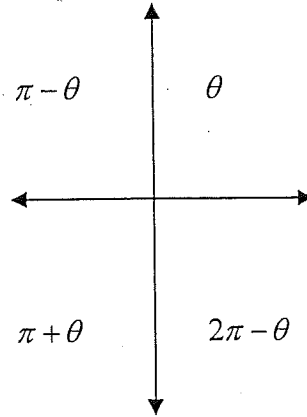
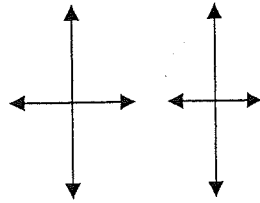
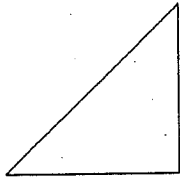
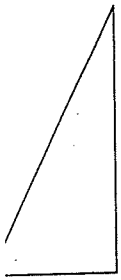


## Algebraic Solutions of Trigonometric Equations



$$\left. \begin{array}{l} \sin \\ \cos \end{array} \right\} \text{add } \pm 2k\pi$$

$$\{\tan\} \text{add } \pm k\pi$$

Sin=      cos=      tan=

**Example 5** Solving Basic Equations with Special Values

Solve  $\sin u = \frac{\sqrt{2}}{2}$  exactly, without using a calculator.

These are like Example 5.

In Exercises 1–8, find the exact solutions.

1.  $\sin x = \frac{\sqrt{3}}{2}$

2.  $2 \cos x = \sqrt{2}$

3.  $\tan x = -\sqrt{3}$

4.  $\tan x = 1$

5.  $2 \cos x = -\sqrt{3}$

6.  $\sin x = 0$

7.  $2 \sin x + 1 = 0$

8.  $\csc x = \sqrt{2}$

**Example 6** Using Substitution and Basic Equations

Solve  $\sin 2x = \frac{\sqrt{2}}{2}$  exactly, without using a calculator.

23.  $\sin 2x = -\frac{\sqrt{3}}{2}$

24.  $\cos 2x = \frac{\sqrt{2}}{2}$

25.  $2 \cos \frac{x}{2} = \sqrt{2}$

26.  $2 \sin \frac{x}{3} = 1$

**Example 1** Solving Basic Cosine EquationsSolve  $\cos x = 0.6$ .**Example 2** Solving Basic Sine EquationsSolve  $\sin x = -0.75$ .**Example 3** Solving Basic Tangent EquationsSolve  $\tan x = 3$ .**Example 4** Using the Solution AlgorithmSolve  $8 \cos x - 1 = 0$ .

These are like Examples 1-4.

In Exercises 13-32, find all the solutions of each equation.

13.  $\sin x = -0.465$

15.  $\cos x = -0.564$

17.  $\tan x = -0.237$

19.  $\cot x = 2.3$  [Hint:  $\cot x = \frac{1}{\tan x}$ ]

21.  $\sec x = -2.65$

22.  $\csc x = 5.27$

28.  $5 \sin 2x = 2$

29.  $5 \cos 3x = -3$

30.  $2 \tan 4x = 16$

31.  $4 \tan \frac{x}{2} = 8$

**Example 7****Factoring Trigonometric Equations**

Find the solutions of  $3 \sin^2 x - \sin x - 2 = 0$  in the interval  $[-\pi, \pi]$ .

In Exercises 33–53, use factoring, the quadratic formula, or identities to solve the equation. Find all solutions in the interval  $[0, 2\pi)$ .

33.  $3 \sin^2 x - 8 \sin x - 3 = 0$

34.  $5 \cos^2 x + 6 \cos x = 8$

$$35. 2 \tan^2 x + 5 \tan x + 3 = 0$$

$$36. 3 \sin^2 x + 2 \sin x = 5$$

**Example 6****Factoring Trigonometric Equations**

Solve  $\tan x \cos^2 x = \tan x$ .

$$37. \cot x \cos x = \cos x$$

$$38. \tan x \cos x = \cos x$$

$$39. \cos x \csc x = 2 \cos x$$

$$41. 4 \sin x \tan x - 3 \tan x + 20 \sin x - 15 = 0$$

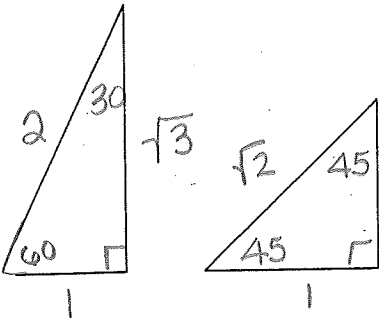
Hint: One factor is  $\tan x + 5$ .

