



Angles and Radian Measure

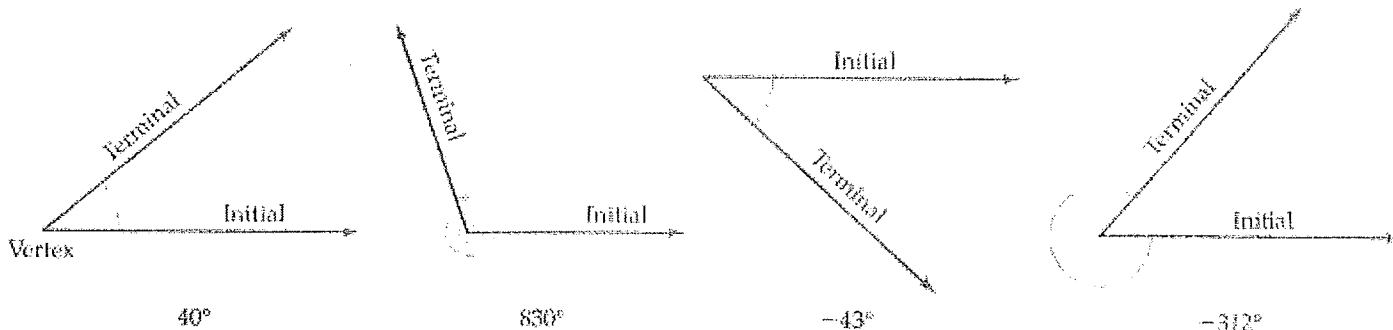
Objectives

- Use a rotating ray to extend the definition of angle measure to negative angles and angles greater than 180°
- Define radian measure and convert angle measures between degrees and radians

Extending Angle Measure

In geometry and triangle trigonometry, an angle is a static figure consisting of two rays that meet at a point. But in modern trigonometry, which will be introduced in the next section, an angle is thought of as being formed dynamically by *rotating* a ray around its endpoint, the **vertex**. The starting position of the ray is called the **initial side** and its final position after the rotation is called the **terminal side**.

The amount the ray is rotated is the measure of the angle. Counterclockwise rotations have positive measure and clockwise rotations have negative measure.



An angle in the coordinate plane is said to be in **standard position** if its vertex is at the origin and its initial side is on the positive x -axis.

