7.4 Phase Shifts Sine/Cos/Csc/Sec

Name____________________________

We have looked at graphs of the form $f(t) = a\sin bt + d$

Where "a" measures the steepness of the graph
if a >1 it's a vertical stretch
if a< 1 it's a vertical compression by a factor of "a"

Where "d" shifts the graph up or down vertically by "d" units.

$f(t) = a\sin bt + d$ **where "b' will effect the period of the graph. The "b" value changes the period. For sine and cosine take 2π /b to find the period. Then divide the new period by 4 to find the increments of the graph!!!**

← new stuff

NOW.... We consider $f(t) = a\sin b(t - c) + d$ where "c" shifts your graph left or right. Be careful that the equation you are working with has a "b" value factored out. If not factor it out otherwise you will not know the true "c" value or horizontal shift left or right!

A phase shift means that we are moving the graph left or right. We are actually beginning in a spot other that the origin. (t - c) is a shift "c" units right
(t + c) is a shift "c" units left

EXAMPLE: Given : $f(t) = \overset{a}{-2}\sin(\overset{b}{3}t - \pi) + \overset{d}{4}$. Find the following and graph 1 full period. Label your graph completely! Be neat!

$3(t - \frac{\pi}{3})$

Rewite the equation $f(t) = -2\sin 3(t - \frac{\pi}{3}) + 4$

"a": Amplitude: 2

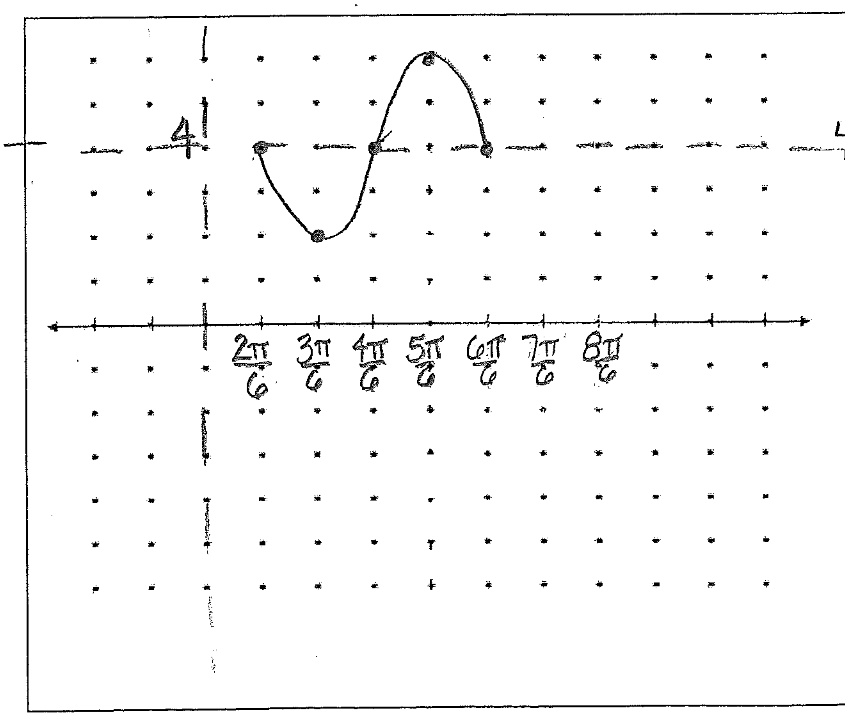
"b": Period: $\frac{2\pi}{3}$

"b": Increments: $\frac{\cancel{2}\pi}{3} \cdot \frac{1}{\cancel{4}2} = \frac{\pi}{6}$

Start

"c" : Phase shift: $\frac{\pi}{3}$ right $\frac{2\pi}{6}$

"d": V. Shift: 4 up



EXAMPLE: Given : $f(t) = \underset{a}{3}\cos(\underset{b}{2}t + 2\pi) \underset{d}{-2}$. Find the following and graph 1 full period. Label your graph completely! Be neat!

Rewite the equation $f(t) = 3\cos 2(t + \underline{\pi}) - 2$.

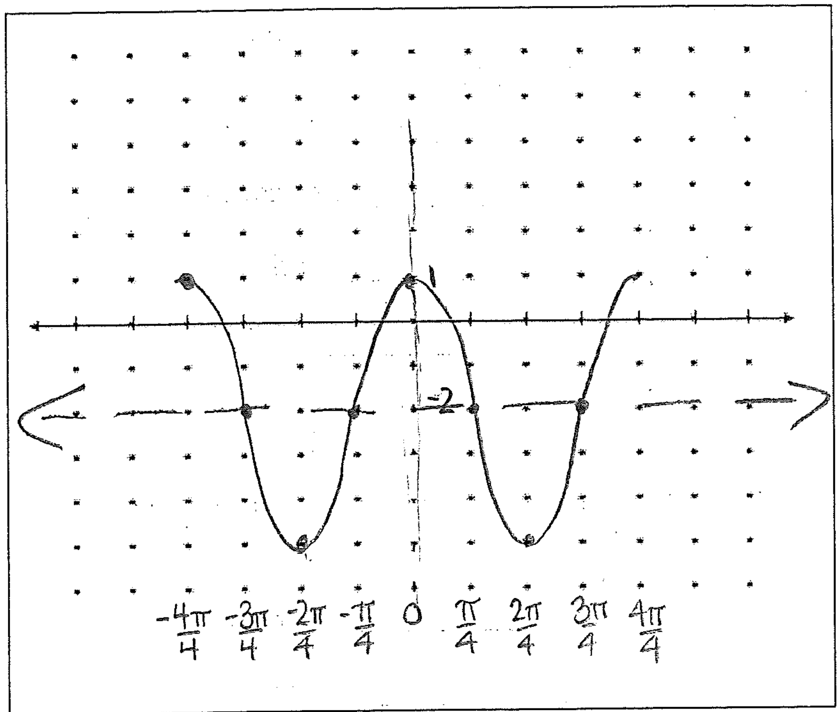
"a": Amplitude: 3

"b": Period: $\frac{\cancel{2}\pi}{\cancel{2}} = \pi$

"b": Increments: $\frac{\pi}{4}$

"c" : Phase shift: π left $-\frac{4\pi}{4}$

"d": V. Shift: 2 down



$-3\sin 2\left(t + \frac{3\pi}{4}\right) - 2$

EXAMPLE: Given : $f(t) = -2\csc(3t + 6\pi) + 2$. Find the following and graph 1 full period. Label your graph completely! Be neat!

