$5 \rightarrow \frac{1}{5}$ $(1,5) \rightarrow (5,1)$

8.2 Inverse Trigonometric Functions

Reciprocals	VS	Inverses
$\sin \rightarrow \frac{1}{\sin} = \csc$		$\sin \rightarrow \sin^{-1}$ or arcsin
$\cos \rightarrow \frac{1}{\cos} = \sec$		$\cos \rightarrow \cos^{-1}$ or arccos
$\tan \rightarrow \frac{1}{\tan} = \cot$		$\tan \rightarrow \tan^{-1}$ or arctan

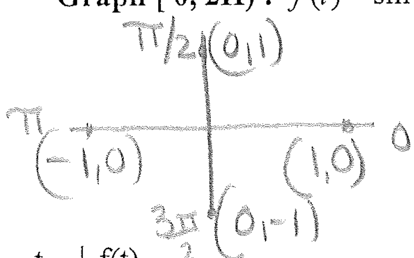
Objectives

- Define the domain and range of the inverse trigonometric functions
- Use inverse trigonometric function notation

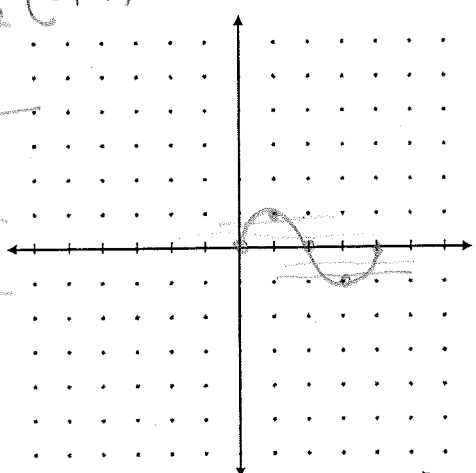
Recall from Section 3.6 that a function cannot have an inverse function unless its graph has the following property.

No horizontal line intersects the graph more than once.

Graph [0, 2Π) : $f(t) = \sin t \rightarrow \frac{y}{r}$



t	f(t)
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	−1
2π	0

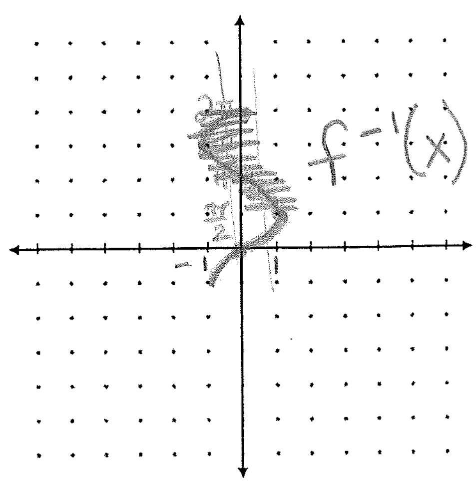


now graph the inverse of $f(t) = \sin t$

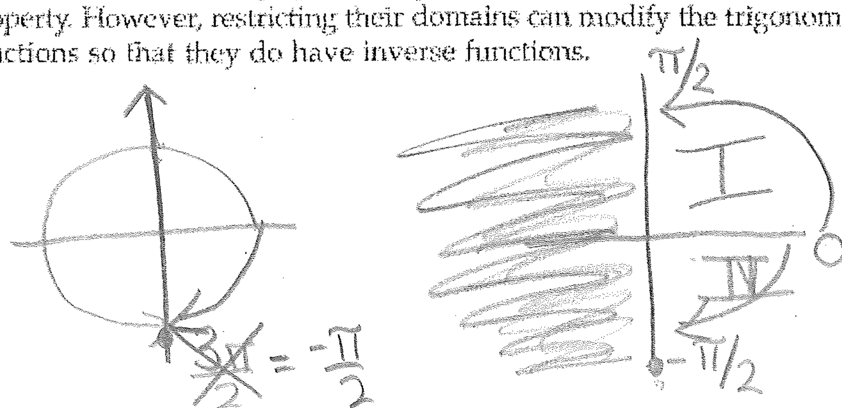
Rewrite the function:

Resolve for y:

t	f(t)
0	0
1	$\frac{\pi}{2}$
0	π
−1	$\frac{3\pi}{2}$
0	2π



You have seen that the graphs of trigonometric functions do not have this property. However, restricting their domains can modify the trigonometric functions so that they do have inverse functions.