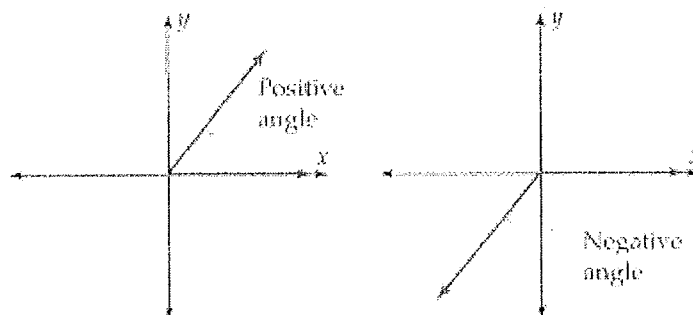


Figure 6.3-2

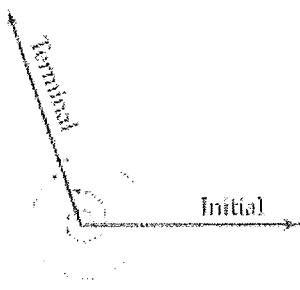


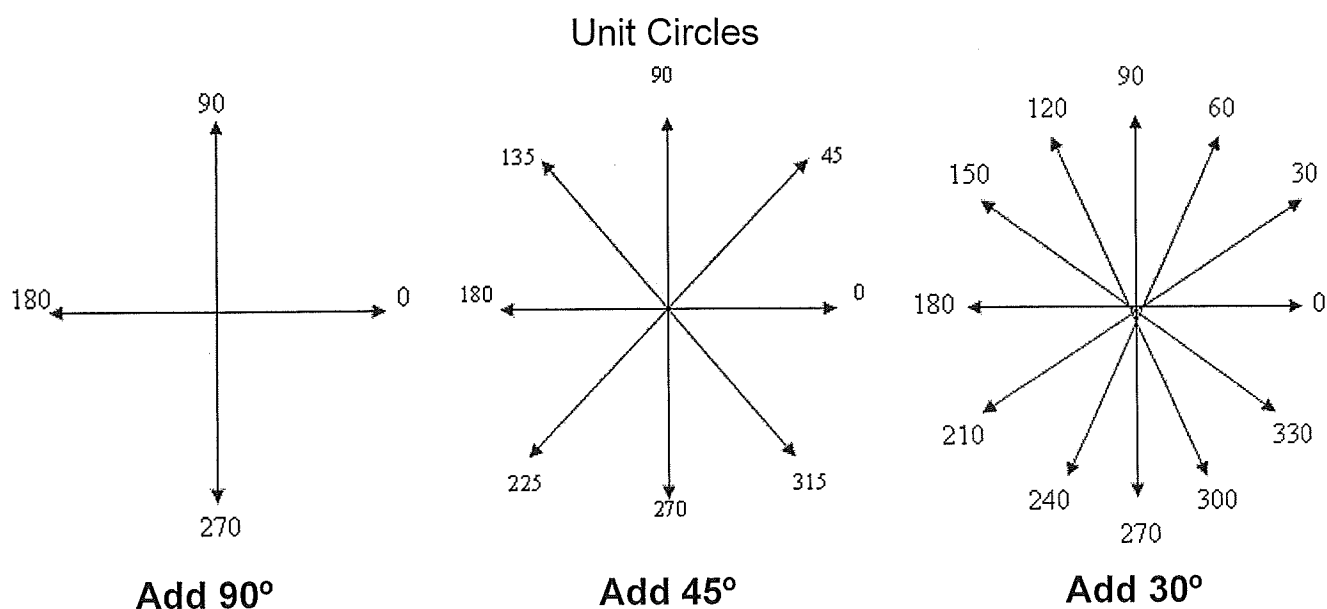
Figure 6.3-3

Angles formed by different rotations that have the same initial and terminal sides are called **coterminal**. (See Figure 6.3-3.) For example, 0° and 360° are coterminal angles.

Increasing or decreasing the angle measure by adding or subtracting any multiple of 360 degrees will result in a coterminal angle

Example 1 Coterminal Angles

Find three angles coterminal with an angle of 60° in standard position.



Radian Measure

The angle found in Example 2 leads to another unit used in finding angle measure called a *radian*. Because it simplifies many formulas in calculus and physics, radians are used as a unit of angle measurement in mathematical and scientific applications.

Definition of Radian Measure

The radian measure of an angle is the distance traveled along the unit circle in a counterclockwise direction by the point P , as it moves from its starting position on the initial side to its final position on the terminal side of the angle.

$$1 \text{ radian} = \left(\frac{180}{\pi}\right)^\circ \approx 57.3^\circ$$

If the vertex of an angle is the center of a circle of radius r , then an angle of 1 radian intercepts an arc of length r .