$-2\sin 3(t + 2\pi) + 2$

Rewite the equation $f(t) = -2\csc 3(t + \underline{2\pi}) + 2$

"a": Amplitude: 2

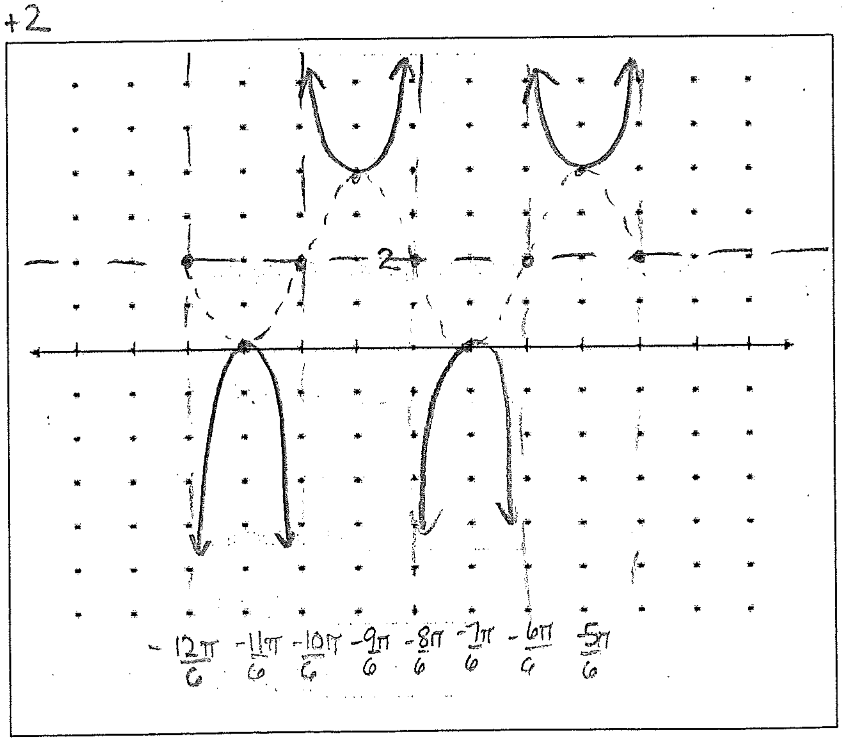
"b": Period: $\frac{2\pi}{3}$

"b": Increments: $\frac{2\pi}{3} \cdot \frac{1}{4_2} = \frac{\pi}{6}$

(2π to left)

"c" : Phase shift: $\frac{-2\pi \cdot 6}{1 \cdot 6} = -\frac{12\pi}{6}$ ← start

"d": V. Shift: 2 up



Find a sin function with an amplitude of 2, period of 8π, phase shift of 2 to the right, and vertical shift 1 down, reflected over the x axis.

$\boxed{y = -2\sin\left(\frac{1}{4}(t-2)\right)-1}$

$$f(t) = a\sin b(t-c)+d$$

$f(x) = -2\sin\frac{1}{4}(t-2)-1$

$f(x) = -2\sin\left(\frac{1}{4}t-\frac{1}{2}\right)-1$

Hint: find "b"

$\frac{2\pi}{b} = \frac{8\pi}{1}$ $\quad \frac{8b}{8}=\frac{2}{8}$

$8\cancel{\pi}b = 2\cancel{\pi}$ $\quad b = 1/4$

The big idea or concept with phase shifts is that any of the trig graphs seen here can be named with more than one equation depending on where you begin. Just altering the phase shift alters the equation.

Let's go back to examples 1 and 2 and rename these graphs with different equations:

If we pick $\frac{3\pi}{6}$ as our new phase shift, the graph is "low" or a "negative" cosine curve.

Example 1: This was the original function $\longrightarrow f(t) = -2\sin 3(t-\frac{\pi}{3})+4$ but depending on where we start, we can change the name of this function to…

a. $-2\cos 3(t-\frac{3\pi}{6})+4$

b. $2\sin 3(t-\frac{4\pi}{6})+4$

c. $2\cos 3(t-\frac{\pi}{6})+4$

…..etc…__________________

So…..what 3 things do you notice can change when we start somewhere else?

+/- , sin/cos , phase shift

Example 2: This was the original function $\longrightarrow f(t) = 3\cos 2(t-\pi)-2$ but depending on where we start, we can change the name of this function to…

d. __________________

e. __________________

f. __________________

…..etc…__________________

So…..what 3 things do you notice can change when we start somewhere else?

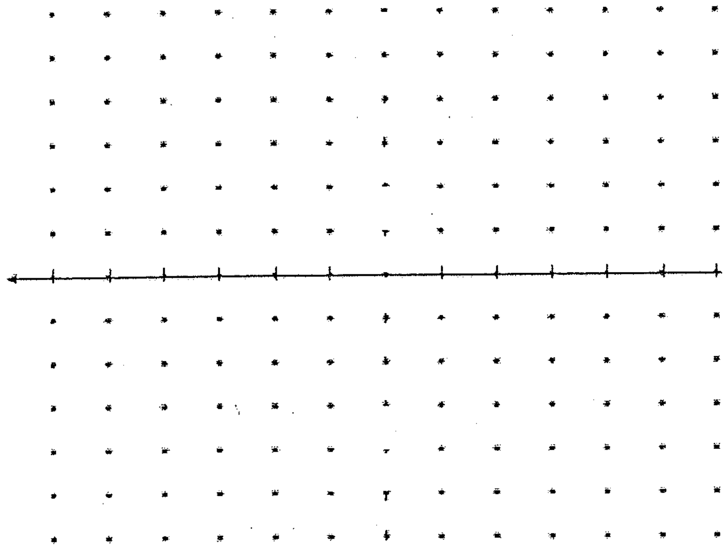
______, ______, ______

PreCalc

7.4 A Homework. Graphs of Sine and Cosine with phase shifts. Name________________________

In 1-6, all graphs must be labeled completely for full credit. Label both the x and y axis. Do not reduce fractions.

