

6.2 Trigonometric Applications

In this section, you will be using your knowledge of right triangle trig (SOH, CAH, TOA) to find missing sides and missing angles on right triangles. Also, recall that the Pythagorean Theorem and your knowledge about proportions is helpful in finding unknown sides of right triangles.

Solving Right Triangles

Objectives

- Solve triangles using trigonometric ratios
- Solve applications using triangles

Triangle Sum Theorem: The sum of the measures of the angles in a triangle is 180° .

Pythagorean Theorem: In a right triangle with legs a and b and hypotenuse c , $a^2 + b^2 = c^2$.

Make sure your calculator is in degree mode!

Example 1 Finding a Side of a Triangle

Find side x of the right triangle in Figure 6.2-1.

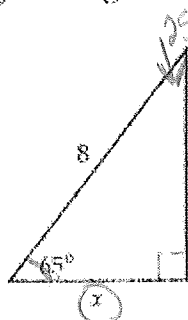


Figure 6.2-1

$$\begin{aligned}90 - 65 &= 25 \\ \cos 65 &= \frac{x}{8} \\ 8 \cos 65 &= x \\ x &= 3.38\end{aligned}$$

$$\begin{aligned}\sin 65 &= \frac{y}{8} \\ 8 \sin 65 &= y \\ y &= 7.25\end{aligned}$$

Example 2 Finding an Angle of a Triangle

Find the measure of angle θ in Figure 6.2-2.

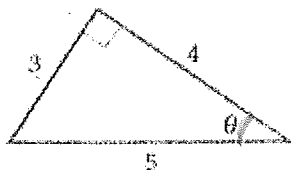


Figure 6.2-2

$$\begin{aligned}\sin \theta &= \frac{3}{5} & \cos \theta &= \frac{4}{5} & \tan \theta &= \frac{3}{4} \\ \theta &= 36.87\end{aligned}$$

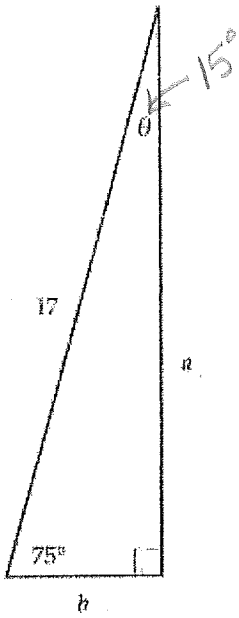


Figure 6.2-4

Example 3 Solving a Right Triangle

Solve the right triangle in Figure 6.2-4.

$$90 - 75 = 15^\circ$$

$$\sin 75 = \frac{a}{17}$$

$$17 \sin 75 = a$$

$$a = 16.42$$

$$\sin 15 = \frac{b}{17}$$

$$17 \sin 15 = b$$

$$b = 4.40$$

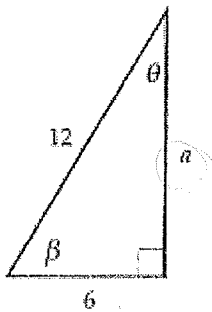


Figure 6.2-5

Example 4 Solving a Right Triangle

Solve the right triangle in Figure 6.2-5.

$$a^2 + 6^2 = 12^2$$

$$a^2 + 36 = 144$$

$$a^2 = 108$$

$$a = \sqrt{108}$$

$$\begin{array}{r} \\ 36 \\ \end{array}$$

$$a = 6\sqrt{3}$$

$$\theta = 30^\circ$$

$$\beta = 60^\circ$$

Solving Right Triangles

A right triangle can be solved if the following information is given.

Case 1:
an acute angle and a side

Sketch the triangle and label the acute angle, the right angle, and the given side.

Find the remaining acute angle by subtracting the known angles from 180° .

Write a trigonometric equation that has an unknown side as the variable, and solve it with a calculator to evaluate the trigonometric ratio of the angle.

Repeat the previous step or use the Pythagorean Theorem to find the third side.

Case 2:
two sides

Sketch the triangle and label the right angle and the two given sides.

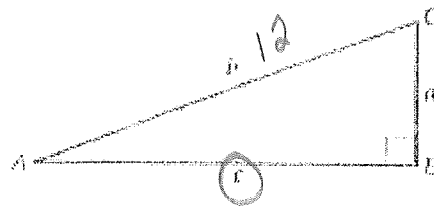
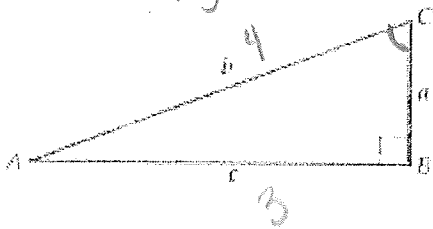
Find the third side by using the Pythagorean Theorem.

