

Prove the following identities. Show all solution steps for full credit!

1. $\cos x(\csc x - \sec x) = \cot x - 1$

$$\cos x \left(\frac{1}{\sin x} - \frac{1}{\cos x} \right)$$

$$\frac{\cos x}{\sin x} - \frac{\cos x}{\cos x}$$

$$\cot x - 1$$

2. $\frac{\tan x}{\sec x - \cos x} = \csc x$

$$\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\cos x}}$$

$$\frac{\frac{\sin x}{\cos x}}{\frac{1 - \cos^2 x}{\cos x}} \rightarrow \frac{\frac{\sin x}{\cancel{\cos x}}}{\frac{1}{\sin x}} = \frac{\cancel{\cos x}}{\sin x} = \csc x$$

3. $\frac{\sec x}{\csc x} + \frac{\sin x}{\cos x} = 2 \tan x$

$$\frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} + \frac{\sin x}{\cos x}$$

$$\frac{1}{\cos x} \cdot \frac{\sin x}{1} + \frac{\sin x}{\cos x}$$

$$\frac{\sin x}{\cos x} + \frac{\sin x}{\cos x}$$

$$\tan x + \tan x = 2 \tan x$$

4. $[\cos(x+y) + \cos(x-y)] = 2 \cos x \cos y$

$$(\cos x \cos y - \sin x \sin y) + (\cos x \cos y + \sin x \sin y)$$

$$\cos x \cos y + \cos x \cos y$$

$$2 \cos x \cos y$$

5. Simplify: $\cos 10 \cos 4 - \sin 10 \sin 4$

$$\cos(10+4)$$

$$\cos(14)$$

6. Simplify: $\sin(3x-y) \cos y + \sin y \cos(3x-y)$

$$\sin(3x-y+y)$$

$$\sin(3x)$$

5. _____

6. _____

7. Simplify: $\tan(x-\pi)$

$$\frac{\tan x - \tan \pi}{1 + \tan x \tan \pi}$$

$(-1,0)$ $\tan \frac{\pi}{2}$
 $\tan \pi = 0$

$$\frac{\tan x - 0}{1 + \tan x \cdot 0} = \frac{\tan x}{1}$$

7. tan x

8. Simplify: $\sin(\frac{\pi}{2} + x)$

$$\sin \frac{\pi}{2} \cos x + \sin x \cos \frac{\pi}{2}$$

$$1 \cdot \cos x + \sin x \cdot 0$$

$$\cos x$$

8. cos x

For #9-10, use Addition and Subtraction Identities to find the exact values. **RATIONALIZE!!!**

9. $\sin \frac{\pi}{12}$

$$\frac{\pi}{4} = \frac{3\pi}{12} - \frac{2\pi}{12}$$

$$\sin(\frac{3\pi}{12} - \frac{2\pi}{12})$$

$$\sin(\frac{\pi}{4} - \frac{\pi}{6})$$

$$\sin \frac{\pi}{4} \cos \frac{\pi}{6} - \sin \frac{\pi}{6} \cos \frac{\pi}{4}$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

9. $\frac{\sqrt{6} - \sqrt{2}}{4}$

10. $\tan 15^\circ$

$$\tan(45^\circ - 30^\circ)$$

$$\frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$\frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \cdot \frac{\sqrt{3}}{3}}$$

$$\frac{3 - \sqrt{3}}{3 + \sqrt{3}}$$

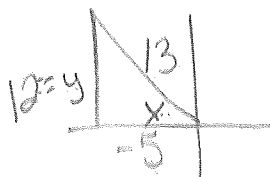
$$\frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$$

$$\frac{9 - 6\sqrt{3} + 3}{12 - 6\sqrt{3}} = \frac{12 - 6\sqrt{3}}{12 - 6\sqrt{3}} = 2 - \sqrt{3}$$

10. $2 - \sqrt{3}$

11. Given: $\cos x = -\frac{5}{13}$, where $\frac{\pi}{2} < x < \pi$, evaluate and simplify $\sin(\frac{\pi}{6} + x)$. Find the exact value.

2nd quad.



$$y^2 + (-5)^2 = 13^2$$

$$y^2 = 144$$

$$y = 12$$

$$\sin(\frac{\pi}{6} + x)$$

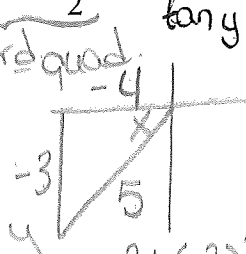
$$\sin \frac{\pi}{6} \cos x + \sin x \cos \frac{\pi}{6}$$

$$\frac{1}{2} \cdot -\frac{5}{13} + \frac{12}{13} \cdot \frac{\sqrt{3}}{2} = \frac{-5 + 12\sqrt{3}}{26}$$

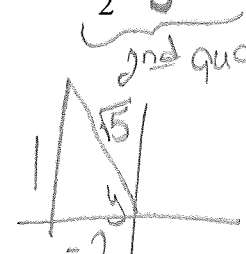
11. $\frac{-5 + 12\sqrt{3}}{26}$

12. Given: $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$ and $\tan y = -\frac{1}{2}$, where $\frac{\pi}{2} < y < \pi$. Find $\cos(x-y)$. Find the exact value.

3rd quad.



2nd quad.



$$\cos(x-y)$$

$$\cos x \cos y + \sin x \sin y$$

$$-\frac{4}{5} \cdot -\frac{2}{5} + -\frac{3}{5} \cdot \frac{1}{5}$$

$$\frac{8}{25} - \frac{3}{25} = \frac{5}{25} = \frac{1}{5}$$

12. $\frac{\sqrt{5}}{5}$

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4. $[\cos(x+y) + \cos(x-y)] = 2 \cos x \cos y$

5. **Simplify:** $\cos 10 \cos 4 - \sin 10 \sin 4$

6. **Simplify:** $\sin(3x-y)\cos y + \sin y \cos(3x-y)$

5. _____

6. _____

7. Simplify: $\tan(x - \pi)$

7. _____

8. Simplify: $\sin\left(\frac{\pi}{2} + x\right)$

8. _____

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10. $\tan 15^\circ$

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12. _____