

# 8-6

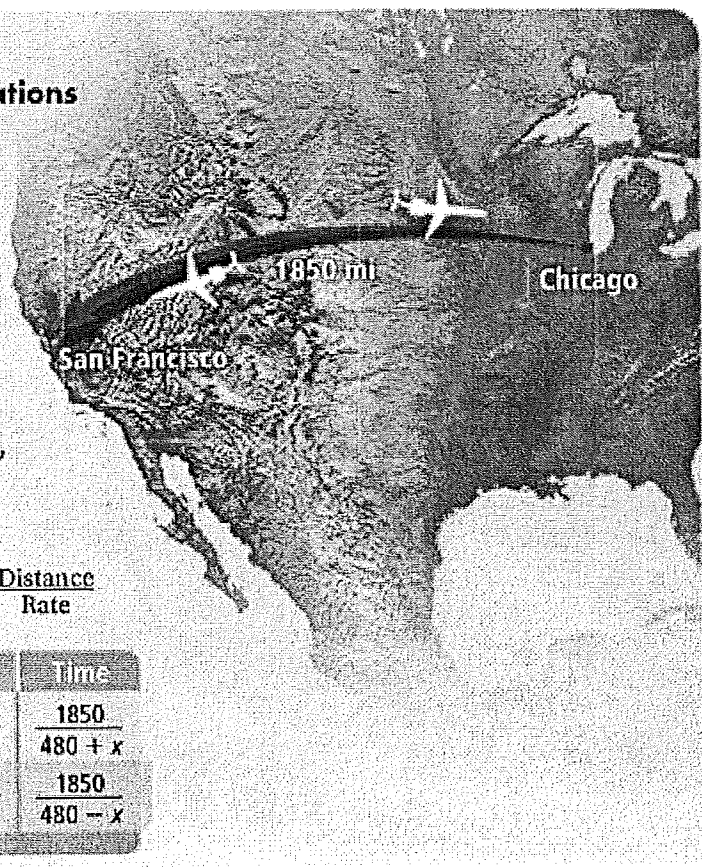
## Solving Rational Equations

© Content Standards  
 A.APR.7 ... Add, subtract, multiply, and divide rational expressions.  
 Also A.APR.6, A.CED.1, A.REI.11

**Objectives** To solve rational equations  
 To use rational equations to solve problems

### Problem 2 Using Rational Equations

**Flight** A flight across the U.S. takes longer east to west than it does west to east. Assume that winds are constant in the eastward direction. When flying westward, the headwind decreases the airplane's speed. When flying eastward, the tailwind increases its speed. The time for a round trip shown at the right is  $7\frac{3}{4}$  h. If the airplane cruises at 480 mi/h, what is the speed of the wind?



Let  $x$  = the wind speed.

Rate  $\times$  Time = Distance, so Time =  $\frac{\text{Distance}}{\text{Rate}}$

	Distance	Rate	Time
Going west to east	1850	$480 + x$	$\frac{1850}{480 + x}$
Going east to west	1850	$480 - x$	$\frac{1850}{480 - x}$

Total time = Time west to east + Time east to west

$$7.75 = \frac{1850}{480 + x} + \frac{1850}{480 - x}$$

Multiply both sides by the LCD,  $(480 + x)(480 - x)$ .

$$(480 + x)(480 - x)7.75 = (480 + x)(480 - x)\frac{1850}{480 + x} + (480 + x)(480 - x)\frac{1850}{480 - x}$$

$$7.75(480 + x)(480 - x) = 1850(480 - x) + 1850(480 + x)$$

$$1,785,600 - 7.75x^2 = 888,000 - 1850x + 888,000 + 1850x$$

$$-7.75x^2 = -9600$$

$$x^2 = \frac{-9600}{-7.75}$$

$$x \approx \pm 35$$

Wind speed is positive, so  $x \approx 35$ . The west-to-east wind speed is about 35 mi/h.

