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6-4 Reteaching

Rational Exponents

You can simplify a number with a rational exponent by converting the expression to a radical expression:

$x^{\frac{1}{n}} = \sqrt[n]{x}$, for $n > 0$ $\qquad 9^{\frac{1}{2}} = \sqrt[2]{9} = 3 \qquad 8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

You can simplify the product of numbers with rational exponents m and n by raising the number to the sum of the exponents using the rule

$$a^m \cdot a^n = a^{m+n}$$

Problem

What is the simplified form of each expression?

a. $36^{\frac{1}{4}} \cdot 36^{\frac{1}{4}}$

$36^{\frac{1}{4}} \cdot 36^{\frac{1}{4}} = 36^{\frac{1}{4}+\frac{1}{4}}$ Use $a^m \cdot a^n = a^{m+n}$.

$= 36^{\frac{1}{2}}$ Add.

$= \sqrt[2]{36}$ Use $x^{\frac{1}{n}} = \sqrt[n]{x}$.

$= 6$ Simplify.

b. Write $\left(6x^{\frac{2}{3}}\right)\left(2x^{\frac{3}{4}}\right)$ in simplified form.

$\left(6x^{\frac{2}{3}}\right)\left(2x^{\frac{3}{4}}\right) = 6 \cdot 2 \cdot x^{\frac{2}{3}} \cdot x^{\frac{3}{4}}$ Commutative and Associative Properties of Multiplication

$= 6 \cdot 2 \cdot x^{\frac{2}{3}+\frac{3}{4}}$ Use $x^m \cdot x^n = x^{m+n}$.

$= 12x^{\frac{17}{12}}$ Simplify.

Exercises

Simplify each expression. Assume that all variables are positive.

1. $5^{\frac{1}{3}} \cdot 5^{\frac{2}{3}}$

2. $\left(2y^{\frac{1}{4}}\right)\left(3y^{\frac{1}{3}}\right)$

3. $(-11)^{\frac{1}{3}} \cdot (-11)^{\frac{1}{3}} \cdot (-11)^{\frac{1}{3}}$

4. $-y^{\frac{2}{3}}y^{\frac{1}{5}}$

5. $5^{\frac{1}{4}} \cdot 5^{\frac{1}{4}}$

6. $\left(-3x^{\frac{1}{6}}\right)\left(7x^{\frac{2}{6}}\right)$

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6-4 Reteaching (continued)

Rational Exponents

To write an expression with rational exponents in simplest form, simplify all exponents and write every exponent as a positive number using the following rules for $a \neq 0$ and rational numbers m and n:

$$a^{-n} = \frac{1}{a^n} \qquad \frac{1}{a^{-m}} = a^m \qquad (a^m)^n = a^{mn} \qquad (ab)^m = a^m b^m$$

Problem

What is $(8x^9y^{-3})^{-\frac{2}{3}}$ in simplest form?

$(8x^9y^{-3})^{-\frac{2}{3}} = (2^3x^9y^{-3})^{-\frac{2}{3}}$	Factor any numerical coefficients.
$= (2^3)^{-\frac{2}{3}}(x^9)^{-\frac{2}{3}}(y^{-3})^{-\frac{2}{3}}$	Use the property $(ab)^m = a^mb^m$.
$= 2^{-2}x^{-6}y^2$	Multiply exponents, using the property $(a^m)^n = a^{mn}$.
$= \frac{y^2}{2^2x^6}$	Write every exponent as a positive number.
$= \frac{y^2}{4x^6}$	Simplify.

Exercises

Write each expression in simplest form. Assume that all variables are positive.

7. $(16x^2y^8)^{-\frac{1}{2}}$

8. $(z^{-3})^{\frac{1}{9}}$

9. $(2x^{\frac{1}{4}})^4$

10. $(25x^{-6}y^2)^{\frac{1}{2}}$

11. $(8a^{-3}b^9)^{\frac{2}{3}}$

12. $\left(\frac{16z^4}{25x^8}\right)^{-\frac{1}{2}}$

13. $\left(\frac{x^2}{y^{-1}}\right)^{\frac{1}{5}}$

14. $(27m^9n^{-3})^{-\frac{2}{3}}$

15. $\left(\frac{32r^2}{2s^4}\right)^{\frac{1}{4}}$

16. $(9z^{10})^{\frac{3}{2}}$

17. $(-243)^{-\frac{1}{5}}$

18. $\left(\frac{x^{\frac{2}{5}}}{y^{\frac{1}{2}}}\right)^{10}$

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6-5 Reteaching

Solving Square Root and Other Radical Equations

Equations containing radicals can be solved by isolating the radical on one side of the equation, and then raising both sides to the same power that would undo the radical.

Problem

What is the solution of the radical equation? $2\sqrt{2x+2} - 2 = 10$

$2\sqrt{2x+2} - 2 = 10$	
$2\sqrt{2x+2} = 12$	Add 2 to each side.
$\sqrt{2x+2} = 6$	Divide each side by 2.
$(\sqrt{2x+2})^2 = 6^2$	Square each side to undo the radical.
$2x + 2 = 36$	Simplify.
$2x = 34$	Subtract 2 from each side.
$x = 17$	Divide each side by 2.