So we have to restrict the Domain in order to keep It a function:

Restrict the domain: [ ]

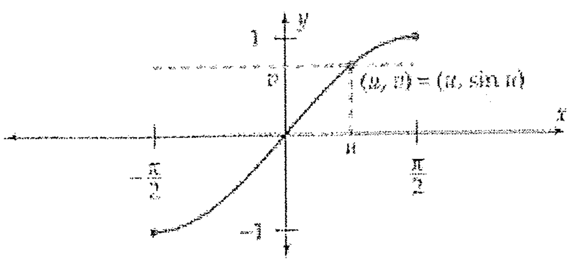


Figure 8.2-1

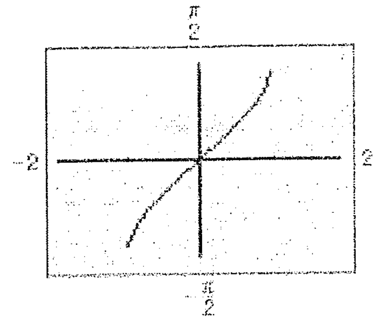


Figure 8.2-2

This is the regular sine graph

This is the inverse sine graph
Notice it is a reflection over y=x

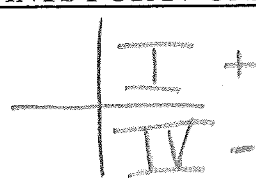
This inverse function is called the
inverse sine (or arcsine) function and is denoted by

$$g(x) = \sin^{-1} x \text{ or } g(x) = \arcsin x.$$

The domain of $g(x) = \sin^{-1} x$ is the interval $[-1, 1]$, and its range is the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

YOU WILL BE IN QUADRANTS I OR IV ONLY FOR INVERSE SINE or Arcsin

Remember : $\sin^{-1} = \arcsin$



Examples:

1. $\arcsin(-\frac{\sqrt{3}}{2}) =$ $-\frac{\pi}{3}$

2. $\sin^{-1}(\frac{1}{2}) =$ $\frac{\pi}{6}$

3. $\arcsin(-1) =$ $-\frac{\pi}{2}$

4. $\arcsin(0) =$ 0

5. $(\sin(\sin^{-1}(-\frac{1}{2}))) =$ $-\frac{1}{2}$

6. $\cos(\arcsin(-\frac{\sqrt{2}}{2})) =$ $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

7. $\sin(\sin^{-1}(-1)) =$ -1

10. $\sin^{-1}(\sin(\frac{\pi}{3})) =$ $\frac{\pi}{3}$

11. $\arcsin(\sin(\frac{2\pi}{3})) =$ $\frac{\pi}{3}$

12. $\sin^{-1}(\sin(\frac{7\pi}{4})) =$ $-\frac{\pi}{4}$

Properties of Inverse Sine

$\sin^{-1}(\sin u) = u$ if $-\frac{\pi}{2} \le u \le \frac{\pi}{2}$

$\sin(\sin^{-1} v) = v$ if $-1 \le v \le 1$

Explain why $\sin^{-1}\left(\sin \frac{\pi}{6}\right) = \frac{\pi}{6}$ is true but $\sin^{-1}\left(\sin \frac{5\pi}{6}\right) = \frac{5\pi}{6}$ is *not* true.

Solution

You know that $\sin \frac{\pi}{6} = \frac{1}{2}$, so by substitution

$$\sin^{-1}\left(\sin\frac{\pi}{6}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

because $\frac{\pi}{6}$ is in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Although $\sin \frac{5\pi}{6}$ is also $\frac{1}{2}$, by substitution

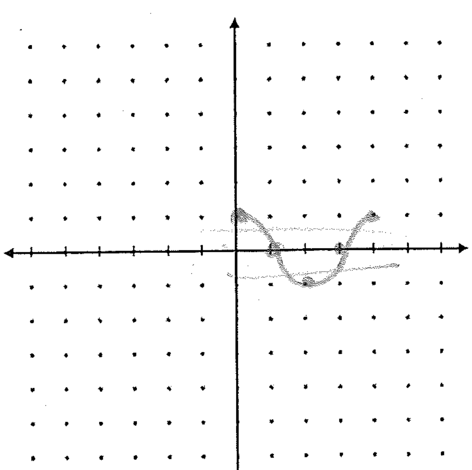
$$\sin^{-1}\left(\sin \frac{5\pi}{6}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6},$$

not $\frac{5\pi}{6}$, because $\frac{5\pi}{6}$ is not in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Now let's look at the cosine graph and its' inverse:

