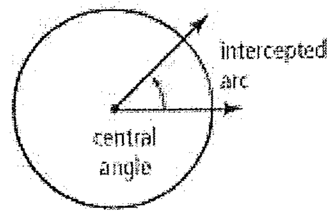
13-3 Radian Measure

Content Standard

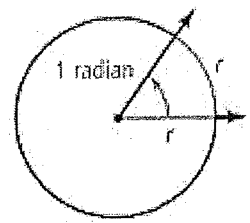
F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Objectives To use radian measure for angles
To find the length of an arc of a circle

A **central angle** of a circle is an angle with a vertex at the center of a circle. An **intercepted arc** is the portion of the circle with endpoints on the sides of the central angle and remaining points within the interior of the angle.



A **radian** is the measure of a central angle that intercepts an arc with length equal to the radius of the circle. Radians, like degrees, measure the amount of rotation from the initial side to the terminal side of an angle.



Essential Understanding An angle with a full circle rotation measures 2π radians. An angle with a semicircle rotation measures π radians.

take note

Key Concept Proportion Relating Radians and Degrees

You can use the proportion $\frac{d^\circ}{180^\circ} = \frac{r \text{ radians}}{\pi \text{ radians}}$ to convert between radians and degrees.

Here's Why It Works

Because the circumference of a circle is $2\pi r$, there are 2π radians in any circle. Since 2π radians $= 360^\circ$, it follows that π radians $= 180^\circ$. This equality leads to the following *conversion factors* for converting between radian measure and degree measure.

take note

Key Concept Converting Between Radians and Degrees

To convert degrees to radians, multiply by $\frac{\pi \text{ radians}}{180^\circ}$.

To convert radians to degrees, multiply by $\frac{180^\circ}{\pi \text{ radians}}$.

You can use the conversion factors and dimensional analysis to convert between angle measurement systems.

$\frac{180^\circ}{\pi}$ or $\frac{\pi}{180^\circ}$ $\quad 180^\circ = \pi$ radians

Problem 1 Using Dimensional Analysis

A What is the degree measure of an angle of $-\frac{3\pi}{4}$ radians?

$$-\frac{3\cancel{\pi}}{\cancel{4}} \cdot \frac{\overset{45}{\cancel{180^\circ}}}{\cancel{\pi}} = -135^\circ$$

$$\begin{array}{r} -135^\circ \\ +360 \\ \hline 225^\circ \end{array}$$

B What is the radian measure of an angle of 27°?

$$\frac{\overset{3}{\cancel{27^\circ}}}{1} \cdot \frac{\pi}{\underset{20}{\cancel{180^\circ}}} = \frac{3\pi}{20} \text{ radians}$$

Got It? 1. What is the degree measure of each angle expressed in radians? What is the radian measure of each angle expressed in degrees? (Express radian measures in terms of π.)

a. $\frac{\pi}{2}$ radians **b.** 225° ~~**c.** 2 radians~~ **d.** 150°

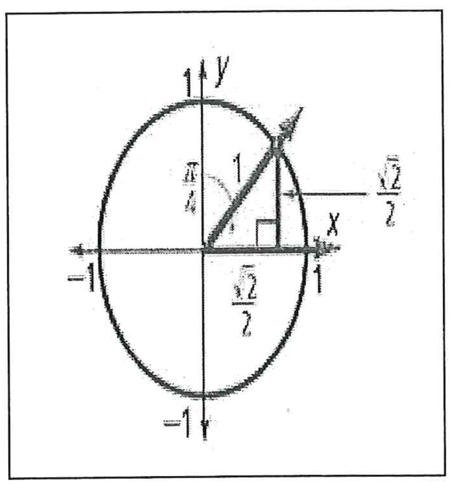
a. $\frac{\cancel{\pi}}{\cancel{2}} \cdot \frac{\overset{90}{\cancel{180}}}{\cancel{\pi}}$ $\quad 90^\circ$

b. $225^\circ \cdot \frac{\pi}{180^\circ}$ $\quad \frac{5}{4}\pi$

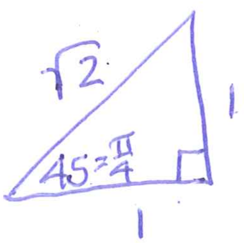
d. $\overset{5}{\cancel{150^\circ}} \cdot \frac{\pi}{\underset{6}{\cancel{180^\circ}}}$ $\quad \frac{5\pi}{6}$

Problem 2 Finding Cosine and Sine of a Radian Measure

What are the exact values of $\cos\left(\frac{\pi}{4}\text{ radians}\right)$ and $\sin\left(\frac{\pi}{4}\text{ radians}\right)$?