$$42. 25 \sin x \cos x - 5 \sin x + 20 \cos x = 4$$

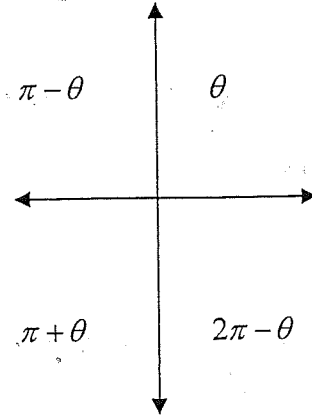
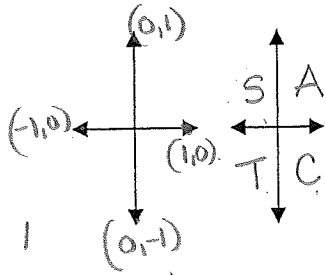
$$43. \sin^2 x + 2 \sin x - 2 = 0$$

$$44. \cos^2 x + 5 \cos x = 1$$

# Algebraic Solutions of Trigonometric Equations



$$\sin = \frac{y}{r} \quad \cos = \frac{x}{r} \quad \tan = \frac{y}{x}$$

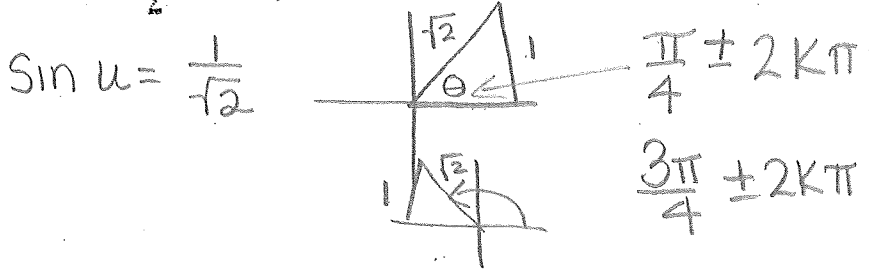


$$\left. \begin{matrix} \sin \\ \cos \end{matrix} \right\} \text{add } \pm 2k\pi$$

$$\{\tan\} \text{add } \pm k\pi$$

## Example 5 Solving Basic Equations with Special Values

Solve  $\sin u = \frac{\sqrt{2}}{2}$  exactly, without using a calculator.



These are like Example 5.

In Exercises 1-8, find the exact solutions.

1.  $\sin x = \frac{\sqrt{3}}{2}$

2.  $\cos x = \frac{\sqrt{2}}{2}$

3.  $\tan x = -\sqrt{3}$

4.  $\tan x = 1$

5.  $\frac{1}{2} \cos x = -\frac{\sqrt{3}}{2}$


6.  $\sin x = 0$

7.  $2 \sin x + 1 = 0$

8.  $\csc x = \sqrt{2}$

**Example 6** Using Substitution and Basic Equations

Solve  $\sin(2x) = \frac{\sqrt{2}}{2}$  exactly, without using a calculator.

$\sin x = \frac{1}{\sqrt{2}}$  

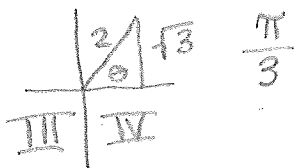
1.  $\frac{\pi}{4} \pm \frac{2k\pi}{2} = \frac{2x}{2}$

$\frac{\pi}{8} \pm k\pi = x$

1.  $\frac{3\pi}{4} \pm \frac{2k\pi}{2} = \frac{2x}{2}$

$\frac{3\pi}{8} \pm k\pi = x$

23.  $\sin 2x = \frac{\sqrt{3}}{2}$



1.  $\frac{4\pi}{3} \pm 2k\pi = \frac{2x}{2}$

$\frac{2\pi}{3} \pm k\pi = x$

1.  $\frac{5\pi}{3} \pm 2k\pi = \frac{2x}{2}$

$\frac{5\pi}{6} \pm k\pi = x$

24.  $\cos 2x = \frac{\sqrt{2}}{2}$

1.  $\frac{\pi}{4} \pm \frac{2k\pi}{2} = \frac{2x}{2}$

$\frac{\pi}{8} \pm k\pi = x$

1.  $\frac{7\pi}{4} \pm 2k\pi = \frac{2x}{2}$

$\frac{7\pi}{8} \pm k\pi = x$

25.  $2 \cos\left(\frac{x}{2}\right) = \sqrt{2}$

$\cos \frac{x}{2} = \frac{\sqrt{2}}{2}$

2.  $\frac{\pi}{4} \pm 2k\pi = \frac{x}{2} \cdot 2$

$\frac{\pi}{2} \pm 4k\pi = x$

2.  $\frac{7\pi}{4} \pm 2k\pi = \frac{x}{2} \cdot 2$

$\frac{7\pi}{2} \pm 4k\pi = x$

26.  $2 \sin \frac{x}{3} = 1$

$\sin \frac{x}{3} = \frac{1}{2}$



$\frac{\pi}{6} \pm 2k\pi = \frac{x}{3}$

$\frac{\pi}{2} \pm 6k\pi = x$

3.  $\frac{5\pi}{6} \pm 2k\pi = \frac{x}{3}$

$\frac{5\pi}{2} \pm 6k\pi = x$



# Calc in radian

## Example 1 Solving Basic Cosine Equations

Solve  $\cos x = 0.6$ .