Graph [0, 2Π) : $f(t) = \cos t$

t	f(t)
0	1
$\frac{\pi}{2}$	0
π	−1
$\frac{3\pi}{2}$	0
2π	1

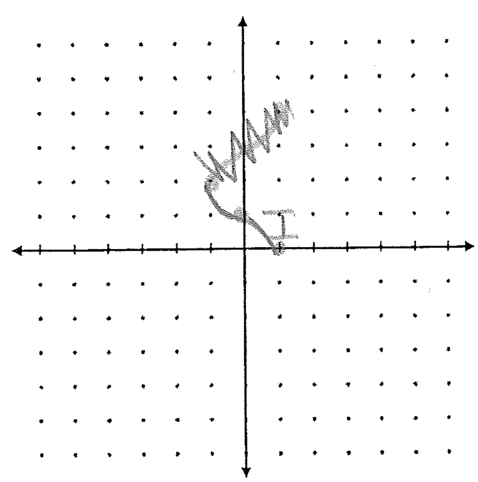


now graph the inverse of $f(t) = \cos t$

Rewrite the function:

Resolve for y:

t	f(t)
1	0
0	π/2
−1	π
0	$\frac{3\pi}{2}$
1	2π



You have seen that the graphs of trigonometric functions do not have this property. However, restricting their domains can modify the trigonometric functions so that they do have inverse functions.

Again we have to restrict the Domain in order to keep it a function:

Because the graph of the restricted cosine function passes the horizontal line test, it has an inverse function. This inverse function is called the **inverse cosine (or arccosine) function** and is denoted by

$$g(x) = \cos^{-1} x \text{ or } g(x) = \arccos x.$$

Restrict the domain: [ ]

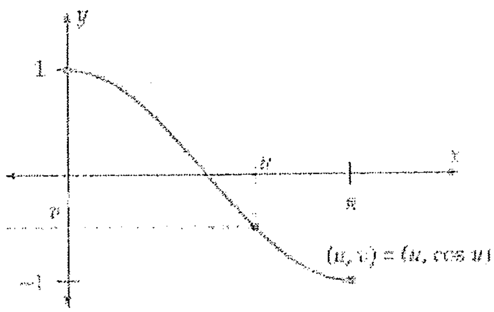


Figure 8.2-3

This is the regular cosine graph with the restricted domain

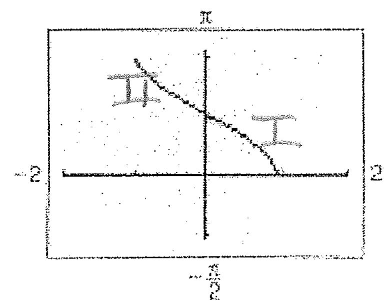


Figure 8.2-4

This is the inverse of cosine graph

The domain of $g(x) = \cos^{-1} x$ is the interval $[-1, 1]$ and its range is $[0, \pi]$.

YOU WILL BE IN QUADRANTS I OR II ONLY FOR INVERSE COSINE!!!!!

Examples:

1. $\arccos(-\frac{\sqrt{3}}{2}) =$

2. $\cos^{-1}(\frac{1}{2}) =$

3. $\arccos(-1) =$

4. $\cos(\cos^{-1}(-\frac{1}{2})) =$

5. $= \cos(\arccos(\frac{\sqrt{2}}{2}))$

6. $\sin(\cos^{-1}(\frac{\sqrt{3}}{2})) =$

13. $\cos^{-1}(\cos(\frac{\pi}{3})) =$

14. $\cos^{-1}(\cos(\frac{2\pi}{3})) =$

15. $\cos^{-1}(\cos(\frac{5\pi}{6})) =$

Properties of Inverse Cosine

$$\cos^{-1}(\cos u) = u \quad \text{if} \quad 0 \le u \le \pi$$
$$\cos(\cos^{-1} v) = v \quad \text{if} \quad -1 \le v \le 1$$

Inverse Tangent Function

The *restricted tangent function* is $f(x) = \tan x$, when its domain is restricted to the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Its graph in Figure 8.2-6 shows that for every real number v, there is exactly *one* number u between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ such that $\tan u = v$.