Write a trigonometric equation that has an unknown angle as the variable. If the angle is a special angle, you can solve it by recognizing the value of the trigonometric ratio.

If the angle is not one of the special angles, use the technique explained in Example 2.

Example 5: Solve for c .

$\sin C = \frac{3}{4}$ *opp* $b = 12$
hyp.



These are like 1-5 in Homework!

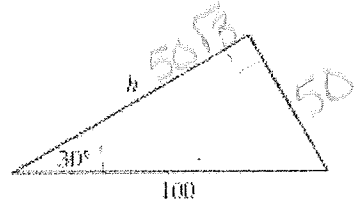
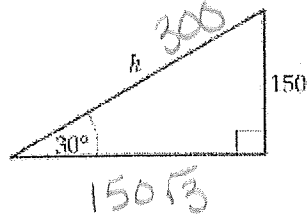
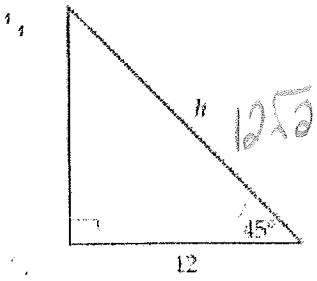
$$\frac{3}{4} = \frac{c}{12}$$

$$4c = 36$$

$$c = 9$$

Example 6: Solve for h.

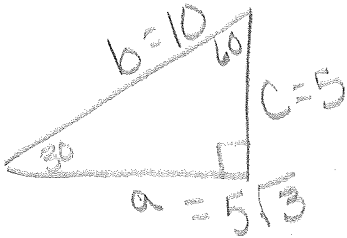
These are like 7-12.



Example 7: Find the indicated value without using a calculator.

These are like 13-16.
Hint... draw a triangle.

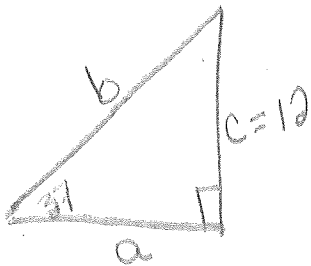
$c = 5$ $m\angle A = 60^\circ$ Find a .



In Exercises 17-24, solve the triangle with the given conditions.

18. $c = 12$

$m\angle C = 37^\circ$



$$90 - 37 = 53^\circ$$

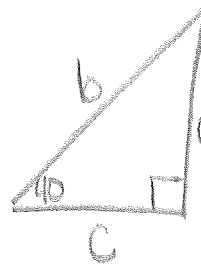
$$\tan 37 = \frac{12}{a} \quad \sin 37 = \frac{12}{b}$$

$$a = \frac{12}{\tan 37} = 15.92$$

$$b = \frac{12}{\sin 37} = 19.93$$

20. $a = 8$

$m\angle A = 40^\circ$



$$90 - 40 = 50^\circ$$

$$\sin 40 = \frac{8}{b}$$

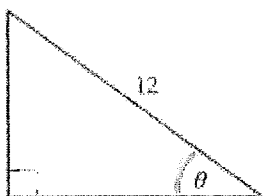
$$b = \frac{8}{\sin 40} = 12.4$$

$$\tan 40 = \frac{8}{c}$$

$$c = \frac{8}{\tan 40} = 9.53$$

In Exercises 25-28, find angle θ .

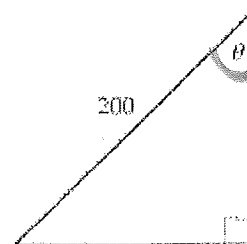
26.



$$\cos \theta = \frac{10}{12}$$

$$\theta = 33.56^\circ$$

28.

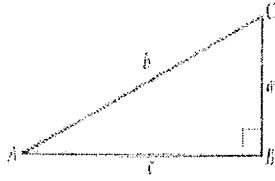
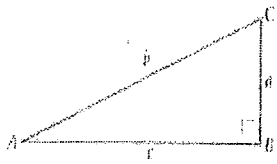
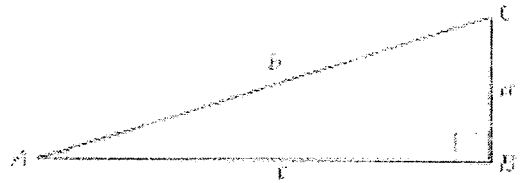


$$\sin \theta = \frac{144}{200}$$

$$\theta = 46.1^\circ$$

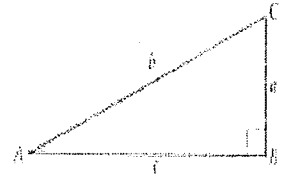
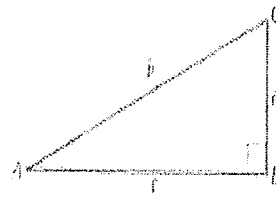
Exercises 6.2

In Exercises 1–6, find side c in the figure below by using the given conditions.



