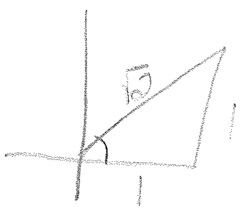
9.4 PreCalculus Notes
Using Trigonometric Identities to Solve Equations

Warm-up:

Goes back to 8.3!

Solve $\sin 2x = \frac{\sqrt{2}}{2}$ exactly, without using a calculator. Find all solutions!



$2x = \frac{\pi}{4}$ $\quad$ $2x = \frac{\pi}{4} \pm 2k\pi$

$x = \frac{\pi}{8}$ $\quad$ $x = \frac{\pi}{8} \pm k\pi$

$2x = \frac{3\pi}{4}$ $\quad$ $2x = \frac{3\pi}{4} \pm 2k\pi$

$x = \frac{3\pi}{8}$ $\quad$ $x = \frac{3\pi}{8} \pm k\pi$

In this section we will be utilizing the double angle identities covered in 9.3 to solve trigonometric equations.

Examples:

Find <u>all solutions</u> in the interval [0, 2π).

1. $2\cos x + \sin 2x = 0$

$2\cos x + 2\sin x\cos x = 0$

$2\cos x(1 + \sin x) = 0$

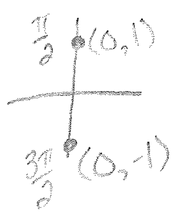
$2\cos x = 0$ $\quad$ $1 + \sin x = 0$

$\cos x = 0$ $\quad$ $\sin x = -1$

where is $\cos = 0$?

$\cos = \frac{x}{r}$

$\sin = \frac{y}{r}$



$\frac{\pi}{2} + 2k\pi \rightarrow \frac{\pi}{2}$ $\quad$ $\frac{3\pi}{2} + 2k\pi \rightarrow \frac{3\pi}{2}$

$\frac{3\pi}{2} + 2k\pi \rightarrow \frac{3\pi}{2}$

$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

2. $\sin 4x = -2\sin 2x$

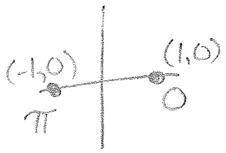
$2\sin 2x\cos 2x + 2\sin 2x = 0$

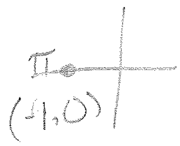
$2\sin 2x(\cos 2x + 1) = 0$

$2\sin 2x = 0$ $\qquad$ $\cos 2x + 1 = 0$

$\sin 2x = 0$ $\qquad$ $\cos 2x = -1$ $\qquad$ where is $x = -1$?

$\sin = \frac{y}{r}$ where is $y = 0$?





$0 + 2k\pi = 2x$ $\qquad$ $\pi + 2k\pi = 2x$

$\pi + 2k\pi = 2x$ $\qquad$ $\frac{\pi}{2} + k\pi \rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$

$0 + k\pi = x \rightarrow 0, \pi$

$\frac{\pi}{2} + k\pi = x \rightarrow \frac{\pi}{2}, \frac{3\pi}{2}$

$\boxed{0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}}$

3. $\cos 2x - \cos x = 0$

$\cos 2x$: $\cos^2 x - \sin^2 x$, $\quad 2\cos^2 x - 1$, $\quad 1 - 2\sin^2 x$

We ultimately want to factor or pull something out. Look for the identity that gives you the least # of trig. functions.

$2\cos^2 x - 1 - \cos x = 0$

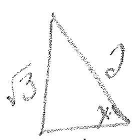
$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

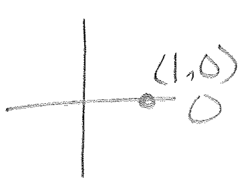
$2\cos x + 1 = 0$ $\qquad$ $\cos x - 1 = 0$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = 1$

where is $\cos x$ negative?