1. Graph **one period** of $f(t) = 2\sin(3t+\pi)-1$.



Amplitude: __________

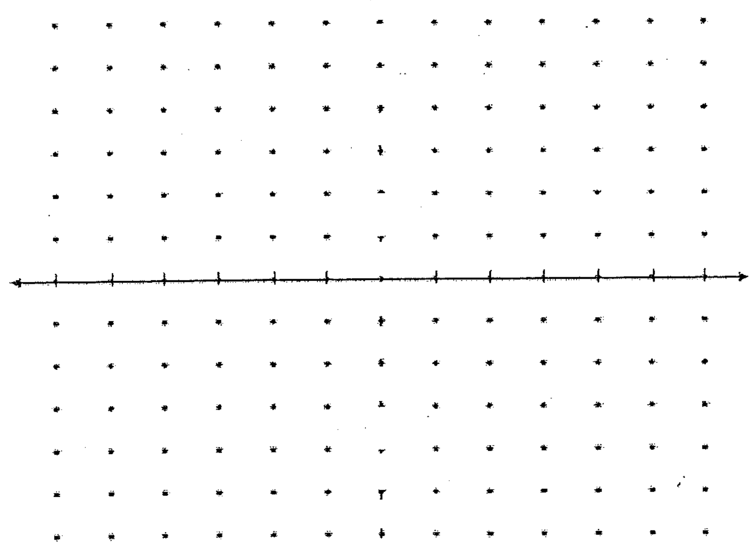
Period: __________

Increments: __________

P. Shift: __________

V. Shift: __________

2. Graph **one period** of $f\left(t\right) = 2+\cos(2t-\frac{\pi}{2})$.



Amplitude: __________

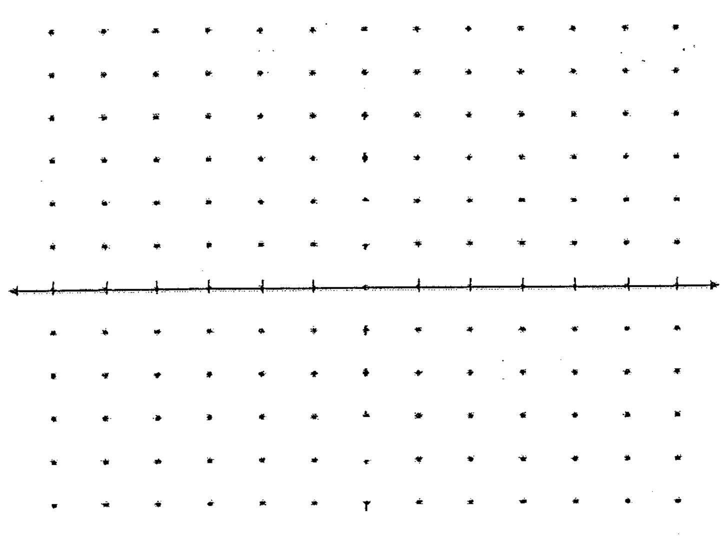
Period: __________

Increments: __________

P. Shift: __________

V. Shift: __________

3. Graph **two full periods** of $f\left(t\right) = 2\cos\left(\frac{\pi t}{3}-1\right)+5$.



Amplitude: __________

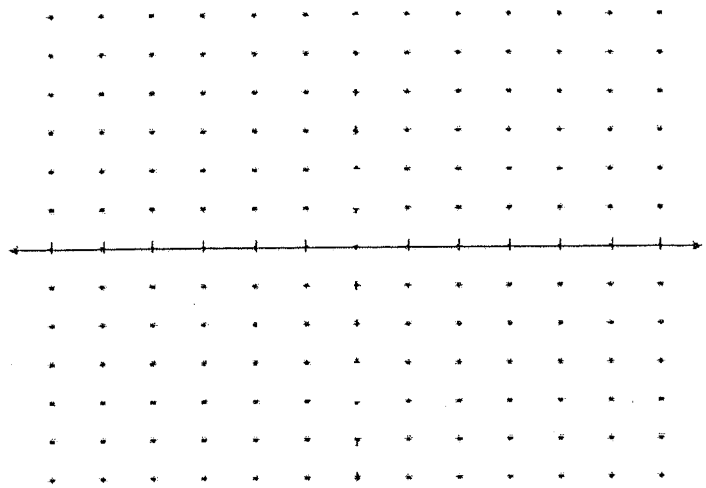
Period: __________

Increments: __________

V. Shift: __________

P. Shift: __________

4. Graph **two full periods** of $f(t) = -5\sin\left(\frac{t}{4}+3\right)-1$.



Amplitude: ___________

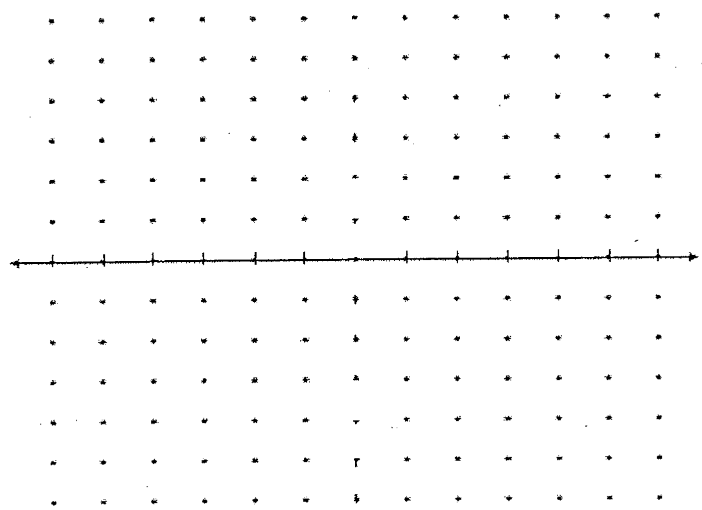
Period: ___________

Increments: ___________

P. Shift: ___________

V. Shift: ___________

5. Graph **two full periods** of $f(t) = 3\sin(2t-\pi)$.



Amplitude: ___________

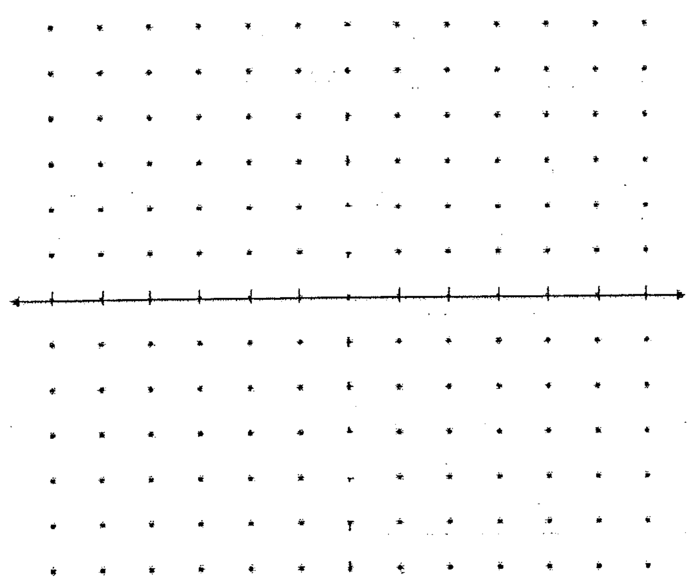
Period: ___________

Increments: ___________

P. Shift: ___________

V. Shift: ___________

6. Graph **two full periods** of $f(t) = 4\cos(t-5)+2$.



Amplitude: ___________

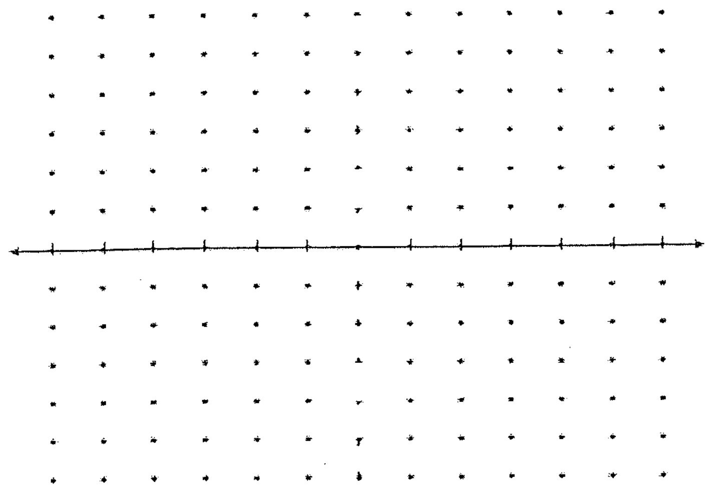
Period: ___________

Increments: ___________

P. Shift: ___________

V. Shift: ___________

7. Graph **two full periods** of $f(t) = 3\sec\left(2t + \frac{\pi}{2}\right) - 1$.



Amplitude: ___________

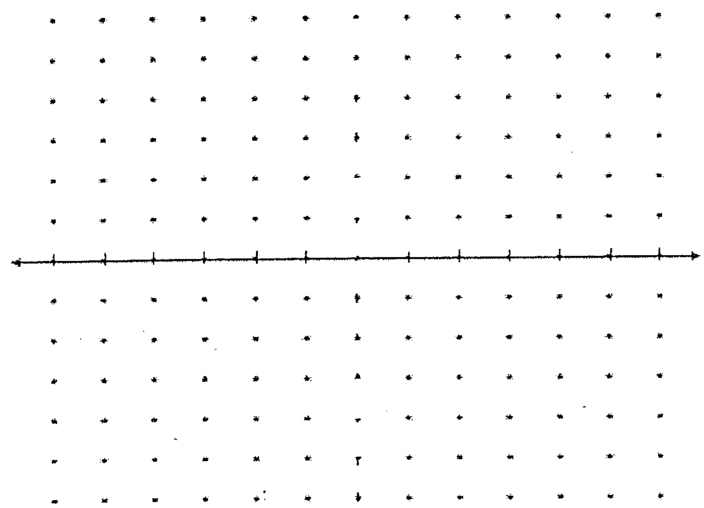
Period: ___________

Increments: ___________

P. Shift: ___________

V. Shift: ___________

8. Graph **two full periods** of $f(t) = -3\csc(3t - \pi)$.



Amplitude: ___________

Period: ___________

Increments: ___________

P. Shift: ___________

V. Shift: ___________