Check the solution in the original equation.

Check

$2\sqrt{2x+2} - 2 = 10$	Write the original equation.
$2\sqrt{2(17)+2} - 2 \stackrel{?}{=} 10$	Replace *x* by 17.
$2\sqrt{36} - 2 \stackrel{?}{=} 10$	Simplify.
$12 - 2 \stackrel{?}{=} 10$	
$10 = 10$ ✓	

The solution is 17.

Exercises

Solve. Check your solutions.

1. $x^{\frac{1}{2}} = 13$

2. $3\sqrt{2x} = 12$

3. $\sqrt{3x} + 5 = 11$

4. $(3x + 4)^{\frac{1}{2}} - 1 = 4$

5. $(6 - x)^{\frac{1}{2}} + 2 = 5$

6. $\sqrt{3x + 13} = 4$

7. $(x + 2)^{\frac{1}{2}} - 5 = 0$

8. $\sqrt{3 - 2x} - 2 = 3$

9. $\sqrt[3]{5x + 2} - 3 = 0$

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6-5 Reteaching (continued)

Solving Square Root and Other Radical Equations

An extraneous solution may satisfy equations in your work, but it does not make the original equation true. Always check possible solutions in the original equation.

Problem

What is the solution? Check your results. $\sqrt{17 - x} - 3 = x$

$\sqrt{17 - x} - 3 = x$

$\sqrt{17 - x} = x + 3$ — Add 3 to each side to get the radical alone on one side of the equal sign.

$(\sqrt{17 - x})^2 = (x + 3)^2$ — Square each side.

$17 - x = x^2 + 6x + 9$

$0 = x^2 + 7x - 8$ — Rewrite in standard form.

$0 = (x - 1)(x + 8)$ — Factor.

$x - 1 = 0 \quad \text{or} \quad x + 8 = 0$ — Set each factor equal to 0 using the Zero Product Property.

$x = 1 \text{ or} \qquad x = -8$

Check

$$\sqrt{17 - x} - 3 \stackrel{?}{=} x \qquad\qquad \sqrt{17 - x} - 3 \stackrel{?}{=} x$$
$$\sqrt{17 - 1} - 3 \stackrel{?}{=} 1 \qquad\qquad \sqrt{17 - (-8)} - 3 \stackrel{?}{=} -8$$
$$\sqrt{16} - 3 \stackrel{?}{=} 1 \qquad\qquad \sqrt{25} - 3 \stackrel{?}{=} -8$$
$$1 = 1 \checkmark \qquad\qquad 2 \neq -8$$

The only solution is 1.

Exercises

Solve. Check for extraneous solutions.

10. $\sqrt{5x + 1} = \sqrt{4x + 3}$

11. $\sqrt{x^2 + 3} = x + 1$

12. $\sqrt{3x} = \sqrt{x + 6}$

13. $x = \sqrt{x + 7} + 5$

14. $x - 3\sqrt{x} - 4 = 0$

15. $\sqrt{x + 2} = x - 4$

16. $\sqrt{2x - 10} = x - 5$

17. $\sqrt{3x - 6} = 2 - x$

18. $\sqrt{x - 1} + 7 = x$

19. $\sqrt{5x + 1} = \sqrt{3x + 15}$

20. $\sqrt{x + 9} = x + 7$

21. $x - \sqrt{x + 2} = 40$

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6-7 Reteaching

Inverse Relations and Functions

- Inverse operations "undo" each other. Addition and subtraction are inverse operations. So are multiplication and division. The inverse of cubing a number is taking its cube root.
- If two functions are inverses, they consist of inverse operations performed in the opposite order.

Problem

What is the inverse of the relation described by $f(x) = x + 1$?

$f(x) = x + 1$	
$y = x + 1$	Rewrite the equation using y, if necessary.
$x = y + 1$	Interchange x and y.
$x - 1 = y$	Solve for y.
$y = x - 1$	The resulting function is the inverse of the original function.

So, $f^{-1}(x) = x - 1$.

Exercises

Find the inverse of each function.

1. $y = 4x - 5$

2. $y = 3x^3 + 2$

3. $y = (x + 1)^3$

4. $y = 0.5x + 2$

5. $f(x) = x + 3$

6. $f(x) = 2(x - 2)$

7. $f(x) = \frac{x}{5}$

8. $f(x) = 4x + 2$

9. $y = x$

10. $y = x - 3$

11. $y = \frac{x - 1}{2}$

12. $y = x^3 - 8$

13. $f(x) = \sqrt{x + 2}$

14. $f(x) = \frac{2}{3}x - 1$

15. $f(x) = \frac{x + 3}{5}$

16. $f(x) = 2(x - 5)^2$

17. $y = \sqrt{x} + 4$

18. $y = 8x + 1$

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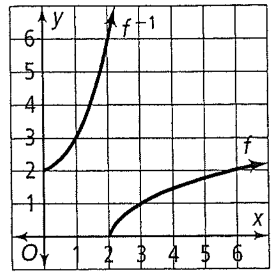
6-7 Reteaching (continued)

Inverse Relations and Functions



Examine the graphs of $f(x) = \sqrt{x - 2}$ and its inverse, $f^{-1}(x) = x^2 + 2$, at the right.

