9-3 Geometric Sequences

Content Standard

Prepares for A.SSE.4 Derive the formula for the sum of a geometric series (when the common ratio is not 1), and use the formula to solve problems.

Objective To define, identify, and apply geometric sequences

4-14-14

You build a *geometric sequence* by multiplying each term by a constant.

Essential Understanding In a *geometric sequence*, the ratio of any term to its preceding term is a constant value.

Take note

Key Concept Geometric Sequence

A **geometric sequence** with a starting value a and a **common ratio** r is a sequence of the form

$$a, ar, ar^2, ar^3, \dots$$

A recursive definition for the sequence has two parts:

$a_1 = a$ initial condition

$a_n = a_{n-1} \cdot r$, for $n > 1$ recursive formula

An explicit definition for this sequence is a single formula:

$a_n = a_1 \cdot r^{n-1}$, for $n \geq 1$

explicit

$a_n = a_1 \cdot r^{n-1}$

$a_n = 3 \cdot 2^{n-1}$

recursive

$a_1 = 3$

$a_n = a_{n-1} \cdot 2$

Problem 1 Identifying Geometric Sequences

Is the sequence geometric? If it is, what are a_1 and r?

Think

How do I find the ratios between consecutive terms? Divide the second term by the first term, then the third term by the second term, and so on.

A 3, 6, 12, 24, 48, ... $r = 2$

Find the ratios between consecutive terms.

3 6 12 24 48

$$\frac{6}{3} = \frac{12}{6} = \frac{24}{12} = \frac{48}{24} = 2$$

The common ratio is 2. The sequence is geometric with $a_1 = 3$ and $r = 2$.

B 3, 6, 9, 12, 15, ... arithmetic

Find the ratio between consecutive terms.

3 6 9 12 15

$$\frac{6}{3} \neq \frac{9}{6} \neq \frac{12}{9} \neq \frac{15}{12}$$

The ratios are different. With no common ratio, the sequence is not geometric.

C $3^5, 3^{10}, 3^{15}, 3^{20}, \dots$

$\frac{3^{10}}{3^5} = 3^5$

$\frac{3^{15}}{3^{10}} = 3^5$

Use the properties of exponents to simplify the ratios of successive terms.

3^5 3^{10} 3^{15} 3^{20}

$$\frac{3^{10}}{3^5} = \frac{3^{15}}{3^{10}} = \frac{3^{20}}{3^{15}} = 3^5$$

The common ratio is 3^5. The sequence is geometric with $a_1 = 3^5$ and $r = 3^5$.

Got It? 1. Is the sequence geometric? If it is, what are a_1 and r?

a. $2, 4, 8, 16, \ldots$

$r = 2$

$a_1 = 2$

b. $1, 5, 9, 13, 17, \ldots$

not geo.

c. $2^3, 2^7, 2^{11}, 2^{15}, \ldots$

yes!

$r = \frac{2^7}{2^3} = 2^4$

Problem 2 Analyzing Geometric Sequences

What are the indicated terms of the geometric sequence?

A **the 10th term of the geometric sequence 4, 12, 36, . . .**

The first term a_1 is 4. The common ratio r is $12 \div 4 = 3$.

$a_n = a_1 r^{n-1}$	Use the explicit formula.
$a_{10} = 4 \cdot 3^{10-1}$	Substitute 10 for n, 4 for a_1, and 3 for r.
$a_{10} = 78{,}732$	Simplify.

The 10th term is 78,732.

Plan

What do you need to find the second term given the first term?
You need the common ratio.

B **the second and third terms of the geometric sequence 2, ▪, ▪, −54, . . .**

The first term a_1 is 2. The fourth term a_4 is −54.

$a_n = a_1 r^{n-1}$	Use the explicit formula.
$a_4 = 2r^{4-1}$	Substitute 2 for a_1 and 4 for n.
$-54 = 2r^3$	Substitute −54 for a_4. Simplify.
$-27 = r^3$	Solve for r.
$-3 = r$	

The common ratio is −3. Begin with 2 and multiply by −3.

$2, -6, 18, -54, \ldots$

The second and third terms are −6 and 18.