In 9-13, state the rule of a sine function with the given amplitude, reflections, period, phase shift and vertical shift.

	Amplitude	**reflection**	**Period**	**Phase Shift**	**Vertical Shift**	**Function/Rule**
9.	5	none	$\frac{\pi}{4}$	$\frac{-2\pi}{3}$	2 up	
10.	2	Over x axis	3π	$\frac{\pi}{4}$	3 down	
11.	1	None	$\frac{5\pi}{3}$	1.5	0 units	
12.	3	Over x axis	8π	$\frac{\pi}{2}$	.5 down	
13.	.5	None	$\frac{1}{2}$	$\frac{-3\pi}{4}$	4 up	

7.4 Phase Shifts Tan/Cot

Name Key

We have looked at graphs of the form $f(t) = a\tan bt + d$

Where "a" measures the steepness of the graph
if a >1 it's a vertical stretch
if a< 1 it's a vertical compression by a factor of "a"

Where "d" shifts the graph up or down vertically by "d" units.

$f(t) = a\tan bt + d$ **where "b' will effect the period of the graph. The "b" value changes the period. For sine and cosine take π/b to find the period. Then divide the new period by 2 to find the increments of the graph!!!**

NOW.... We consider $f(t) = a\tan b(t-c) + d$ where "c" shifts your graph left or right. Be careful that the equation you are working with has a "b" value factored out. If not factor it out otherwise you will not know the true "c" value or horizontal shift left or right!

For tan and cot, instead of a phase shift, we will find the location of the "first asymptote" and begin there. This would be like our phase shift. For the tan graph since the first asymptote begins at $\frac{\pi}{2}$, you will use this to find the first asymptote. For cot graphs, since the first asymptote begins at 0, you will use that to find the first asymptote. Once you find the first asymptote, add the two increments to find the next asymptote. Then construct your graph.

EXAMPLE: Graph : $f(t) = 2\tan(3t-\pi)+4$. Find the following and graph 2 full periods. Label your graph completely! Be neat!