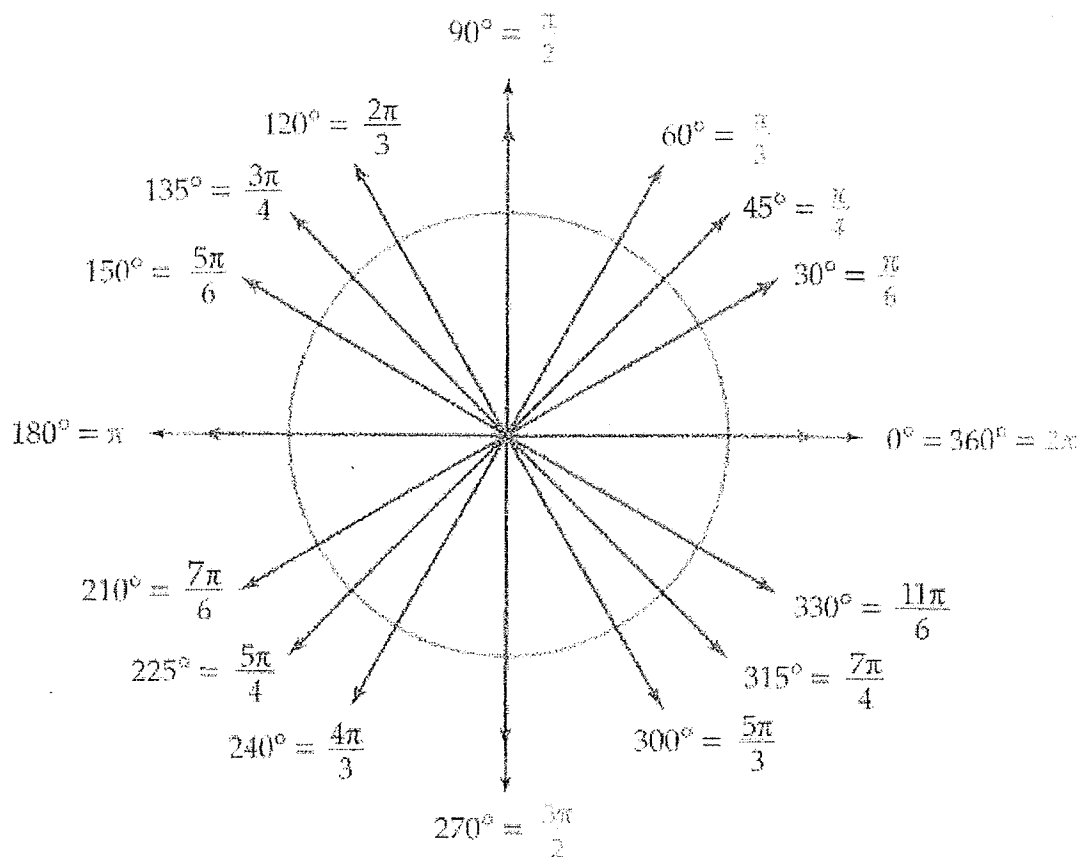
Movement along the unit circle is counterclockwise for positive measure and clockwise for negative measure.

NOTE Radian measurements are usually given in terms of π ; however, it is useful to know the decimal equivalents for common measurements when using a calculator.

$$\pi \approx 3.14 \quad 2\pi \approx 6.28$$

$$\frac{\pi}{2} \approx 1.57 \quad \frac{\pi}{4} \approx 0.79$$

$$\frac{\pi}{6} \approx 0.52 \quad \frac{\pi}{3} \approx 1.05$$



Converting Between Degrees and Radians

$$\pi \text{ radians} = 180^\circ$$

$$1 \text{ radian} = \left(\frac{180}{\pi}\right)^\circ \approx 57.3^\circ$$

$$\frac{\pi}{180} \text{ radians} = 1^\circ$$

Radian/Degree Conversion

To convert radians to degrees, multiply by $\frac{180}{\pi}$.

To convert degrees to radians, multiply by $\frac{\pi}{180}$.

Example 3 Converting From Radians to Degrees

Convert the following radian measurements to degrees.

a. $\frac{\pi}{5}$

b. $\frac{4\pi}{9}$

c. 6π

Example 4 Converting From Degrees to Radians

Convert the following degree measurements to radians.

a. 75°

b. 220°

c. 400°

Increasing or decreasing the radian measure of an angle by an integer multiple of 2π results in a coterminal angle.

Example 5: Find one positive and one negative coterminal angle to $-\frac{\pi}{3}$

Example 6: State the radian measure of an angle between 0 and 2π that is coterminal to $-\frac{3\pi}{4}$

Name _____
Date _____ Hour _____

Exercises 6.3

In Exercises 1–10, find the degree and radian measure of the angle in standard position formed by rotating the terminal side by the given amount.

1. $\frac{1}{9}$ of a circle

3. $\frac{1}{18}$ of a circle

5. $\frac{1}{36}$ of a circle

7. $\frac{2}{3}$ of a circle

9. $\frac{4}{5}$ of a circle

In Exercises 11–22, convert the given radian measure to degrees.