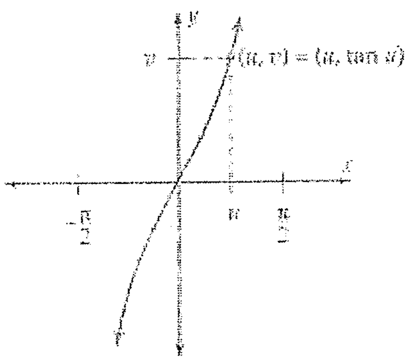


Figure 8.2-6

This is the regular tan graph

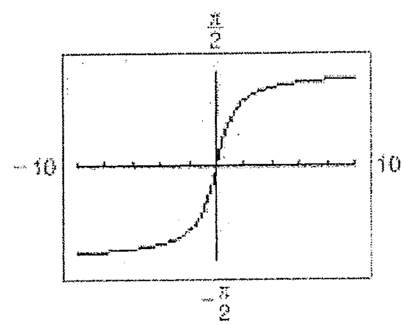


Figure 8.2-7

This is the inverse tan graph

Because the graph of the restricted tangent function passes the horizontal line test, it has an inverse function. This inverse function is called the **inverse tangent (or arctangent) function** and is denoted

$$g(x) = \tan^{-1} x \text{ or } g(x) = \arctan x.$$

The domain of $g(x) = \tan^{-1} x$ is the set of all real numbers and its range is the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Properties of Inverse Tangent

$$\tan^{-1}(\tan u) = u \quad \text{if} \quad -\frac{\pi}{2} \le u \le \frac{\pi}{2}$$
$$\tan(\tan^{-1} v) = v \text{ for every real number } v.$$

YOU WILL BE IN QUADRANTS I OR II ONLY FOR INVERSE TAN!!!!!

Examples:

1. $\arctan\sqrt{3} =$

2. $\tan^{-1}(\frac{\sqrt{3}}{3}) =$

3. $\arctan(-1) =$

4. $(\tan(\tan^{-1}(-\sqrt{3})) =$

5. $\tan(\arctan(1)) =$

6. $\sin(\tan^{-1}(-\frac{\sqrt{3}}{3})) =$

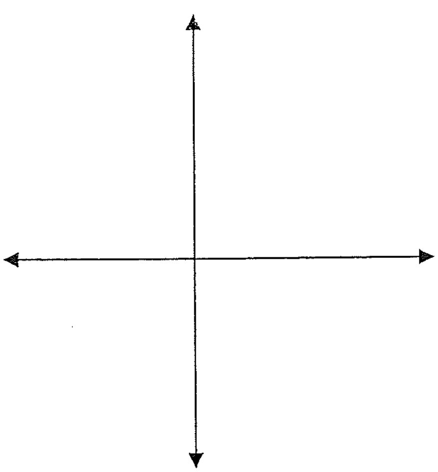
7. $\tan^{-1}(\tan(\frac{\pi}{3})) =$

8. $\tan^{-1}(\tan(\frac{2\pi}{3})) =$

9. $\tan^{-1}(\tan(\frac{3\pi}{4}))$

Example 4 Equivalent Algebraic Expressions

Write $\sin(\cos^{-1} v)$ as an algebraic expression in v.



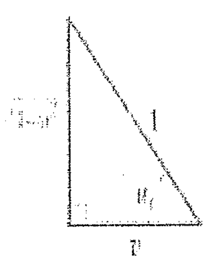
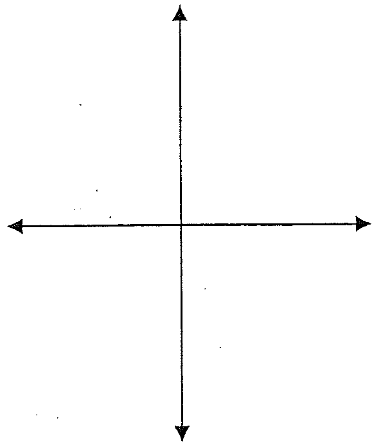


Figure 8.2-5

Example 5 Evaluating Inverse Tangent Expressions

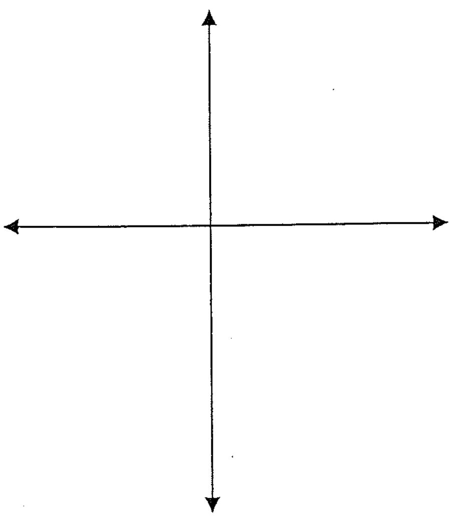
Evaluate:

$\tan^{-1}(\tan(-\frac{2}{5}))$



Example 6 Exact Values

Find the exact value of $\cos\left(\tan^{-1}\frac{\sqrt{5}}{2}\right)$.