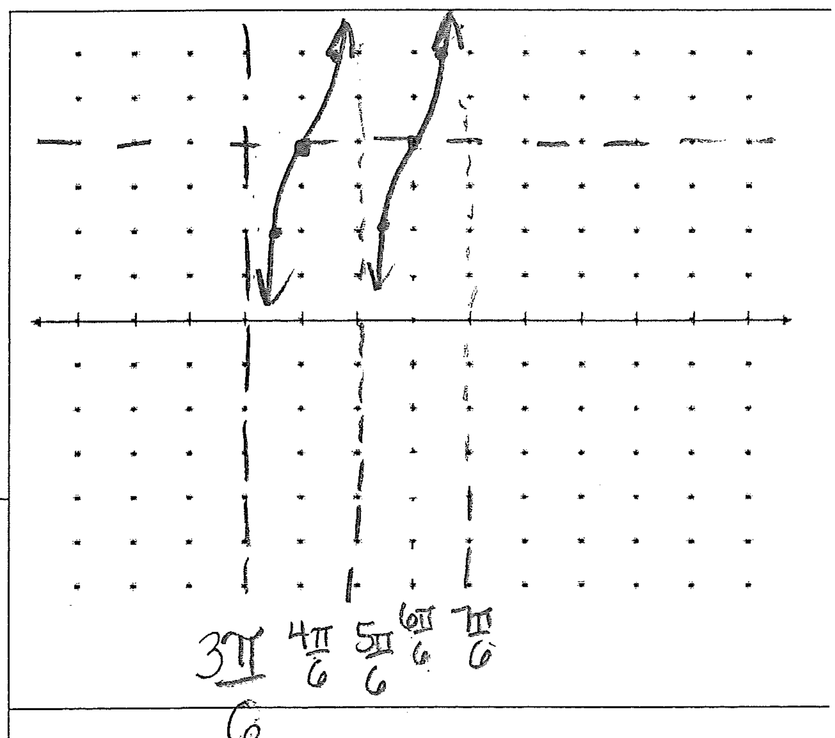
"a": Steepness: 2

"b": Period: $\frac{\pi}{3} \cdot \frac{1}{2}$

"b": Increments: $\frac{\pi}{6}$

"c" : 1st asymptote: $\frac{\pi}{2} = \frac{3\pi}{6}$ start

"d": V. Shift: 4



To find the 1st asymptote, set $(3t-\pi) = \frac{\pi}{2}$ and solve for t. This will be your first asymptote. Then add increments to that number. You will want to get the first asymptote and the increments in the same denominator.

$3t = \frac{\pi}{2} + \frac{\pi}{1} \cdot \frac{2}{2}$

$\frac{1}{3} \cdot 3t = \frac{3\pi}{2} \cdot \frac{1}{3}$

$t = \frac{\pi}{2}$

EXAMPLE: Given : $f(t) = 3\cot(\underbrace{2t + 2\pi}_{=0}) - 2$. Find the following and graph 2 full periods. Label your graph completely! Be neat!

"a": Steepness: 3

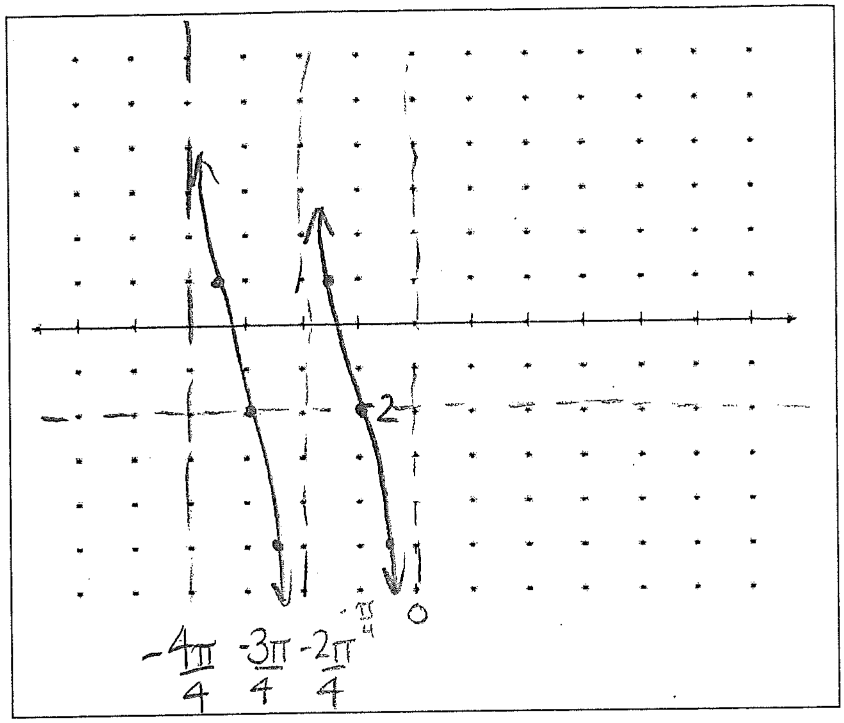
"b": Period: $\frac{\pi}{2}$

"b": Increments: $\frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$

Start

"c" : 1st asymptote: $\frac{-\pi}{1} = \frac{-4\pi}{4}$

"d": V. Shift: -2



To find the 1st asymptote, $2t + 2\pi = 0$ and solve for t. This is the first asymptote. Add increments accordingly. You may need to get the 1st asymptote and the increments in the same denominator.

$$2t + 2\pi = 0$$
$$\frac{2t}{2} = \frac{-2\pi}{2}$$
$$t = -\pi$$

EXAMPLE: Given : $f(t) = -3\tan\left(2t - \frac{\pi}{2}\right) + 2$. Find the following and graph 2 full periods. Label your graph completely! Be neat!

$$2t - \frac{\pi}{2} = \frac{\pi}{2}$$
$$\frac{2t}{2} = \frac{\pi}{2}$$
$$t = \frac{\pi}{2}$$