11. $\frac{\pi}{5}$

13. $-\frac{\pi}{10}$

15. $\frac{3\pi}{4}$

17. $\frac{\pi}{45}$

19. $-\frac{5\pi}{12}$

21. $\frac{27\pi}{5}$

In Exercises 23–34, convert the given degree measure to radians. Write your answer in terms of π .

23. 6°

25. -12°

27. 75°

29. 135°

31. -225°

33. 930°

In Exercises 35–42, state the radian measure of an angle in standard position between 0 and 2π that is coterminal with the given angle in standard position.

35. $-\frac{\pi}{3}$

37. $\frac{19\pi}{4}$

39. $-\frac{7\pi}{5}$

41. 7

In Exercises 43–46, find the radian measure of four angles in standard position that are coterminal with the given angle in standard position.

43. $\frac{\pi}{4}$

45. $-\frac{\pi}{6}$

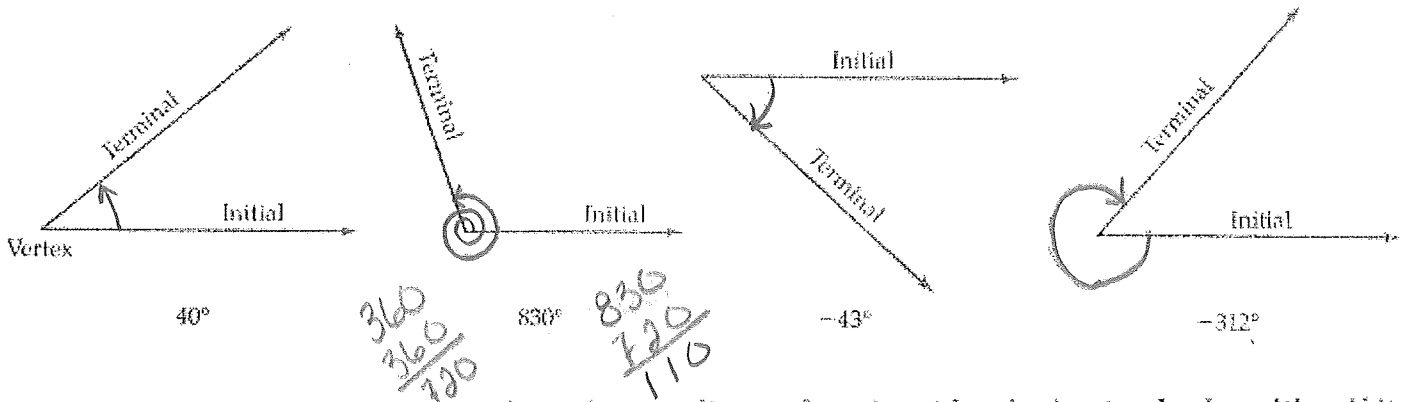
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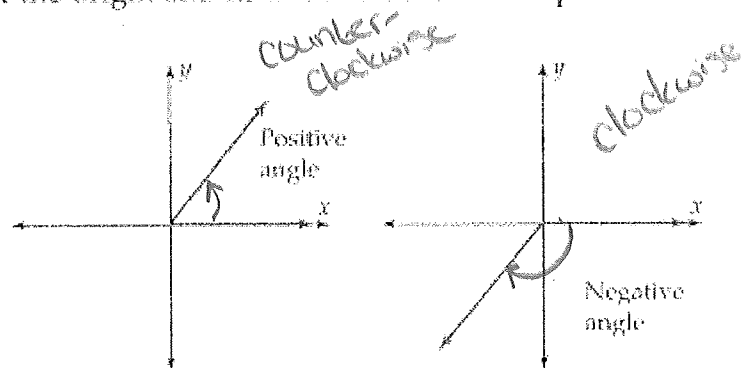