$x = \frac{\pi}{3}$



$Q2: \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$Q3: \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ $\qquad$ $0 + 2k\pi \rightarrow 0$

$\frac{2\pi}{3} + 2k\pi \rightarrow \frac{2\pi}{3}$ $\qquad$ $\boxed{0, \frac{2\pi}{3}, \frac{4\pi}{3}}$

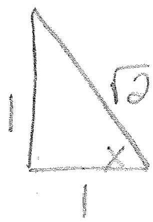
$\frac{4\pi}{3} + 2k\pi \rightarrow \frac{4\pi}{3}$

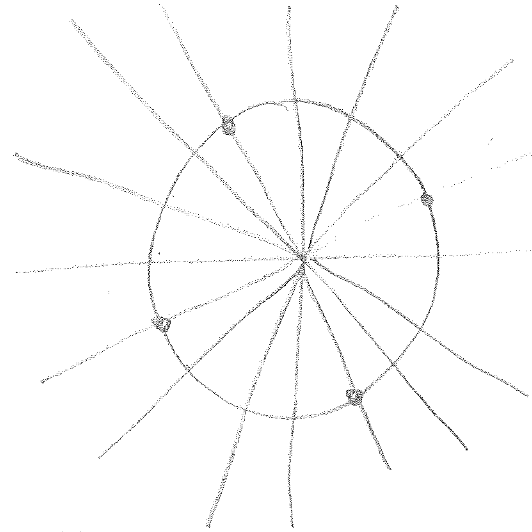
4. $\sin 2x - \cos 2x = 0$

$$\frac{\sin 2x}{\cos 2x} - \frac{\cos 2x}{\cos 2x} = \frac{0}{\cos 2x}$$

$\tan 2x - 1 = 0$

$\tan 2x = 1$





$\frac{\pi}{4} + k\pi = 2x$

$\frac{\pi}{8} + \frac{k\pi}{2} = x$

$$\boxed{\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}}$$

$\frac{\pi}{8} + \frac{\pi}{2} = \frac{5\pi}{8}$

$\frac{5\pi}{8} + \frac{\pi}{2} = \frac{9\pi}{8}$

$\frac{9\pi}{8} + \frac{\pi}{2} = \frac{13\pi}{8}$

$\frac{13\pi}{8} + \frac{\pi}{2} = \frac{17\pi}{8}$ Too big!

$[0, 2\pi)$

This is where we stop!

9.4 Exercises

In Exercises 1-27, find all solutions of the equation in the interval [0, 2π].

1. $\sin^2 x + 3\cos^2 x = 0$

$$\frac{\sin^2 x}{\cos^2 x} = \frac{-3\cos^2 x}{\cos^2 x}$$

$\tan^2 x = -3$

No solution since anything squared cannot be negative!

2. $\sin 2x + \cos x = 0$

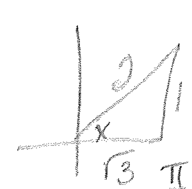
$2\sin x \cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ $\qquad$ $2\sin x + 1 = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $\sin x = -\frac{1}{2}$

$\frac{7\pi}{6}, \frac{11\pi}{6}$



$\frac{\pi}{6}$

3. $\cos 2x - \sin x = 1$

$1 - 2\sin^2 x - \sin x = 1$

$-2\sin^2 x - \sin x = 0$

$\sin x(-2\sin x - 1) = 0$

$\sin x = 0 \qquad -2\sin x - 1 = 0$

$0, \pi \qquad \sin x = -\frac{1}{2}$

$\frac{7\pi}{6}, \frac{11\pi}{6}$

6. $\sin 4x - \sin 2x = 0$

$2\sin 2x \cos 2x - \sin 2x = 0$

$\sin 2x(2\cos 2x - 1) = 0$

$\sin 2x = 0 \qquad 2\cos 2x - 1 = 0$

$0 \pm 2k\pi = 2x \qquad \cos 2x = \frac{1}{2}$

$0 \pm k\pi = x \qquad \frac{\pi}{3} \pm 2k\pi = 2x$

$\pi \pm 2k\pi = 2x \qquad \frac{\pi}{6} \pm k\pi = x$

$\frac{\pi}{2} \pm k\pi = x \qquad \frac{5\pi}{3} \pm 2k\pi = 2x$

$\frac{5\pi}{6} \pm k\pi = x$

$$\boxed{\begin{array}{cc} 0 & \frac{\pi}{6} \\ \pi & \frac{5\pi}{6} \\ 2\pi & \frac{7\pi}{6} \\ \frac{\pi}{2} & \frac{11\pi}{6} \\ \frac{3\pi}{2} & \end{array}}$$

8. $\sin 2t \cos t - \cos 2t \sin t = 0$

$\sin(2t + t) = 0$

$\sin t = 0$

$0, \pi, 2\pi$

9. $\sin 2x \sin x + \cos x = 0$

$2\sin x \cos x(\sin x) + \cos x = 0$

$\cos x(2\sin^2 x + 1) = 0$

$\cos x = 0 \qquad 2\sin^2 x + 1 = 0$

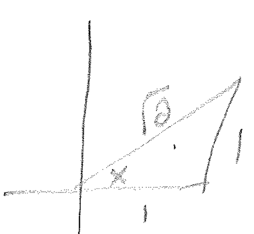
$\frac{\pi}{2}, \frac{3\pi}{2} \qquad \sin^2 x = -\frac{1}{2}$

No sol.