Notice that the range of f and the domain of f^{-1} are the same: the set of all real numbers $x \geq 0$.

Similarly, the domain of f and the range of f^{-1} are the same: the set of all real numbers $x \geq 2$.

This inverse relationship is true for all relations whenever both f and f^{-1} are defined.

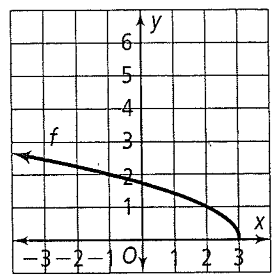
Problem

What are the domain and range of the inverse of the function $f(x) = \sqrt{3 - x}$?

f is defined for $3 - x \geq 0$ or $x \leq 3$.

Therefore, the domain of f and the range of f^{-1} is the set of all $x \leq 3$.

The range of f is the set of all $x \geq 0$. So, the domain of f^{-1} is the set of all $x \geq 0$.



Exercises

Name the domain and range of the inverse of the function.

19. $y = 2x - 1$

20. $y = 2 - \frac{1}{x}$

21. $y = \sqrt{x + 5}$

22. $y = \sqrt{-x} + 8$

23. $y = 3\sqrt{x} + 2$

24. $y = (x - 6)^2$

25. $y = x^2 - 6$

26. $y = \frac{1}{x + 4}$

27. $y = \frac{1}{(x + 4)^2}$

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6-8 Reteaching

Graphing Radical Functions

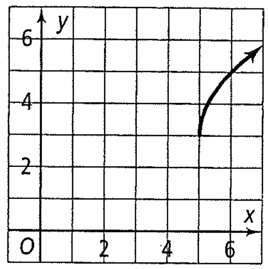
The graph of $y = a\sqrt{x - h} + k$ is a translation h units horizontally and k units vertically of $y = a\sqrt{x}$. The value of a determines a vertical stretch or compression of $y = \sqrt{x}$.

Problem

What is the graph of $y = 2\sqrt{x - 5} + 3$?

$$y = 2\sqrt{x - 5} + 3$$

$a = 2$ $h = 5$ $k = 3$



Translate the graph of $y = 2\sqrt{x}$ right five units and up three units. The graph of $y = 2\sqrt{x}$ looks like the graph of $y = \sqrt{x}$ with a vertical stretch by a factor of 2.

Exercises

Graph each function.

1. $y = \sqrt{x - 4} + 1$

2. $y = \sqrt{x} - 4$

3. $y = \sqrt{x + 1}$

4. $y = -\sqrt{x + 2} - 3$

5. $y = 2\sqrt{x - 1}$

6. $y = -2\sqrt{x + 3} + 4$

7. $y = -\sqrt{x} + 1$

8. $y = \sqrt{x + 3} - 4$

9. $y = 3\sqrt{x} + 2$

10. $y = -\sqrt{x - 2}$

6-8 Reteaching (continued)

Graphing Radical Functions

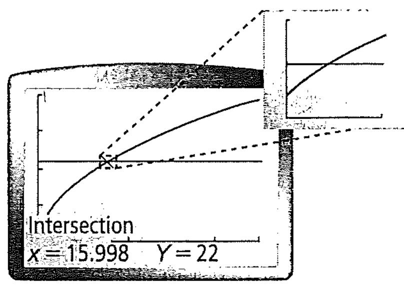
Graphs can be used to find solutions of equations containing radical expressions.

Problem

What is the minimum braking distance of a bicycle with a speed of 22 mph?

You can find the minimum braking distance d, in feet, of a bicycle travelling s miles per hour using the equation $s = 5.5\sqrt{d + 0.002}$.

We want to find the value of d when $s = 22$. In other words, solve the equation $5.5\sqrt{d + 0.002} = 22$. Graph `Y1=5.5√(X+0.002)` and `Y2=22`. Try different values until you find an appropriate window. Then use the `intersect` feature to find the coordinates of the point of intersection.



The minimum braking distance will be about 16 ft.

Exercises

Solve the equation by graphing. Round the answer to the nearest hundredth, if necessary. If there is no solution, explain why.

11. $\sqrt{3x + 1} = 5$

12. $\sqrt{4x + 1} = 9$

13. $\sqrt{2 - 5x} = 4$

14. $\sqrt{3x + 5} = 7$

15. $\sqrt{7x + 2} = 11$

16. $\sqrt{2x - 1} = \sqrt{1 - 2x}$

17. $\sqrt{x - 2} = \sqrt{2 - 3x}$

18. $7\sqrt{x - 3} = 2\sqrt{2x + 1}$

19. $\sqrt{2x - 5} = \sqrt{4 - x}$

20. $\sqrt{2x + 7} = 3\sqrt{5x + 2}$