Figure 6.3-2

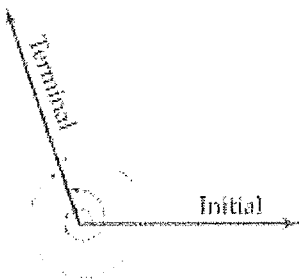


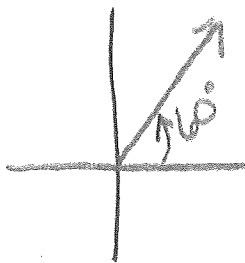
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Increasing or decreasing the angle measure by adding or subtracting any multiple of 360 degrees will result in a coterminal angle

Example 1 Coterminal Angles

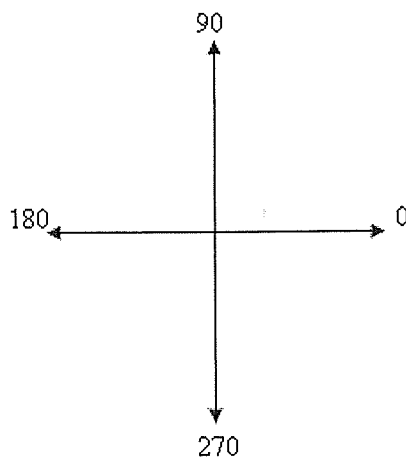
Find three angles coterminal with an angle of 60° in standard position.



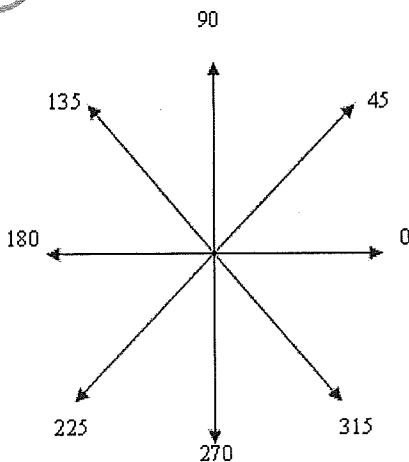
$$\begin{array}{r} 60 \\ 360 \\ \hline 420 \\ 360 \\ \hline 780 \\ 360 \\ \hline 1140 \end{array}$$

$$\begin{array}{r} 60 \\ -360 \\ \hline -300 \\ -360 \\ \hline -660 \end{array}$$