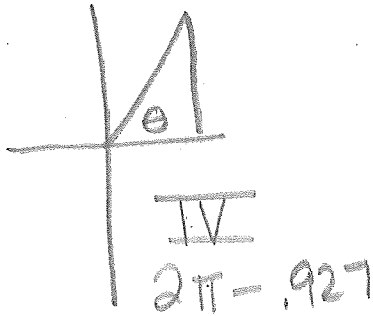
$$\cos^{-1}(.6)$$

$$\theta = .927$$

$$5.356$$

$$\pm 2k\pi$$

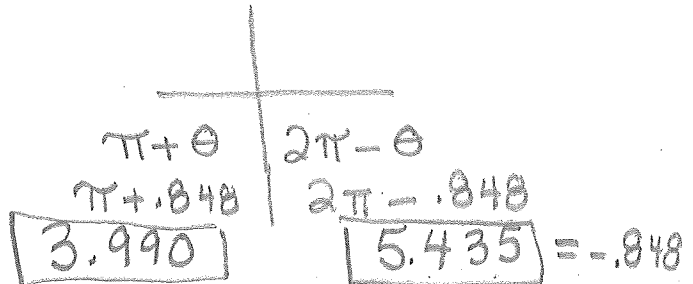
$$.927 \pm 2k\pi$$



## Example 2 Solving Basic Sine Equations

Solve  $\sin x = -0.75$ .

$$\sin^{-1}(.75) = .848$$



## Example 3 Solving Basic Tangent Equations

Solve  $\tan x = 3$ .

$$\tan^{-1}(3) = \underline{\underline{1.249}}$$

$$1.249 \pm k\pi$$

$$(1.249 + \pi)$$



## Example 4 Using the Solution Algorithm

Solve  $8 \cos x - 1 = 0$ .

$$\frac{8 \cos x}{8} = \frac{1}{8}$$

$$1.445 \pm 2k\pi$$

$$-1.445 \pm 2k\pi$$

$$\cos x = \frac{1}{8}$$

$$\text{2nd } \cos(1/8) \quad 2\pi - 1.445$$

$$\cos^{-1}(1/8) = 1.445$$

These are like Examples 1-4.

In Exercises 13-32, find all the solutions of each equation.

13.  $\sin x = -0.465$

$$\sin^{-1}(.465) = .484$$



$$\pi + .484$$

$$2\pi - .484$$

$$\boxed{3.626, 5.80}$$

15.  $\cos x = -0.564$

$$\theta = .972$$



$$\pi - .972$$

$$\boxed{2.170}$$

$$\pi + .972$$

$$\boxed{4.114}$$

17.  $\tan x = -0.237$

$$\theta = .233$$

$$\pi - .233 = \boxed{2.909}$$

$$2\pi - .233$$

$$\boxed{-.233}$$

19.  $\cot x = 2.3$  [Hint:  $\cot x = \frac{1}{\tan x}$ ]

$$\frac{1}{\cot x} = \frac{1}{2.3}$$

$$\tan x = .435$$

$$.410 \pm k\pi$$

21.  $\sec x = -2.65$

$$\frac{1}{\sec x} = \frac{1}{-2.65}$$

$$\cos x = .377$$

$$\theta = 1.184$$



$$\pi - 1.184$$

$$\pi + 1.184$$

22.  $\csc x = 5.27$

$$\sin x = \frac{1}{5.27}$$

$$\theta = .191$$

$$\pi - .191 = \boxed{2.951}$$

28.  $5 \sin 2x = 2$

29.  $5 \cos 3x = -3$

30.  $2 \tan 4x = 16$

31.  $4 \tan \frac{x}{2} = 8$

**Example 7** Factoring Trigonometric Equations

Find the solutions of  $3 \sin^2 x - \sin x - 2 = 0$  in the interval  $[-\pi, \pi]$ .

In Exercises 33–53, use factoring, the quadratic formula, or identities to solve the equation. Find all solutions in the interval  $[0, 2\pi)$ .

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34.  $5 \cos^2 x + 6 \cos x = 8$

$$35. 2 \tan^2 x + 5 \tan x + 3 = 0$$

$$36. 3 \sin^2 x + 2 \sin x = 5$$

**Example 8****Factoring Trigonometric Equations**

Solve  $\tan x \cos^2 x = \tan x$ .

$$37. \cot x \cos x = \cos x$$

$$38. \tan x \cos x = \cos x$$

$$39. \cos x \csc x = 2 \cos x$$

$$41. 4 \sin x \tan x - 3 \tan x + 20 \sin x - 15 = 0$$

Hint: One factor is  $\tan x + 5$ .

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