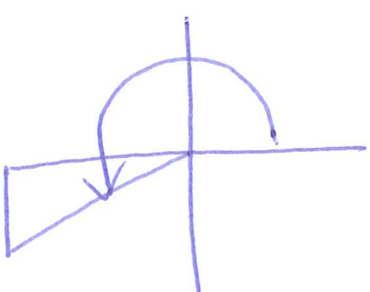
$\cos 45^\circ = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

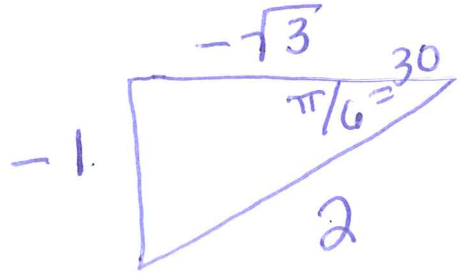


$\sin\frac{\pi}{4} = \sin 45^\circ = \frac{1}{\sqrt{2}}$

$\tan\frac{\pi}{4} = \tan 45 = \frac{1}{1}$

Got It? 2. What are the exact values of $\cos\left(\frac{7\pi}{6}\text{ radians}\right)$ and $\sin\left(\frac{7\pi}{6}\text{ radians}\right)$?





$\sin\frac{7\pi}{6} = -\frac{1}{2}$

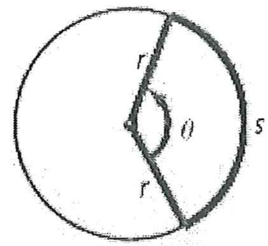
$\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

$\tan\frac{7\pi}{6} = \frac{1}{\sqrt{3}}$

take note

Key Concept Length of an Intercepted Arc

For a circle of radius r and a central angle of measure θ (in radians), the length s of the intercepted arc is $s = r\theta$.



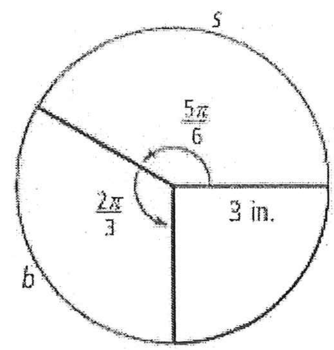
Here's Why It Works The length of the intercepted arc is the same fraction of the circumference of the circle as the central angle is of 2π. So, $\frac{\theta}{2\pi} = \frac{s}{C}$. Since $C = 2\pi r$, then $\frac{\theta}{2\pi} = \frac{s}{2\pi r}$. This simplifies to $\theta = \frac{s}{r}$. Multiplying by r results in $s = r\theta$.

Problem 3 Finding the Length of an Arc

Use the circle at the right. What is length s to the nearest tenth?

$s = r\theta$	Use the formula.
$= 3 \cdot \frac{5\pi}{6}$	Substitute 3 for r and $\frac{5\pi}{6}$ for θ.
$= \frac{5\pi}{2}$	Simplify.
≈ 7.9	Use a calculator.

The arc has a length of about 7.9 in.



Got It? 3. **a.** What is length b in Problem 3 to the nearest tenth?

b. Reasoning If the radius of the circle doubled, how would the arc length change?

Practice and Problem-Solving Exercises

MATHEMATICAL PRACTICES

$\frac{\pi}{180}$

A Practice Write each measure in radians. Express your answer in terms of π and as a decimal rounded to the nearest hundredth. See Problem 1.

6. $-300°$

7. $150°$

8. $-90°$

9. $-60°$

10. $160°$

11. $20°$

$\frac{180}{\pi}$

Write each measure in degrees. Round your answer to the nearest degree, if necessary.

12. 3π radians

13. $\frac{11\pi}{10}$ radians

14. $-\frac{2\pi}{3}$ radians

15. -3 radians

-3π radians

~~16. 1.57 radians~~

~~17. 4.71 radians~~

The measure θ of an angle in standard position is given. Find the exact values of $\cos\theta$ and $\sin\theta$ for each angle measure.

See Problem 2.

18. $\frac{\pi}{6}$ radians

19. $\frac{\pi}{3}$ radians

20. $\frac{\pi}{2}$ radians

21. $-\frac{\pi}{4}$ radians

22. $\frac{2\pi}{3}$ radians

23. $-\frac{\pi}{2}$ radians

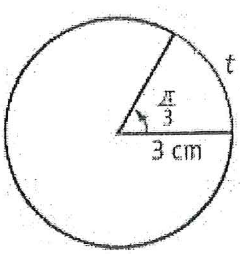
24. $\frac{5\pi}{4}$ radians

25. $\frac{7\pi}{6}$ radians

Use each circle to find the length of the indicated arc. Round your answer to the nearest tenth.

See Problem 3.

26.



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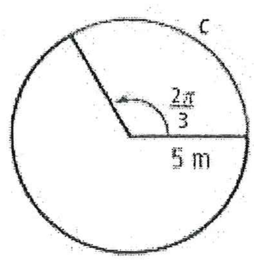

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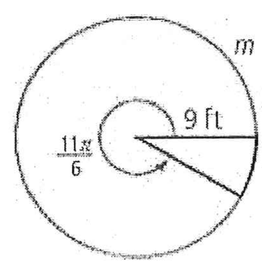

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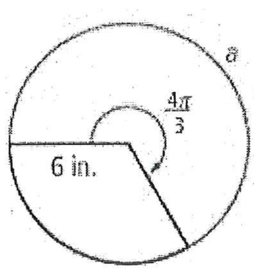

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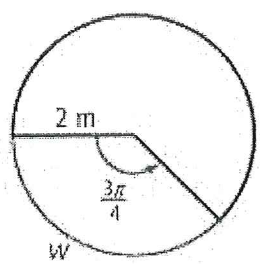

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