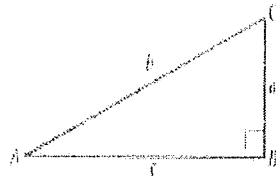
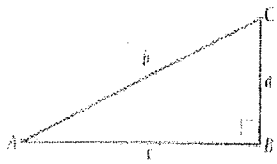
1. $\cos A = \frac{12}{13}$

$b = 39$



5. $\cot A = 6$

$a = 1.4$

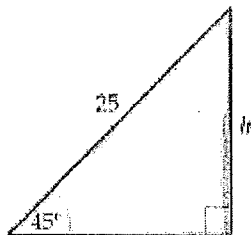


3. $\tan A = \frac{5}{12}$

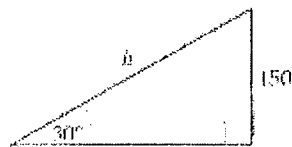
$a = 15$

In Exercises 7–12, find the exact value of h without using a calculator.

7.



9.

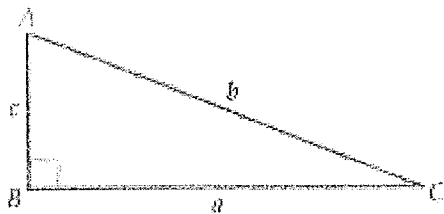


11.



Use the figure below for Exercises 13–24.

In Exercises 13–16, find the indicated value without using a calculator.

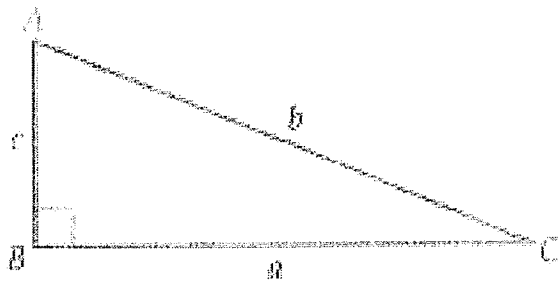


13. $a = 4$ $m\angle A = 60^\circ$ Find c .

15. $c = 10$

$m\angle A = 30^\circ$

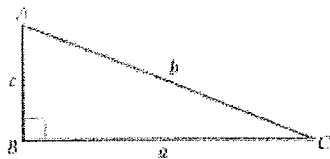
Find a .



In Exercises 17–24, solve the triangle with the given conditions.

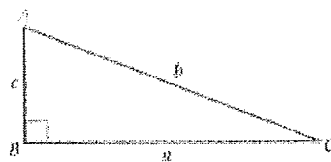
17. $b = 10$

$m\angle C = 50^\circ$



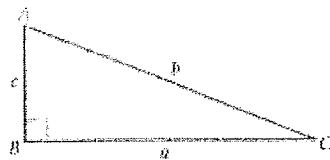
19. $a = 6$

$m\angle A = 14^\circ$



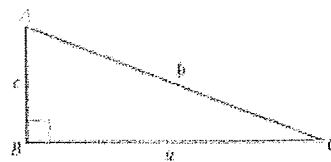
21. $c = 5$

$m\angle A = 65^\circ$



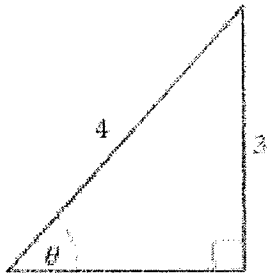
23. $b = 3.5$

$m\angle A = 72^\circ$

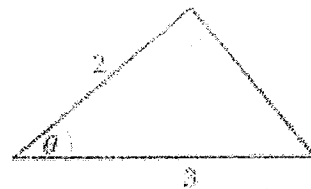


In Exercises 25–28, find angle θ .

25.



27.



In Exercises 29–36, use the figure for Exercises 13–24 to find angles A and C under the given conditions.

Hint..... Draw a triangle!

