6-5 Solving Square Root and Other Radical Equations

Content Standards

A.REI.2 Solve simple rational and radical equations in one variable, and . . . show how extraneous solutions may arise.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Objective To solve square root and other radical equations

A **radical equation** is an equation that has a variable in a radicand or a variable with a rational exponent. If the radical has index 2, the equation is a **square root equation**. In this lesson, assume that all radicals and expressions with rational exponents represent real numbers.

Essential Understanding Solving a square root equation may require that you square each side of the equation. This can introduce extraneous solutions.

To solve a radical equation, isolate the radical on one side of the equation. Then raise each side to the power suggested by the index.

Problem 1 Solving a Square Root Equation

What is the solution of $3 + \sqrt{2x - 3} = 8$?

Got It? 1. What is the solution of $\sqrt{4x + 1} - 5 = 0$?

To solve equations of the form $x^{\frac{m}{n}} = k$, raise each side of the equation to the power $\frac{n}{m}$, the reciprocal of $\frac{m}{n}$. If either m or n is even, then $\left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} = |x|$.

Problem 2 Solving Other Radical Equations

A What is the solution of $3(x + 1)^{\frac{3}{5}} = 12$?

B What is the solution of $3\sqrt[5]{(x+1)^3} + 1 = 25$?

Got It? 2. What are the solution(s) of $2(x+3)^{\frac{2}{3}} = 8$?

When you raise each side of an equation to a power, it is possible to introduce extraneous solutions. Therefore, it becomes very important that you check all solutions in the original equation. A correct solution will give a true statement. An extraneous solution will give a false statement.

Problem 4 Checking for Extraneous Solutions

What is the solution of $\sqrt{x+7} - 5 = x$?

$(\sqrt{x+7})^2 = (x+5)^2$

$x+7 = x^2+10x+25$

$-x-7 \qquad -x-7$

$0 = x^2+9x+18$

$(x+3)(x+6)$

$x=-3$ ~~$x=-6$~~

$\sqrt{5x-1} + 3 = x$?

$(\sqrt{5x-1})^2 = (x-3)^2$

$5x-1 = x^2-6x+9$

$-5x+1 \qquad -5x+1$

$0 = x^2-11x+10$

$(x-10)(x-1)$

$x=10$ ~~$x=1$~~

If an equation contains two radical expressions (or two terms with rational exponents), isolate one of the radicals (or one of the terms), then eliminate it (or its rational exponent). Isolate the more complicated radical expression first. In the resulting equation, simplify the expressions before you eliminate the second radical.

Problem 5 Solving an Equation With Two Radicals

$(2x+1)^{1/2} - (x)^{1/2} = 1$

$\sqrt{2x+1} - \sqrt{x} = 1$?

$(\sqrt{2x+1})^2 = (1+\sqrt{x})^2$

$2x+1 = 1+1\sqrt{x}+1\sqrt{x}+x$

$2x+1 = 1+2\sqrt{x}+x$

$-x \qquad -1$

$(x)^2 = (2\sqrt{x})^2$

$x^2 = 4x$

$x^2-4x=0$

$x(x-4)=0$

$x=0 \quad x=4$

$\sqrt{5x+4} - \sqrt{x} = 4$?

$(\sqrt{5x+4})^2 = (4+\sqrt{x})^2$

$5x+4 = 16+8\sqrt{x}+x$

$-x \; -16 \qquad -16 \qquad -x$

$\frac{4x-12}{4} = \frac{8\sqrt{x}}{4}$

$(x-3)^2 = (2\sqrt{x})^2$

$x^2-6x+9 = 4x$

$x^2-10x+9=0$

$(x-9)(x-1)=0$

$x=9$ ~~$x=1$~~

Solve. See Problem 1.

9. $3\sqrt{x} + 3 = 15$

10. $4\sqrt{x} - 1 = 3$

11. $\sqrt{x + 3} = 5$

15. $\sqrt{3x + 4} = 4$

16. $\sqrt{2x + 3} - 7 = 0$

17. $\sqrt{6 - 3x} - 2 = 0$

Solve. See Problem 2.

18. $(x + 5)^{\frac{2}{3}} = 4$

19. $(x + 2)^{\frac{2}{3}} = 9$

20. $3(x - 2)^{\frac{3}{4}} = 24$

21. $3(x + 3)^{\frac{3}{4}} = 81$

22. $(x + 1)^{\frac{3}{2}} - 2 = 25$

23. $3 + (4 - x)^{\frac{3}{2}} = 11$

Solve. Check for extraneous solutions. See Problem 4.

26. $\sqrt{3x + 7} = x - 1$

27. $(5 - x)^{\frac{1}{2}} = x + 1$

28. $\sqrt{-3x - 5} = x + 3$

32. $\sqrt{x + 7} + 5 = x$

33. $(x + 3)^{\frac{1}{2}} - 1 = x$

34. $\sqrt{x + 7} - x = 1$

Solve. Check for extraneous solutions.

35. $\sqrt{3x} = \sqrt{x + 6}$

36. $(2x)^{\frac{1}{2}} = (x + 5)^{\frac{1}{2}}$

37. $(7x + 6)^{\frac{1}{2}} - (9 + 4x)^{\frac{1}{2}} = 0$

38. $\sqrt{3x + 2} - \sqrt{2x + 7} = 0$

39. $(x + 5)^{\frac{1}{2}} - (5 - 2x)^{\frac{1}{2}} = 0$

40. $(x - 2)^{\frac{1}{3}} - (28 - 2x)^{\frac{1}{3}} = 0$

41. $\sqrt{5 - x} - \sqrt{x} = 1$

42. $\sqrt{3x + 1} - \sqrt{x + 1} = 2$

43. $\sqrt{2x + 6} - \sqrt{x - 1} = 2$

44. $\sqrt{3 - x} + \sqrt{x + 2} = 3$