"a": Amplitude: 3

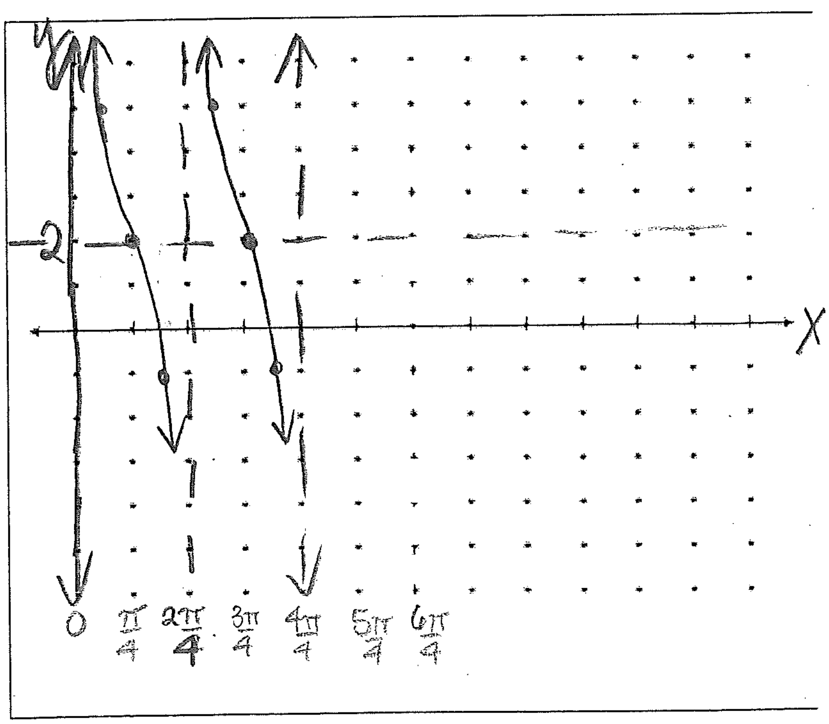
"b": Period: $\frac{\pi}{2}$

$\frac{\pi}{2} \cdot \frac{1}{2}$

"b": Increments: $\frac{\pi}{4}$

"c" : 1st asymptote: $\frac{\pi}{2} = \frac{2\pi}{4}$

"d": V. Shift: 2 up

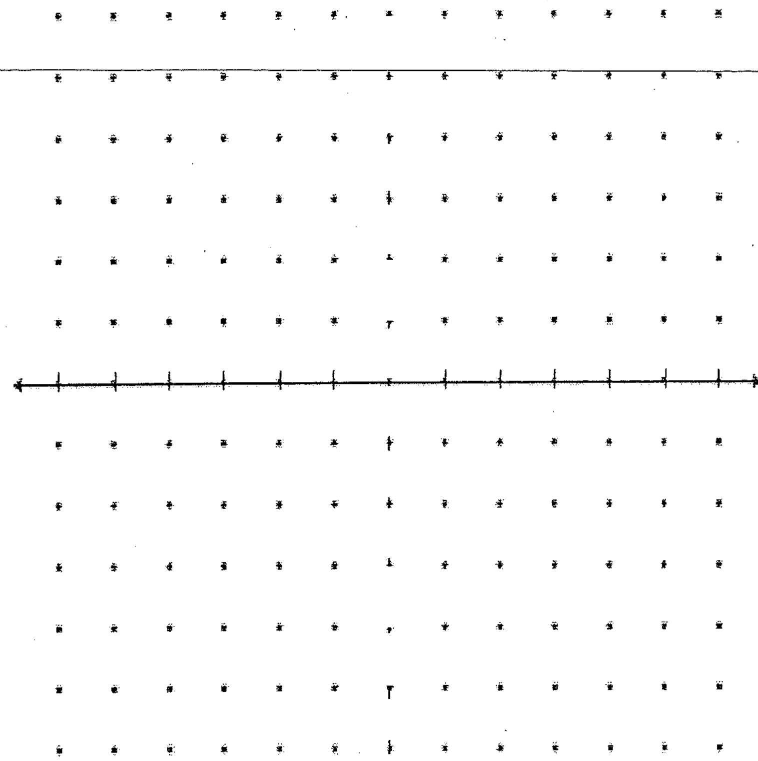


To find the first asymptote, set $(2t - \frac{\pi}{2}) = \frac{\pi}{2}$ and solve for t.

PreCalc
7.4 B Homework. Graphs of Tan and Cot. Name____________________________

In 1-4, all graphs must be labeled completely for full credit. Label both the x and y axis. Do not reduce fractions. Graph 2 full periods.