29. $a = 4$ and $c = 6$

31. $a = 7$ and $b = 10$

33. $b = 18$ and $c = 12$

35. $a = 2.5$ and $c = 1.4$

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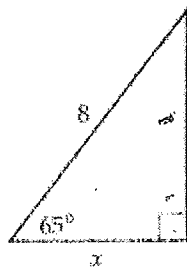


Figure 6.2-1

Example 2 Finding an Angle of a Triangle

Find the measure of angle θ in Figure 6.2-2.

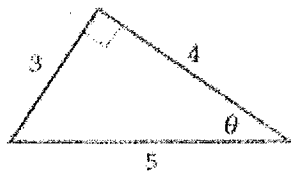


Figure 6.2-2

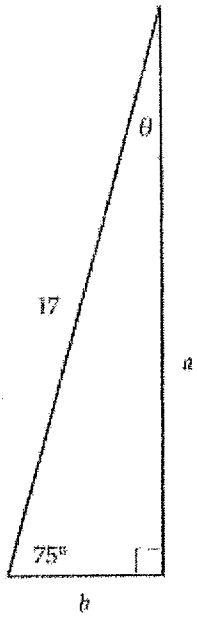


Figure 6.2-4

Example 3 Solving a Right Triangle

Solve the right triangle in Figure 6.2-4.

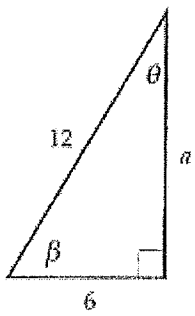


Figure 6.2-5

Example 4 Solving a Right Triangle

Solve the right triangle in Figure 6.2-5.

Solving Right Triangles

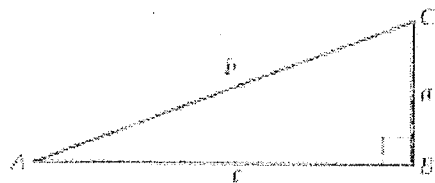
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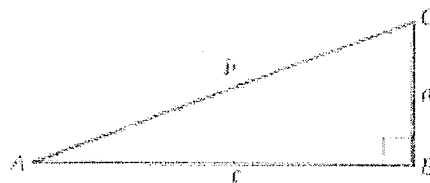
Example 5: Solve for c .

$$\sin C = \frac{3}{4}$$

$$b = 12$$

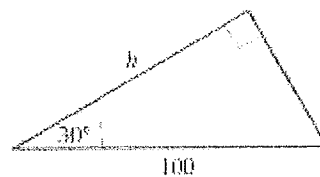
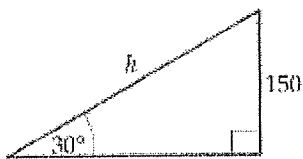
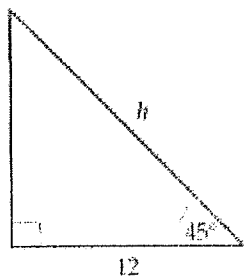


These are like 1-5 in Homework!



Example 6: Solve for h .

These are like 7-12.



Example 7: Find the indicated value without using a calculator.

$$c = 5$$

$$m\angle A = 60^\circ$$

Find a .

These are like 13-16.
Hint... draw a triangle.

In Exercises 17-24, solve the triangle with the given conditions.

18. $c = 12$

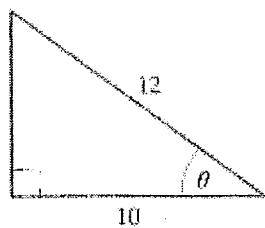
$$m\angle C = 37^\circ$$

20. $a = 8$

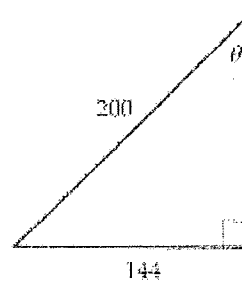
$$m\angle A = 40^\circ$$

In Exercises 25-28, find angle θ .

26.

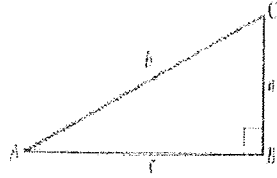
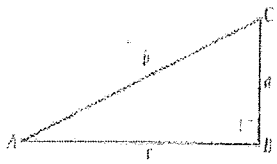
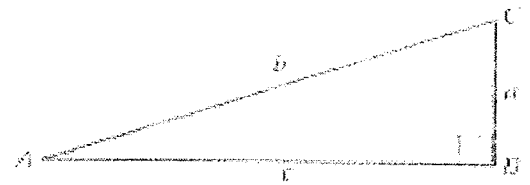


28.