Got It? 2. What is the 2nd term of the geometric sequence 3, ▪, 12, . . . ?

6

$\sqrt{3 \cdot 12}$

$\sqrt{36}$

In a geometric sequence, the square of the middle term of any three consecutive terms is equal to the product of the other two terms. For example, examine the sequence $2, -6, 18, -54, \ldots$.

$$(-6)^2 = 2 \cdot 18 = 36$$

$$2, -6, 18, -54, \ldots$$

$$18^2 = (-6)(-54) = 324$$

In an arithmetic sequence, recall that the middle term of any three consecutive terms is the arithmetic mean of the other two terms.

The **geometric mean** of two positive numbers x and y is $\sqrt{xy}$.

Note that the geometric mean is positive by definition. While there are two possible values for the missing term in the geometric sequence $3, \blacksquare, 12, \ldots$, there is only one geometric mean. The geometric mean is one possible value to fill in the geometric sequence. The opposite of the geometric mean is the other.

Problem 4 Using the Geometric Mean

Multiple Choice What are the possible values of the missing term of the geometric sequence?

$48, \blacksquare, 3, \ldots$

$\sqrt{48 \cdot 3} = \sqrt{144}$

Ⓐ ± 4 Ⓑ ± 9 Ⓒ ± 12 Ⓓ ± 20

Find the geometric mean of 48 and 3.

$$\sqrt{48 \cdot 3} = \sqrt{144}$$
$$= 12$$

The possible values for the missing term are ± 12. The correct answer choice is C.

9th 11th

Got It? 4. The 9th and 11th terms of a geometric sequence are 45 and 80. What are possible values for the 10th term?

$\sqrt{45 \cdot 80}$

$\sqrt{3600}$

60

9.3 Algebra 2
Homework

Name______________________

Determine whether each sequence is geometric. If so, find the common ratio. See Problem 1.

7. 1, 2, 4, 8, ...

yes

$r=2$

8. 1, 2, 3, 4, ...

no

9. 1, −2, 4, −8, ...

yes

$r=-2$

13. $18, -6, 2, -\frac{2}{3}, \ldots$

$\frac{-6}{18} = -\frac{1}{3}$

$r = -\frac{1}{3}$

14. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

no

15. 10, 15, 22.5, 33.75, ...

$\frac{15}{10} = \frac{3}{2} = 1.5$

$\frac{22.5}{15} = 1.5$

yes $r=1.5$

Find the eighth term of each geometric sequence. See Problem 2.

19. 3, 9, 27, ...

$a_8 = a_1 \cdot r^{8-1}$

$3 \cdot 3^7$

$a_8 = 6561$

20. −3, 6, −12, ...

$a_8 = -3 \cdot (-2)^7$

$a_8 = 384$

21. 10, 5, 2.5, ...

$a_8 = 10\left(\frac{1}{2}\right)^7$

.078

Find the missing term of each geometric sequence. It could be the geometric mean or its opposite. See Problem 4.

26. 5, ▪, 911.25, ...

$\sqrt{5 \cdot 911.25}$

67.5

27. 9180, ▪, 255, ...

$\sqrt{9180 \cdot 255}$

1530

28. $\frac{2}{5}$, ▪, $\frac{8}{45}$, ...

$\sqrt{\frac{2}{5} \cdot \frac{8}{45}} = \sqrt{\frac{16}{225}} = \frac{4}{15}$

Write an explicit formula for each sequence. Then generate the first five terms.

32. $a_1 = 1, r = 0.5$

$a_n = 1 \cdot .5^{n-1}$

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$

33. $a_1 = 100, r = -20$

$a_n = 100(-20)^{n-1}$

100, −2000, 40,000, −800,000, 16,000,000

34. $a_1 = 7, r = 1$

$a_n = 7 \cdot 1^{n-1}$

7, 7, 7, 7, 7

Identify each sequence as *arithmetic*, *geometric*, or *neither*. Then find the next two terms.

38. 45, 90, 180, 360, ...

geo.

$r=2$

720, 1440

39. 25, 50, 75, 100, ...

arithmetic

$d=25$

125, 150

40. 3, −3, 3, −3, ...

geometric $r=-1$

3, −3

Done!