

1-6

Absolute Value Equations and Inequalities

Key

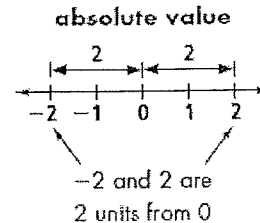
Vocabulary Builder

absolute value (noun) AB suh loof VAL yoo

Definition: The absolute value of a real number x is its distance from zero on the number line.

Main Idea: If x is positive, then $|x| = x$. If x is negative, then $|x| = -x$.

Examples: $|5| = 5$, $|-5| = 5$



$|3x+2| = -4$

NO SOL

Use Your Vocabulary

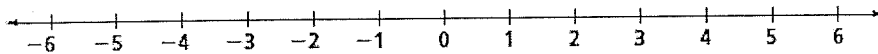
Graph each absolute value on the number line.

2. $|3| = 3$

3. $|-2| = 2$

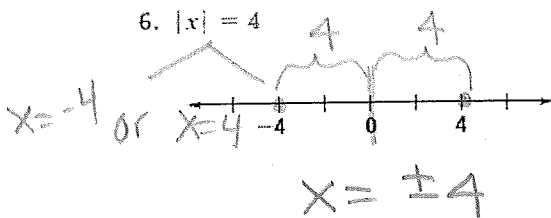
4. $-|1| = -1$

5. $2|-2| = 4$

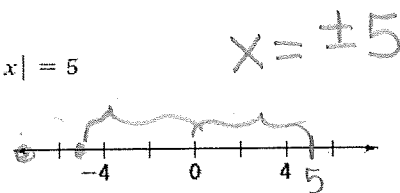


Complete each graph.

6. $|x| = 4$



7. $|x| = 5$



NO SOL



Problem 1 Solving an Absolute Value Equation

Got It? What is the solution of $|3x + 2| = 4$? Graph the solution.

$|3 \cdot \frac{2}{3} + 2|$
 $|4| = 4$
 $4 = 4$

$3x+2 = -4$ or $3x+2 = 4$
 $-2 \quad -2$
 $\frac{3x = -6}{3} \quad \frac{3x = 2}{3}$
 $x = -2 \quad x = \frac{2}{3}$

$|-5| = -5$
 $5 = -5$

$|x| = -5$
 $|5| = -5$
 $5 = -5$
 $|0| = -5$
 $0 = -5$

Solve each equation. Check your answers.

10. $|3x| = 18$

$3x = -18$ or $3x = 18$
 $x = -6$ or $x = 6$

11. $|-4x| = 32$

$-4x = 32$ or $-4x = -32$
 $\frac{-4x = 32}{-4} \quad \frac{-4x = -32}{-4}$
 $x = -8 \quad x = 8$

12. $|x - 3| = 9$

$x - 3 = 9$ or $x - 3 = -9$
 $x = 12$ or $x = -6$

$$2x + 3 = 7$$



Problem 2 Solving a Multi-Step Absolute Value Equation

Got It? What is the solution of $2|x + 9| + 3 = 7$? Graph the solution.



$$\frac{2|x+9| + 3}{-3 \quad -3} = 7$$

$$\frac{2|x+9|}{2} = \frac{4}{2}$$

$$|x+9| = 2 \leftarrow$$

$$\begin{cases} x+9 = -2 \\ x = -11 \end{cases} \quad \text{or} \quad \begin{cases} x+9 = 2 \\ x = -7 \end{cases}$$

$$2|-11+9| + 3 \stackrel{?}{=} 7$$

$$2|-2| + 3$$

$$2 \cdot 2 + 3$$

$$4 + 3$$

$$7$$

Solve each equation. Check your answers.

16. $|x + 4| + 3 = 17$
 $\quad \quad \quad -3 \quad -3$

$$|x + 4| = 14$$

$$\begin{cases} x+4 = 14 \\ -4 \quad -4 \end{cases} \quad \text{or} \quad \begin{cases} x+4 = -14 \\ -4 \quad -4 \end{cases}$$

$$x = 10 \quad \text{or} \quad x = -18$$

17. $|y - 5| - 2 = 10$

$$|y - 5| = 12$$

$$\begin{cases} y-5 = -12 \\ +5 \quad +5 \end{cases} \quad \begin{cases} y-5 = 12 \\ +5 \quad +5 \end{cases}$$

$$y = -7$$

$$y = 17$$

18. $|4 - z| - 10 = 1$
 $\quad \quad \quad +10 \quad +10$

$$|4 - z| = 11$$

$$\begin{cases} 4-z = -11 \\ -4 \quad -4 \end{cases}$$

$$-z = -15$$

$$z = 15$$

$$\begin{cases} 4-z = 11 \\ -4 \quad -4 \end{cases}$$

$$-z = 7$$

$$z = -7$$



Problem 3 Solving for Extraneous Solutions

Got It? What is the solution of $|5x - 2| = 7x + 14$? **Check for extraneous solutions.**

$$\begin{array}{r} 5x - 2 = 7x + 14 \\ -5x \quad -5x \\ \hline -2 = 2x + 14 \\ -14 \quad -14 \\ \hline -16 = 2x \end{array}$$

$$x = -8$$

or $5x - 2 = -(7x + 14)$

$$5x - 2 = -7x - 14$$

$$12x = -12$$

$$x = -1$$



Solve each equation. Check for extraneous solutions.