1. Graph **two periods** of $f(t) = 2\tan(3t + \frac{\pi}{4}) - 1$.



Steepness: __________

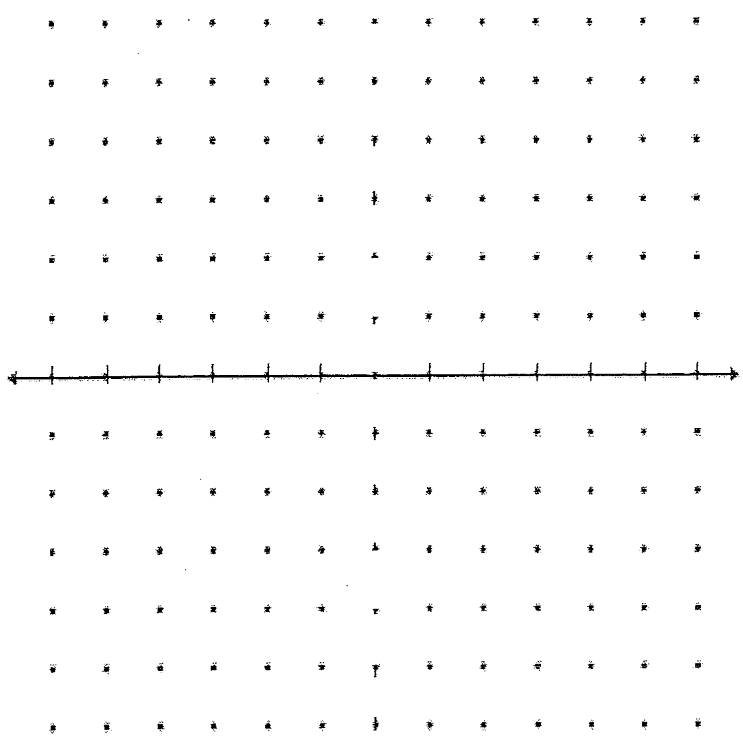
Period: __________

Increments: __________

1st asymptote: __________

V. Shift: __________

2. Graph **two periods** of $f(t) = 2 + \cot(2t - \frac{\pi}{2})$.



Steepness: __________

Period: __________

Increments: __________

1st asymptote: __________

V. Shift: __________

3. Graph **two full periods** of $f(t) = 2\cot\left(\frac{\pi t}{3} - 1\right) + 3$.

$2\cot\frac{\pi}{3}\left(t - \frac{3}{\pi}\right) + 3$

Steepness: 2

Period: $\pi / \frac{\pi}{3} = 3$

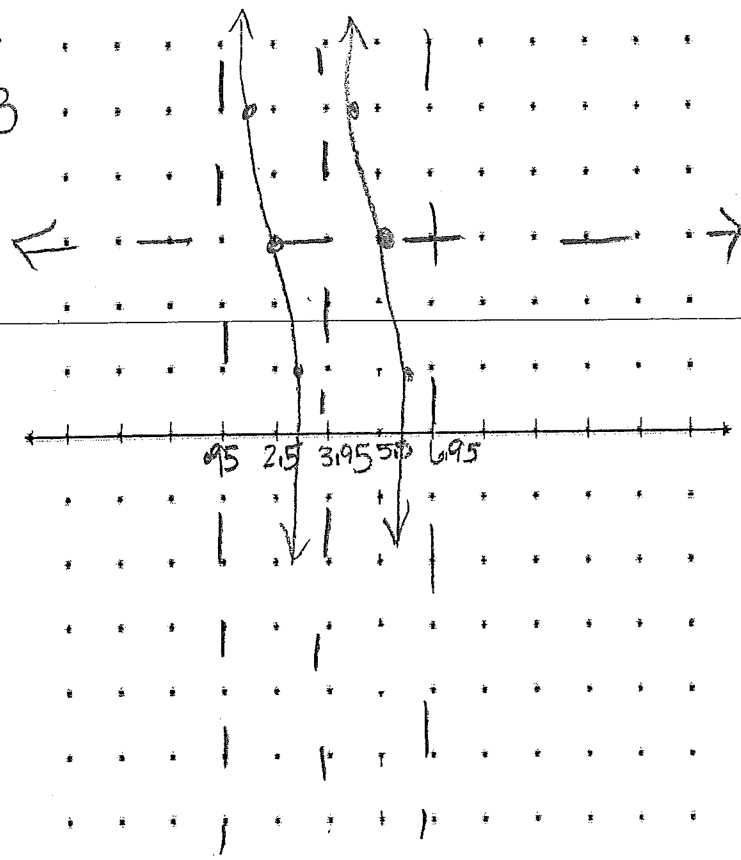
Increments: $\frac{3}{2}$

1st asymptote: $\frac{3}{\pi} = .95$

$\frac{\pi t}{3} - 1 = 0$

$\frac{\pi t}{3} = 1$

V. Shift: Up 3



4. Graph **two full periods** of $f(t) = -2\tan\left(\frac{t}{4} - \pi\right) - 1$.

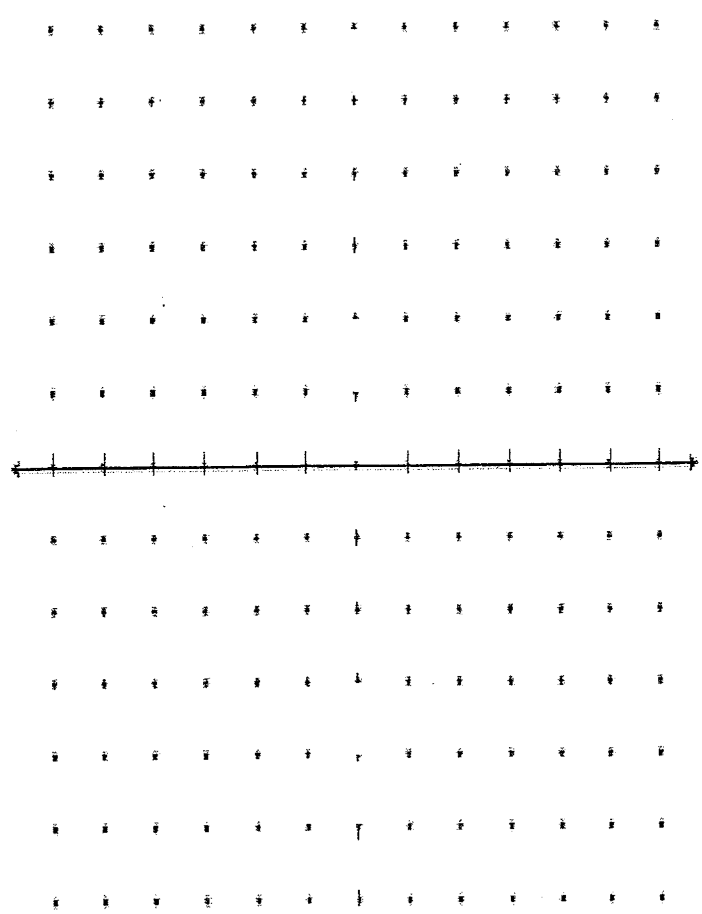
Steepness: __________

Period: __________

Increments: __________

1st asymptote: __________

V. Shift: __________



Precalculus – Worksheet 7-4 B

Writing Equations

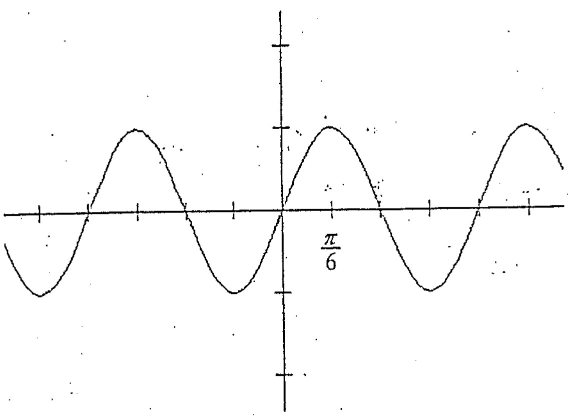
Name: ______________________________

Date: ________________ Hour: ________

Write *two* equations using two ***different*** trigonometric functions that could be the rule for each graph shown.