Using a Calculator, evaluate the following:

7. $\arcsin(-.53) =$

8. $\arccos(.28) =$

9. $\arctan(5.2) =$

10. $\arccos(\sin(.456)) =$

11. $\sin(\tan^{-1}(-9.8))$

PreCalc Homework 8.2 odds

Name______________________

1. $\arcsin\frac{\sqrt{3}}{2}=$

2. $\sin^{-1}(-\frac{1}{2})=$

3. $\arcsin(1)=$

4. $\arcsin(0)=$

5. $\sin^{-1}(-1)=$

6. $\arcsin(-\frac{\sqrt{2}}{2})=$

7. $(\sin(\sin^{-1}(-\frac{1}{2}))$

8. $\cos(\arcsin(-\frac{\sqrt{2}}{2}))=$

9. $\sin(\sin^{-1}(\frac{\sqrt{2}}{2}))=$

10. $\sin(\sin^{-1}(\frac{\sqrt{3}}{2}))=$

11. $\sin(\sin^{-1}(-1))=$

12. $\sin(\arcsin(0))=$

13. $\sin^{-1}(\sin(\frac{\pi}{6}))=$

14. $arc\sin(\sin(\frac{5\pi}{3}))=$

15. $\sin^{-1}(\sin(\frac{3\pi}{4}))=$

1. $\arccos\frac{\sqrt{3}}{2}=$

2. $\cos^{-1}(-\frac{1}{2})=$

3. $\arccos(1)=$

4. $\arccos(0)=$

5. $\cos^{-1}(-1)=$

6. $\arccos(-\frac{\sqrt{2}}{2})=$

7. $(\cos(\cos^{-1}(-\frac{1}{2}))$ =

8. $\cos(\arccos(-\frac{\sqrt{2}}{2}))=$

9. $\sin(\cos^{-1}(\frac{\sqrt{2}}{2}))=$

10. $\sin(\cos^{-1}(\frac{\sqrt{3}}{2}))=$

11. $\cos(\cos^{-1}(-1))=$

12. $\cos(\cos^{-1}(0))=$

13. $\cos^{-1}(\cos(\frac{\pi}{6}))=$

14. $\cos^{-1}(\cos(\frac{5\pi}{3}))=$

15. $\cos^{-1}(\cos(\frac{3\pi}{4}))=$

1. $\arctan\frac{\sqrt{3}}{2}=$

2. $\tan^{-1}(-\frac{\sqrt{3}}{3})=$

3. $\arctan(1)=$

4. $\arctan(0)=$

5. $\tan^{-1}(-1)=$

6. $\arctan(-\sqrt{3})=$

7. $(\tan(\tan^{-1}(\sqrt{3}))=$

8. $\tan(\arctan(-1))=$

9. $\sin(\tan^{-1}(\frac{\sqrt{3}}{3}))=$

10. $\sin(\tan^{-1}(-1))=$

11. $\tan(\tan^{-1}(-1))=$

12. $\tan(\tan^{-1}(0))=$

13. $\tan^{-1}(\tan(\frac{\pi}{6})) =$

14. $\tan^{-1}(\tan(\frac{5\pi}{3})) =$

15. $\tan^{-1}(\tan(\frac{3\pi}{4}))$

1. $\sin(\arccos v) =$

2. $\tan(\arccos u) =$

3. $\cos(\arctan\frac{\sqrt{7}}{2})$

4. $\arctan(\tan(-\frac{5\pi}{6}))$

6. $\tan(\arccos(-\frac{7}{9}))$

7. $\cot(\arctan(-\frac{2}{5}))$

Use a calculator to evaluate the following:

7. $\arccos(-.63) =$

8. $\arcsin(-.28) =$

9. $\arctan(-4.2) =$

10. $\arcsin(\sin(-5)) =$

11. $\arctan(\tan(9.8))$