19. $|x - 1| = 5x + 10$

$$x = -3/2$$

20. $|2z - 3| = 4z - 1$

$$z = 2/3$$

$$\begin{array}{r} 2z - 3 = 4z - 1 \\ -2z \quad -2z \\ \hline -3 = 2z - 1 \\ +1 \quad +1 \\ \hline -2 = 2z \\ z = -1 \end{array}$$

$$2z - 3 = -(4z - 1)$$

22. $|2y - 4| = 12$

$$y = -4, 8$$

$$2y - 4 = -12$$

$$2y - 4 = 12$$

23. $3|4w - 1| - 5 = 10$

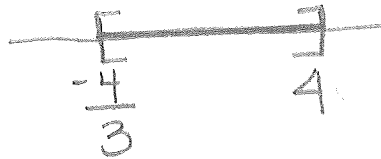
$$w = -1, \frac{3}{2}$$



Problem 4 Solving the Absolute Value Inequality $|A| < b$

Got It? What is the solution of $|3x - 4| \leq 8$? Graph the solution.

$$\begin{aligned}
 -8 &\leq 3x - 4 \leq 8 \\
 +4 &\quad +4 \quad +4 \\
 \hline
 -4 &\leq 3x \leq 12 \\
 \frac{-4}{3} &\leq \frac{3x}{3} \leq \frac{12}{3} \\
 -\frac{4}{3} &\leq x \leq 4
 \end{aligned}$$



Solve each inequality. Graph the solution.

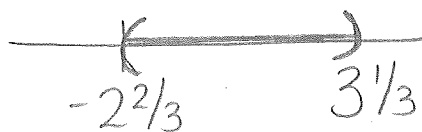
25. $3|y - 9| < 27$

$$0 < y < 18$$



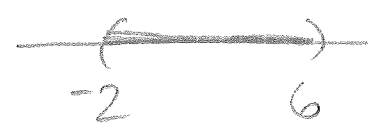
26. $|6y - 2| + 4 < 22$

$$-2\frac{2}{3} < y < 3\frac{1}{3}$$



27. $|3x - 6| + 3 < 15$

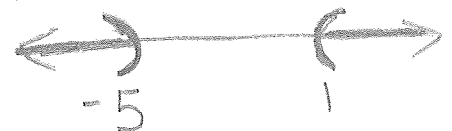
$$-2 < x < 6$$



Problem 5 Solving the Absolute Value Inequality $|A| \geq b$

Got It? What is the solution of $|5x + 10| > 15$? Graph the solution.

$$\begin{aligned}
 5x + 10 &< -15 \quad \text{or} \quad 5x + 10 > 15 \\
 \frac{-10}{5} \quad \frac{-10}{5} &\quad \quad \quad \frac{-10}{5} \quad \frac{-10}{5} \\
 \hline
 5x &< -25 \quad \quad \quad 5x > 5 \\
 \frac{5x}{5} &\quad \quad \quad \frac{5x}{5} \\
 x &< -5 \quad \quad \quad \text{or} \quad x > 1
 \end{aligned}$$

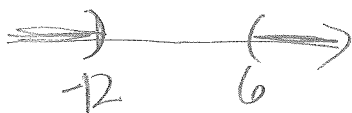


$$x + 3 < -9 \quad \text{or} \quad x + 3 > 9$$

Solve each inequality. Graph the solution.

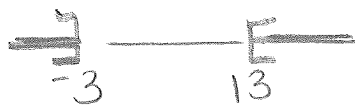
31. $|x + 3| > 9$

$$x < -12 \quad \text{or} \quad x > 6$$



32. $|x - 5| \geq 8$

$$x \leq -3 \quad \text{or} \quad x \geq 13$$

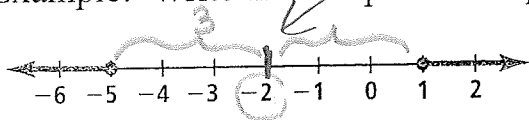


33. $|y - 3| \geq 12$

$$y \leq -9 \quad \text{or} \quad y \geq 15$$



Example: Write the compound inequality as an absolute value inequality.



$$- -2 \rightarrow +2$$

$$x \leq -5 \text{ or } x \geq 1$$

$$x - -2 \leq -5 - -2 \quad x - -2 \geq 1 - -2$$

$$x + 2 \leq -3 \quad \text{or} \quad x + 2 \geq 3$$

$$|x + 2| \geq 3$$

Write each compound inequality as an absolute value inequality.

34. $10 \leq x \leq 16$



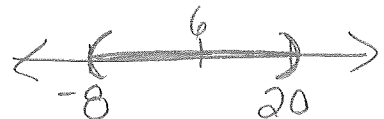
$$|x - 13| \leq 3$$

35. $x < -4 \text{ or } x > 2$



$$|x + 1| > 3$$

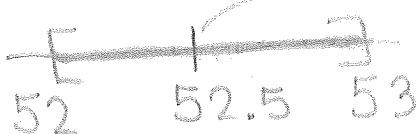
36. $-8 < x < 20$



$$|x - 6| < 14$$

Problem 6 Using an Absolute Value Inequality

Got It? Suppose the smallest allowable height of a racecar is 52 in. and the desirable height is 52.5 in. What absolute value inequality describes heights of a racecar within an indicated tolerance?



$$|x - 52.5| \leq .5$$

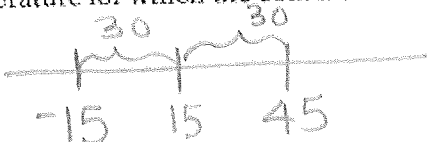
Write an absolute value inequality to represent each situation.

69. **Cooking** Suppose you used an oven thermometer while baking and discovered that the oven temperature varied between +5 and -5 degrees from the setting. If your oven is set to 350°, let t be the actual temperature.



$$|x - 350| \leq 5$$

71. **Climate** A friend is planning a trip to Alaska. He purchased a coat that is recommended for outdoor temperatures from -15°F to 45°F. Let t represent the temperature for which the coat is intended.



$$|x - 15| \leq 30$$