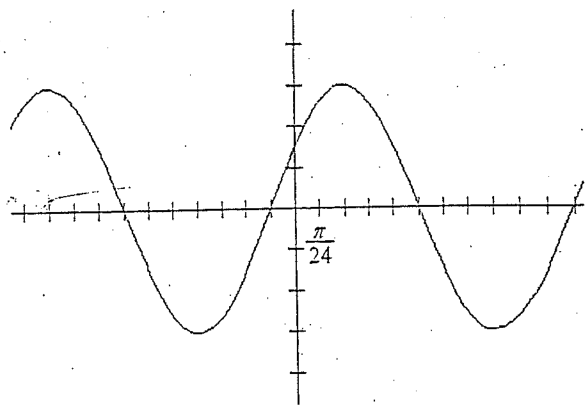
1. ______________________________

____



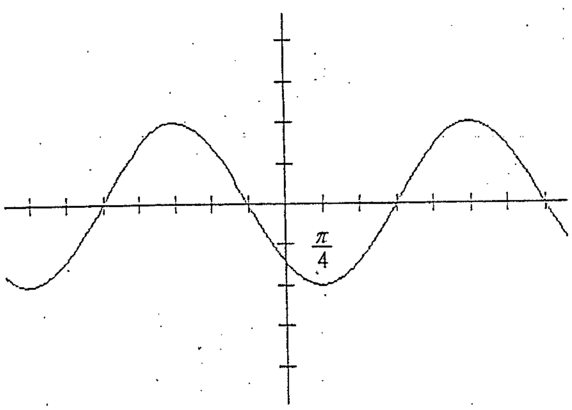
2. ______________________________

____



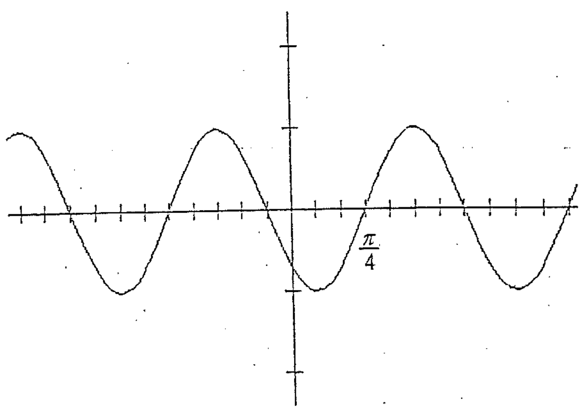
3. ______________________________

____



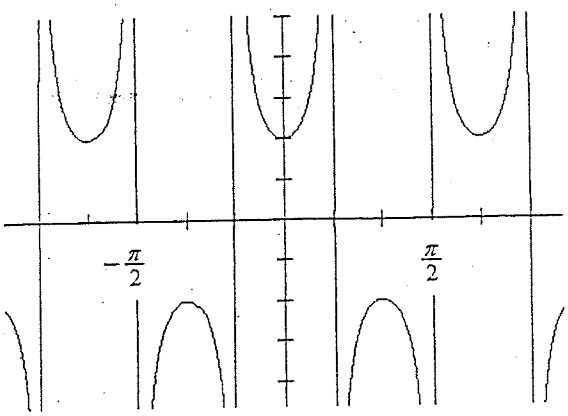
4. ______________________________

____



5. ________________________

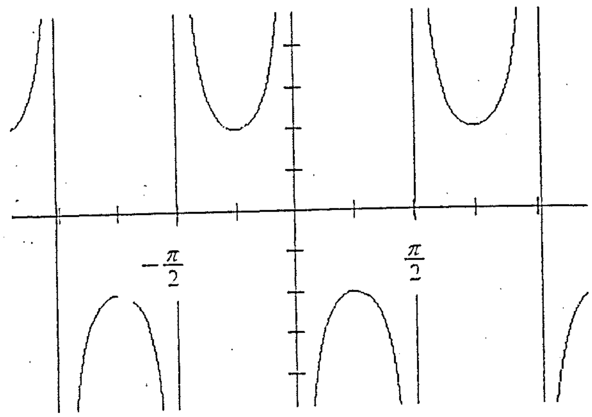
____



$-\frac{\pi}{2}$
$\frac{\pi}{2}$

6. ________________________

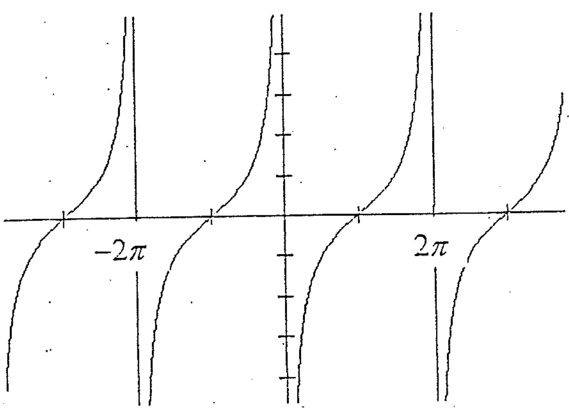
____



$-\frac{\pi}{2}$
$\frac{\pi}{2}$

7. ________________________

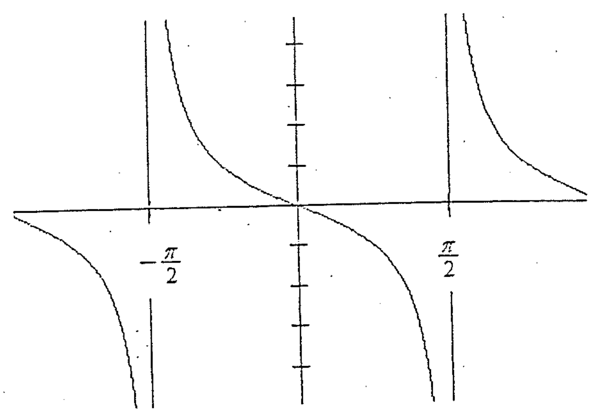
____



-2π
2π

8. ________________________

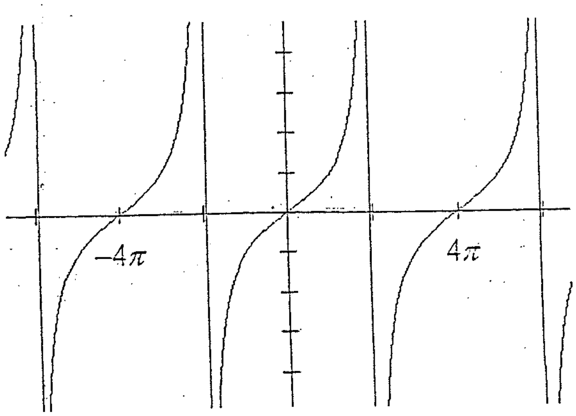
____



$-\frac{\pi}{2}$
$\frac{\pi}{2}$

9. ________________________

____



-4π
4π