PreCalculus

Name:_____

6.3 Arc Length

Date:_____ Hour:_____

Arc Length

Recall from geometry that an **arc** is a part of a circle and that a **central angle** is an angle whose vertex is the center of the circle. The length of an arc depends on the radius of the circle and the measure of the central angle θ that it intercepts, as shown in Figure 6.3-5.

An arc with central angle measure θ radians has length

$$\ell = r\theta$$

In other words, the arc length is the radius times the radian measure of the central angle of the arc.

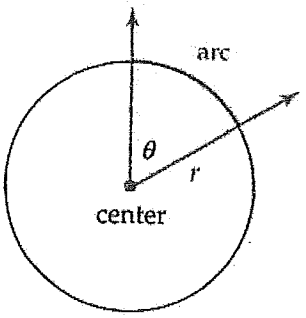

Figure 6.3-5

Example 5 **Arc Length**

The second hand on a clock is 6 inches long. How far does the tip of the second hand move in 15 seconds?

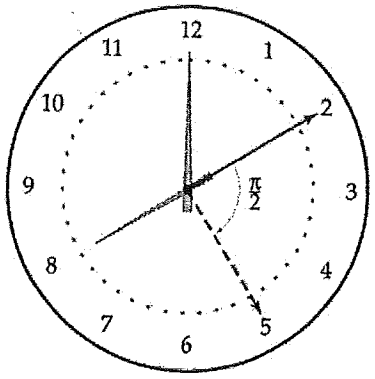

Figure 6.3-12

Example: A car is traveling at the rate of 25 meters per second. If a tire is 40 centimeters in radius, find the radian measure of the angle through which the wheel turns in 2 seconds.

Homework

1. What is the speed of an earth satellite (in kilometers per minute) traveling in a circular orbit 1000 kilometers above the earth's surface, if it completes one orbit every 90 minutes? Use 6400 kilometers for the earth's radius, and give your answer in kilometers per minute.

2. The tip of the minute hand on a watch travels 180 centimeters in 36 hours. What is the length of the minute hand in centimeters?

3. A ferris wheel makes one revolution in 15 seconds. The radius of the wheel is 6 meters. What is the speed in kilometers per hour of a person riding on the ferris wheel?

4. The earth spins on its axis and makes one revolution every 24 hours. What is the speed in kilometers per hour of a point on the equator? Use 6400 kilometers for the earth's radius.

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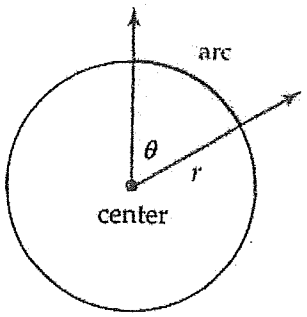


Figure 6.3-5

Example 5 **Arc Length**

The second hand on a clock is 6 inches long. How far does the tip of the second hand move in 15 seconds?

$1 \text{ rev} = 2\pi$

$1 \text{ rev} = 60 \text{ sec}$

$\frac{15}{60} = \frac{1}{4}$

$\frac{1}{4}(2\pi) = \frac{\pi}{2}$

$\ell = r\theta$

$= 6\left(\frac{\pi}{2}\right)$

$= 3\pi$

$\approx 9.4 \text{ in.}$

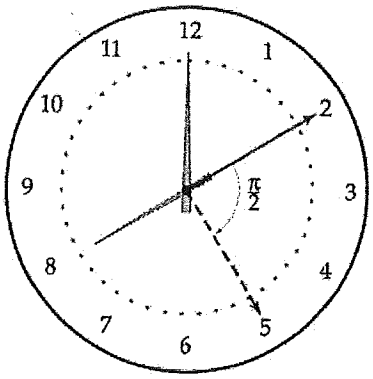


Figure 6.3-12

Example: A car is traveling at the rate of 25 meters per second. If a tire is 40 centimeters in radius, find the radian measure of the angle through which the wheel turns in 2 seconds.

θ?

$d = rt$

$= 25 \frac{\text{m}}{\text{sec}} \cdot 2 \text{ sec}$

$= 50 \text{ m}$

$\frac{50 \text{ m}}{1} \left| \frac{100 \text{ cm}}{1 \text{ m}} \right| = 5000 \text{ cm}$

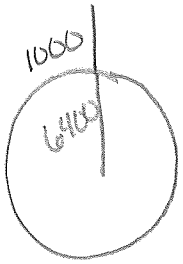
$\ell = r\theta$

$5000 = 40\theta$

$\theta = 125 \text{ radians}$

Homework

1. What is the speed of an earth satellite (in kilometers per minute) traveling in a circular orbit 1000 kilometers above the earth's surface, if it completes one orbit every 90 minutes? Use 6400 kilometers for the earth's radius, and give your answer in kilometers per minute.



$1 \text{ rev} = 2\pi$ $\quad \frac{1 \text{ rev}}{90 \text{ min}}$ $\quad \frac{14800\pi \text{ km}}{90 \text{ min}}$

$r = 6400 + 1000$
$= 7400$

$\ell = r\theta$
$= 7400 \cdot 2\pi$
$= 14800\pi \text{ km}$

$516.6 \frac{\text{km}}{\text{min}}$

2. The tip of the minute hand on a watch travels 180 centimeters in 36 hours. What is the length of the minute hand in centimeters?

ℓ $\quad r$?

$\frac{36 \text{ hr}}{1} \cdot \frac{1 \text{ rev.}}{1 \text{ hr}} = 36 \text{ rev.}$

$\frac{36 \text{ rev}}{1} \cdot \frac{2\pi}{1 \text{ rev}} = 72\pi$

$\ell = r\theta$
$180 \text{ cm} = r(72\pi)$
$r \approx 0.795 \text{ cm}$

3. A ferris wheel makes one revolution in 15 seconds. The radius of the wheel is 6 meters. What is the speed in kilometers per hour of a person riding on the ferris wheel?

r

$1 \text{ rev} = 2\pi$

$\ell = r\theta$
$= 6(2\pi)$
$= 12\pi \text{ m}$

$\frac{12\pi \text{ m}}{15 \text{ sec}} \cdot \frac{1 \text{ km}}{1000 \text{ m}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 9.0477 \frac{\text{km}}{\text{hr}}$

4. The earth spins on its axis and makes one revolution every 24 hours. What is the speed in kilometers per hour of a point on the equator? Use 6400 kilometers for the earth's radius.

r

$1 \text{ rev} \rightarrow 2\pi$

$\ell = r\theta$
$= 6400 \cdot 2\pi$
$= 12800\pi \text{ km}$

$\frac{12800\pi \text{ km}}{24 \text{ hr.}}$
$\approx 1675.5 \frac{\text{km}}{\text{hr.}}$