349	1954

$$P(\text{female} | \text{graduate school}) = \frac{1954}{3303} \approx .59 \text{ or } 59\%$$

$$P(\text{four-year} | \text{male}) = \frac{4324}{7539} \approx .57 \text{ or } 57\%$$

$$\begin{array}{r} 1349 \\ 1954 \\ \hline 3303 \end{array}$$

$$\begin{array}{r} 1866 \\ 4324 \\ 1349 \\ \hline 7539 \end{array}$$

Problem 2 Conditional Probability in Statistics

Multiple Choice Americans recycle increasing amounts through municipal waste collection. The table shows the collection data for 2007. What is the probability that a sample of recycled waste is paper?

- (A) 16% (C) 33%
(B) 28% (D) 57%

The given condition is that the waste is recycled. A favorable outcome is that the recycled waste is paper.

Municipal Waste Collected
(millions of tons)

Material	Recycled	Not Recycled
Paper	45.2	37.8
Metal	7.2	13.6
Glass	3.2	10.4
Plastic	2.1	28.6
Other	21.7	46.3

Source: U.S. Environmental Protection Agency

$$P(\text{paper} | \text{recycled}) = \frac{45.2}{79.4} \approx .57 \text{ or } 57\%$$

Problem 3 Using the Conditional Probability Formula

Market Research A utility company asked 50 of its customers whether they pay their bills online or by mail. What is the probability that a customer pays the bill online, given that the customer is male?

	Online	By Mail
Male	12	8
Female	24	6

$$P(\text{male and online}) = \frac{12}{50}$$

$$P(\text{male}) = \frac{20}{50}$$

$$P(\text{online} | \text{male}) = \frac{\frac{12}{50}}{\frac{20}{50}} = \frac{12}{50} \cdot \frac{50}{20} = \frac{3}{5} = .6 \text{ or } 60\%$$

1. What is the probability that the total of two dice will be greater than 9, given that the first die is a 5?

Event A: First die is 5

Event B: Total of two dice is greater than 9

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

$$P(B) = \frac{6}{36} = \frac{1}{6}$$

$$P(A \text{ and } B) = \frac{2}{36} = \frac{1}{18}$$

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{\frac{1}{18}}{\frac{1}{6}} = \frac{1}{3} \cdot \frac{6}{1} = \frac{1}{3} \approx .33 \text{ or } 33\%$$

		First Die					
		1	2	3	4	5	6
Second Die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

2. Susan took two tests. The probability of her passing both tests is 0.6. The probability of her passing the first test is 0.8. What is the probability of her passing the second test given that she has passed the first test?

$$P(\text{second} | \text{first}) = \frac{P(\text{first and second})}{P(\text{first})} = \frac{0.6}{0.8} = .75 \text{ or } 75\%$$

3. A bag contains red and blue marbles. Two marbles are drawn without replacement. The probability of selecting a red marble and then a blue marble is 0.28. The probability of selecting a red marble on the first draw is 0.5. What is the probability of selecting a blue marble on the second draw, given that the first marble drawn was red?

$$P(\text{blue} | \text{red}) = \frac{P(\text{red and blue})}{P(\text{red})} = \frac{0.28}{0.5} = .56 \text{ or } 56\%$$



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Assignment: Lesson 11.4

Name: _____
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Use the table at the right to find each probability.

Education and Salary of Employees

1. $P(\text{has less than high school education})$

$$\frac{107}{1200} \approx .089 \rightarrow 8.9\%$$

2. $P(\text{earns over } \$30,000 \text{ and has less than high school education})$

$$\frac{2}{1200} \approx 0.002 \rightarrow .2\%$$

3. $P(\text{earns over } \$30,000 \mid \text{has only high school education}) = \frac{P(\text{high school and over } \$30,000)}{P(\text{high school})}$

$$\frac{14}{224} \approx 0.0625 \rightarrow 6.25\%$$

	Under \$20,000	\$20,000 to \$30,000	Over \$30,000
Less than high school	69	36	2
High school	112	98	14
Some college	102	193	143
College degree	13	173	245

107
224
438
431
1200

Use the table below to find each probability. The table gives information about students at one school.

Favorite Leisure Activities

	Sports	Hiking	Reading	Phoning	Shopping	Other
Female	39	48	85	62	71	29
Male	67	58	76	54	68	39

334
362

4. $P(\text{sports} \mid \text{female})$

$$= \frac{P(\text{female and sports})}{P(\text{female})} = \frac{39}{334}$$

$$5. P(\text{female} \mid \text{sports}) = \frac{P(\text{female and sports})}{P(\text{sports})}$$

6. $P(\text{reading} \mid \text{male})$

$$= \frac{P(\text{male and reading})}{P(\text{male})} = \frac{76}{362} \approx .21 \rightarrow 21\%$$

$$7. P(\text{male} \mid \text{reading}) = \frac{P(\text{male and reading})}{P(\text{reading})} = \frac{76}{85+76} \approx .427 \rightarrow 42.7\%$$

8. $P(\text{hiking} \mid \text{female})$

$$= \frac{P(\text{hiking and female})}{P(\text{female})} = \frac{48}{334} \approx .144 \rightarrow 14.4\%$$

9. $P(\text{hiking} \mid \text{male})$

$$= \frac{P(\text{hiking and male})}{P(\text{male})} = \frac{58}{362} \approx .16 \rightarrow 16\%$$

10. The senior class is 55% female, and 32% of the class are females who play a competitive sport. What is the probability that a student plays a competitive sport, given that the student is female?

$$P(\text{competitive} \mid \text{female}) = \frac{P(\text{competitive and female})}{P(\text{female})} = \frac{32}{55} = .582 \rightarrow 58.2\%$$

11. A softball game has an 80% chance of being cancelled if it rains and a 30% chance of being cancelled if there is fog when there is no rain. There is a 70% chance of fog with no rain and a 30% chance of rain. Find the probability that there will be fog and the game will be cancelled.

$$P(\text{fog and cancelled}) = P(\text{fog}) \cdot P(\text{cancelled}) = (.3)(.7) = .21 \rightarrow 21\%$$



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Bill Payment

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P(A): _____

P(B): _____

P(A and B): _____

P(B|A): _____

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