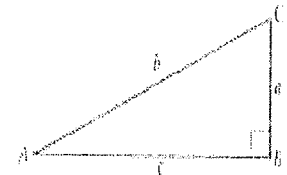
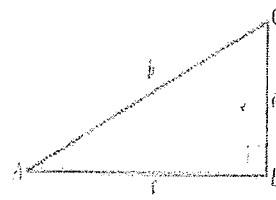
Exercises 6.2

In Exercises 1–6, find side c in the figure below by using the given conditions.



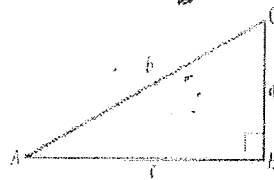
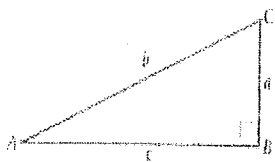
1. $\cos A = \frac{12}{13}$

$b = 39$



5. $\cot A = 6$

$a = 14$

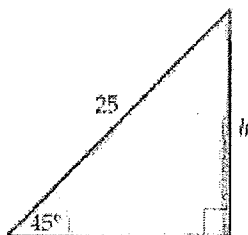


3. $\tan A = \frac{5}{12}$

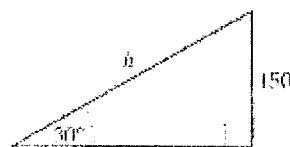
$a = 15$

In Exercises 7–12, find the exact value of h without using a calculator.

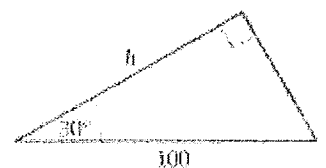
7.



9.

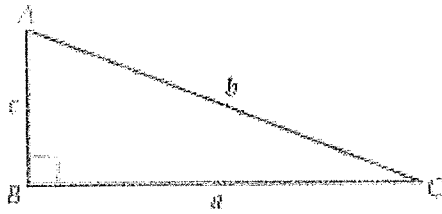


11.



Use the figure below for Exercises 13–24.

In Exercises 13–16, find the indicated value without using a calculator.



15. $c = 10$

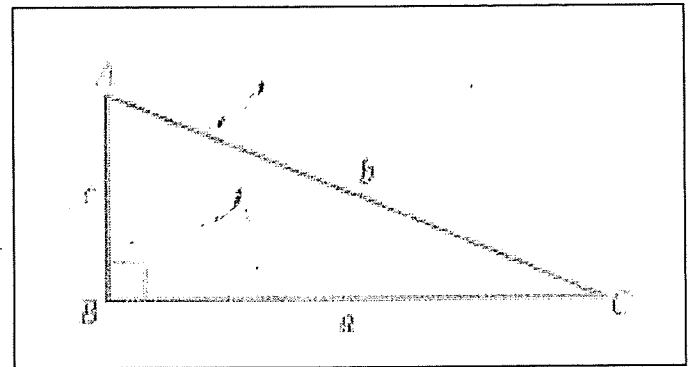
$m\angle A = 30^\circ$

Find a .

13. $a = 4$

$m\angle A = 60^\circ$

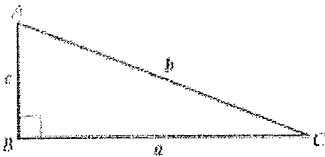
Find c .



In Exercises 17–24, solve the triangle with the given conditions.

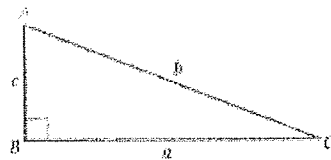
17. $b = 10$

$m\angle C = 50^\circ$



19. $a = 6$

$m\angle A = 14^\circ$

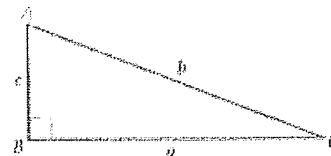
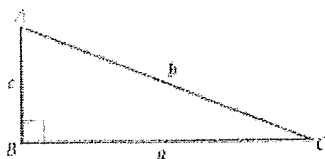


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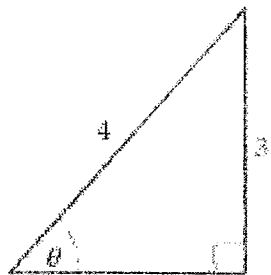
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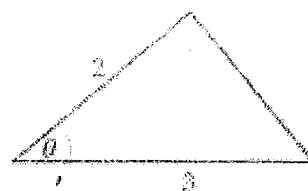


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35. $a = 2.5$ and $c = 1.4$