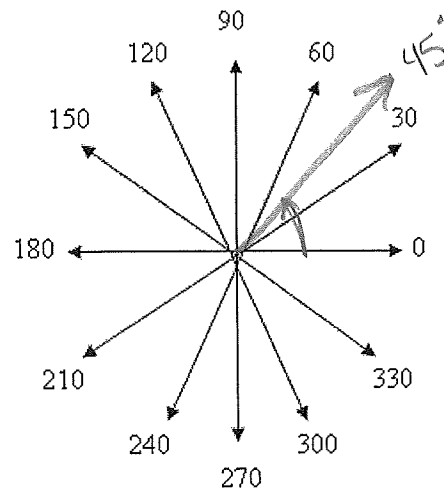
Unit Circles



Add 90°



Add 45°



Add 30°

Radian Measure

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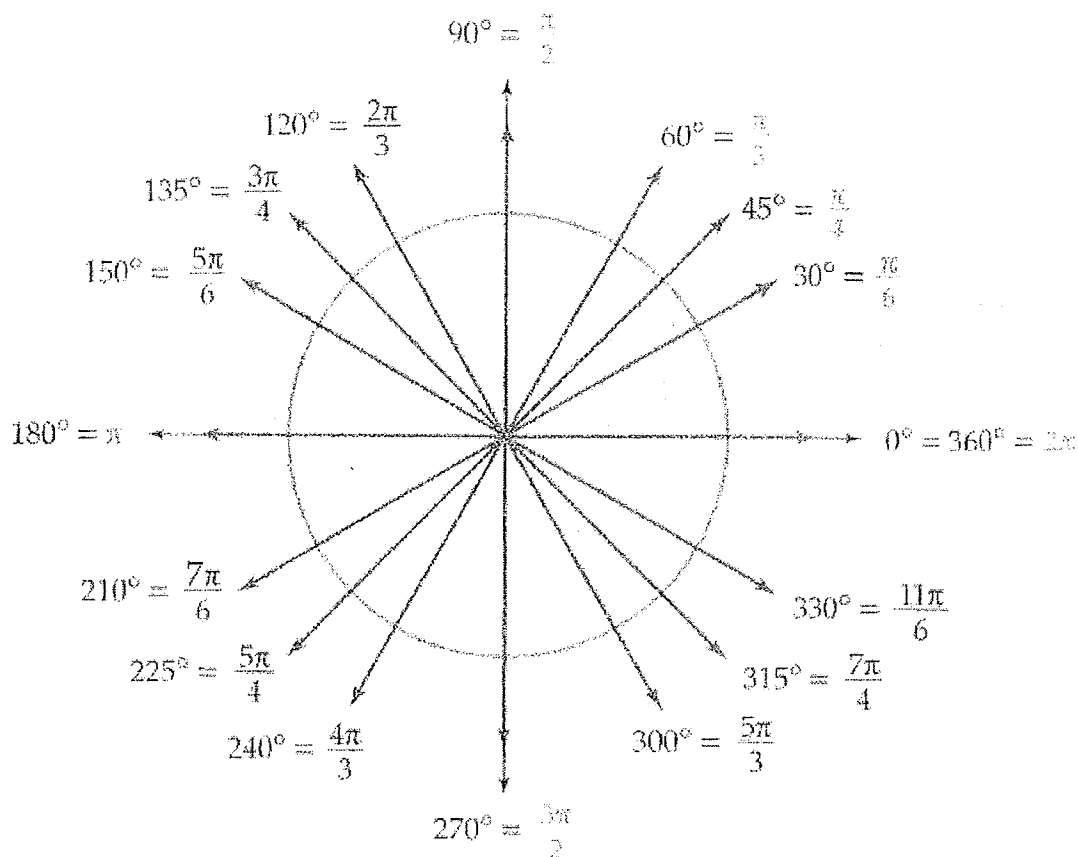
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Example 3 Converting From Radians to Degrees

Convert the following radian measurements to degrees.

a. $\frac{\pi}{5} \cdot \frac{180}{\pi} = 36$

b. $\frac{4\pi}{9} \cdot \frac{180}{\pi} = 80$

c. $6\pi \cdot \frac{180}{\pi} = 1080$

$$36^\circ$$

$$80^\circ$$

$$1080^\circ$$

Example 4 Converting From Degrees to Radians

Convert the following degree measurements to radians.

a. $75^\circ = \frac{\pi}{180}$

b. $220^\circ = \frac{\pi}{180}$

c. $400^\circ = \frac{\pi}{180}$

$$\frac{5\pi}{12}$$

$$\frac{11\pi}{9}$$

$$\frac{20\pi}{9}$$

Increasing or decreasing the radian measure of an angle by an integer multiple of 2π results in a coterminal angle.**Example 5:** Find one positive and one negative coterminal angle to $-\frac{\pi}{3}$

$$-\frac{\pi}{3} + \frac{2\pi}{1} = \frac{5\pi}{3}$$

$$-\frac{\pi}{3} - \frac{2\pi}{1} = -\frac{7\pi}{3}$$

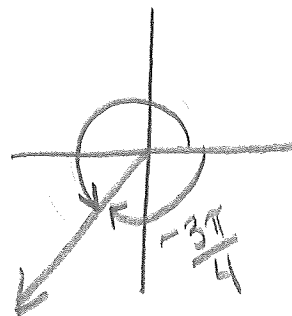
$$-\frac{\pi}{3} + \frac{6\pi}{3} = \frac{5\pi}{3} \text{ positive}$$

$$-\frac{\pi}{3} - \frac{6\pi}{3} = -\frac{7\pi}{3} \text{ negative}$$

Example 6: State the radian measure of an angle between 0 and 2π that is coterminal to $-\frac{3\pi}{4}$

$$-\frac{3\pi}{4} + \frac{2\pi}{1} = \frac{5\pi}{4}$$

$$-\frac{3\pi}{4} + \frac{8\pi}{4} = \frac{5\pi}{4}$$



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