11. $\sin 2x + \cos 2x = 0$

$\frac{\sin 2x}{\cos 2x} = \frac{-\cos 2x}{\cos 2x}$

$\tan 2x = -1$



$\frac{\pi}{4}$

$\frac{3\pi}{4} \pm 2k\pi = 2x$

$\frac{3\pi}{8} \pm k\pi = x$

$\frac{7\pi}{4} \pm 2k\pi = 2x$

$\frac{7\pi}{8} \pm k\pi = x$

$$\boxed{\begin{array}{cc} \frac{3\pi}{8}, & \frac{11\pi}{8} \\ \frac{7\pi}{8}, & \frac{15\pi}{8} \end{array}}$$

13. $\sin 4x = \cos 2x$

$2\sin 2x \cos 2x = \cos 2x$

$2\sin 2x \cos 2x - \cos 2x = 0$

$\cos 2x(2\sin 2x - 1) = 0$

$\cos 2x = 0 \qquad 2\sin 2x - 1 = 0$

$\frac{\pi}{2} \pm 2k\pi = 2x \qquad \sin 2x = \frac{1}{2}$

$\frac{\pi}{4} \pm k\pi = x \qquad \frac{\pi}{6} \pm 2k\pi = 2x$

$\frac{3\pi}{2} \pm 2k\pi = 2x \qquad \frac{\pi}{12} \pm k\pi = x$

$\frac{3\pi}{4} \pm k\pi = x \qquad \frac{5\pi}{6} \pm 2k\pi = 2x$

$\frac{5\pi}{12} \pm k\pi = x$

$$\boxed{\begin{array}{cc} \frac{\pi}{4}, & \frac{5\pi}{4} \\ \frac{3\pi}{4}, & \frac{7\pi}{4} \\ \frac{\pi}{12}, & \frac{13\pi}{12} \\ \frac{5\pi}{12}, & \frac{17\pi}{12} \end{array}}$$

14. $\cos 2x + sin^2 x = 0$

$\cos^2 x - \sin^2 x + \sin^2 x = 0$

$\cos^2 x = 0$

$\cos x = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}$

15. $2cos^2 x - 2\cos 2x = 1$

$2\cos^2 x - 2(\cos^2 x - \sin^2 x) = 1$

$2\cos^2 x - 2\cos^2 x + 2\sin^2 x = 1$

$2\sin^2 x = 1$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

18. $\sin x - \sqrt{3}\cos x = 0$

$\frac{\sin x}{\cos x} = \frac{\sqrt{3}\cos x}{\cos x}$

$\tan x = \sqrt{3}$

$\frac{\pi}{3}, \frac{4\pi}{3}$

19. $\sin x + \cos x = 0$

$\frac{\sin x}{\cos x} = -\frac{\cos x}{\cos x}$

$\tan x = -1$

$\frac{3\pi}{4}, \frac{7\pi}{4}$

20. $\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $\quad$ $2\sin x - 1 = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\sin x = \frac{1}{2}$

$\frac{\pi}{6}, \frac{5\pi}{6}$

21. $\cos 2x + \cos x = 0$

$2\cos^2 x - 1 + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$ $\quad$ $\cos x + 1 = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -1$

$\frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ π

22. $(\sin x - \cos x)^2 = 1$

$\sin^2 x - 2\sin x\cos x + \cos^2 x = 1$

$\underbrace{\sin^2 x + \cos^2 x}_{1} - 2\sin x\cos x = 1$

$1 - 2\sin x\cos x = 1$

$-2\sin x\cos x = 0$

$\sin x\cos x = 0$

$\sin x = 0$ $\quad$ $\cos x = 0$

$0, \pi, 2\pi$ $\quad$ $\frac{\pi}{2}, \frac{3\pi}{2}$

23. $\sin x\cos x + \frac{1}{2} = 0$

$\sin x\cos x = -\frac{1}{2}$

$\frac{1}{2}\sin 2x = -\frac{1}{2}$

$\sin 2x = -1$

$\frac{3\pi}{2} \pm 2k\pi = 2x$

$\frac{3\pi}{4} \pm k\pi = x$

$\boxed{\frac{3\pi}{4}, \frac{7\pi}{4}}$