

Prove the following identities. Show all solution steps for full credit!

1. $\sec^4 x - \tan^4 x = 2 \tan^2 x + 1$

$$\begin{aligned}
 &(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x) \\
 &\quad \downarrow \quad \quad \quad \swarrow \\
 &1(1 + \tan^2 x + \tan^2 x) \\
 &1 + 2\tan^2 x
 \end{aligned}$$

2. $\csc x(\csc x + \cot x) = \frac{1}{1 - \cos x}$

$$\begin{aligned}
 &\frac{1}{\sin x} \left(\frac{1}{\sin x} + \frac{\cos x}{\sin x} \right) \\
 &\frac{1}{\sin^2 x} + \frac{\cos x}{\sin^2 x} \\
 &\frac{1 + \cos x}{\sin^2 x} \\
 &\frac{1 + \cos x}{1 - \cos^2 x} \\
 &\frac{1 + \cos x}{(1 - \cos x)(1 + \cos x)} = \frac{1}{1 - \cos x}
 \end{aligned}$$

3. $\frac{\cot x}{\csc x - \sin x} = \sec x$

$$\begin{aligned}
 &\frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x} - \sin x} \cdot \frac{\sin x}{\sin x} \\
 &\frac{\frac{\cos x}{\sin x}}{\frac{1 - \sin^2 x}{\sin x}} \\
 &\frac{\frac{\cos x}{\sin x}}{\frac{\cos^2 x}{\sin x}} \quad \frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \\
 &= \sec x
 \end{aligned}$$

4. $\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$

$$\begin{aligned}
 &\frac{1}{2} [(\cos x \cos y + \sin x \sin y) - (\cos x \cos y - \sin x \sin y)] \\
 &\frac{1}{2} [2 \sin x \sin y] \\
 &\sin x \sin y
 \end{aligned}$$

5. Simplify: $\sin 7 \cos 3 + \cos 7 \sin 3$

$$\begin{aligned}
 &\sin(7+3) \\
 &\sin(10)
 \end{aligned}$$

5. sin(10)

6. Simplify: $\cos(x - y)\cos y - \sin(x - y)\sin y$

$$\begin{aligned}
 &\cos((x - y) + y) \\
 &\cos x
 \end{aligned}$$

6. cos x

7. Simplify: $\tan(\pi+x)$

$$\frac{\tan \pi + \tan x}{1 - \tan \pi \tan x} = \frac{0 + \tan x}{1 - 0 \cdot \tan x} = \frac{\tan x}{1} = \tan x$$

7. $\tan x$

8. Simplify: $\cos\left(\frac{\pi}{2}-x\right)$

$$\cos \frac{\pi}{2} \cos x + \sin \frac{\pi}{2} \sin x = 0 \cdot \cos x + 1 \cdot \sin x = 0 + \sin x = \sin x$$

8. $\sin x$

For #9-10, use Addition and Subtraction Identities to find the **exact** values. **RATIONALIZE!!!**

9. $\cos \frac{5\pi}{12}$

$$\cos\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$\cos \frac{\pi}{4} \cos \frac{\pi}{6} - \sin \frac{\pi}{4} \sin \frac{\pi}{6}$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

9. $\frac{\sqrt{6} - \sqrt{2}}{4}$

10. $\tan 75^\circ$

$$\tan(30+45)$$

$$\frac{\tan 30 + \tan 45}{1 - \tan 30 \tan 45} = \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}}$$

$$\frac{(3+\sqrt{3}) \cdot \frac{3+\sqrt{3}}{(3+\sqrt{3}) \cdot 3 - \sqrt{3}}}{9-3} = \frac{9+6\sqrt{3}+3}{6} = \frac{12+6\sqrt{3}}{6} = 2 + \sqrt{3}$$

10. $2 + \sqrt{3}$

11. Given: $\cos x = -\frac{4}{5}$, where $\frac{\pi}{2} < x < \pi$, evaluate and simplify $\cos\left(\frac{\pi}{4} + x\right)$. Find the **exact** value.



$$\begin{aligned} (-4)^2 + y^2 &= 5^2 && \text{2nd quad.} \\ 16 + y^2 &= 25 \\ y^2 &= 9 \\ y &= 3 \end{aligned}$$

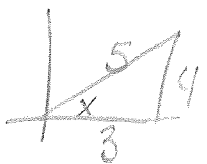
$$\cos \frac{\pi}{4} \cos x - \sin \frac{\pi}{4} \sin x$$

$$\frac{\sqrt{2}}{2} \cdot \left(-\frac{4}{5}\right) - \frac{\sqrt{2}}{2} \cdot \frac{3}{5}$$

$$-\frac{4\sqrt{2}}{10} - \frac{3\sqrt{2}}{10} = -\frac{7\sqrt{2}}{10}$$

11. $-\frac{7\sqrt{2}}{10}$

12. Given: $\sin x = \frac{4}{5}$, where $0 < x < \frac{\pi}{2}$ and $\csc y = \frac{13}{5}$, where $\frac{\pi}{2} < y < \pi$. Find $\sin(x+y)$. Find the exact value.



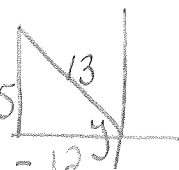
$$\begin{aligned} 4^2 + x^2 &= 5^2 && \text{1st} \\ 16 + x^2 &= 25 \\ x^2 &= 9 \\ x &= 3 \end{aligned}$$

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$

$$= \frac{4}{5} \cdot \frac{-12}{13} + \frac{5}{13} \cdot \frac{3}{5}$$

$$-\frac{48}{65} + \frac{15}{65} = -\frac{33}{65}$$

12. $-\frac{33}{65}$



$$\begin{aligned} 5^2 + x^2 &= 13^2 \\ 25 + x^2 &= 169 \\ x^2 &= 144 \\ x &= 12 \end{aligned}$$

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3. $\frac{\cot x}{\csc x - \sin x} = \sec x$

4. $\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$

5. **Simplify:** $\sin 7 \cos 3 + \cos 7 \sin 3$

6. **Simplify:** $\cos(x - y) \cos y - \sin(x - y) \sin y$

5. _____

6. _____

7. **Simplify:** $\tan(\pi + x)$

7. _____

8. **Simplify:** $\cos\left(\frac{\pi}{2} - x\right)$

8. _____

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10. $\tan 75^\circ$

9. _____

10. _____

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12. _____