- Got It?** 2. a. You ride your bike to a store, 4 mi away, to pick up things for dinner. When there is no wind, you ride at 10 mi/h. Today your trip to the store and back took 1 hour. What was the speed of the wind today?
- b. **Reasoning** Explain why there is no difference between the travel time to and from the store when there is no wind.

Distance = rate x time

$$d = r \cdot t$$

$$\frac{d}{r} = t$$

TRIP	Distance	Rate	Time
To the store <i>Tailwind</i>	4	(R + W) 10 + W	$t_1$
Back home <i>Headwind</i>	4	(R - W) 10 - W	$t_2$

→

To the store :  $t_1 =$

→

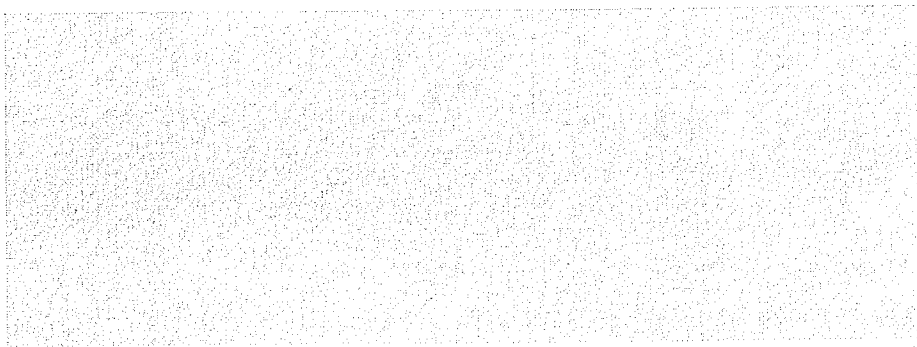
Back home :  $t_2 =$

If.... To the store + Back home = 1 hour

So.....  $t_1 + t_2 = 1$

Then.....

15. Now solve the equation.



16. The speed of the wind was about  mph.

Carpenter A can build a deck in 6 hours. Carpenter B can build the same deck in 8 hours. How long will it take the carpenters working together to build one deck?

Carpenter A: Can do \_\_\_\_\_ of the job in 1 hour working alone.

Carpenter B: Can do \_\_\_\_\_ of the job in 1 hour working alone.

So working together \_\_\_\_\_ + \_\_\_\_\_ = 1

One delivery driver can complete a route in 6 hours. Another driver can complete the same route in 5 hours. If the drivers work together, about how many hours will they take to complete the route?

Delivery Driver A: Can do \_\_\_\_\_ of the job in 1 hour working alone.

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So working together \_\_\_\_\_ + \_\_\_\_\_ = 1



### Problem 3 Using a Graphing Calculator to Solve a Rational Equation

What are the solutions of the rational equation? Use a graphing calculator to solve.


$$\frac{2}{x+2} + \frac{x}{x-2} = 1$$



**Got It?** 3. What are the solutions of the rational equation  $\frac{x+2}{1-2x} = 5$ ?

Use a graphing calculator to solve.

**4. Flight** If the speed of an airplane is 350 mi/h with a tail wind of 40 mi/h, what is the speed of the plane in still air?

**20. Transportation** The speed  $s$  of an airplane is given by  $s = \frac{d}{t}$ , where  $d$  represents the distance and  $t$  is the time.  **See Problem 2.**

- a. A plane flies 700 miles from New York to Chicago at a speed of 360 mi/h. Find the time for the trip.
- b. On the return trip from Chicago to New York, a tail wind helps the plane move faster. The total flying time for the round trip is 3.5 h. Find the speed  $x$  of the tail wind.

**37. Storage** One pump can fill a tank with oil in 4 hours. A second pump can fill the same tank in 3 hours. If both pumps are used at the same time, how long will they take to fill the tank?

**38. Teamwork** You can stuff envelopes twice as fast as your friend. Together, you can stuff 6750 envelopes in 4.5 hours. How long would it take each of you working alone to complete the job?

# 8-6

## Solving Rational Equations

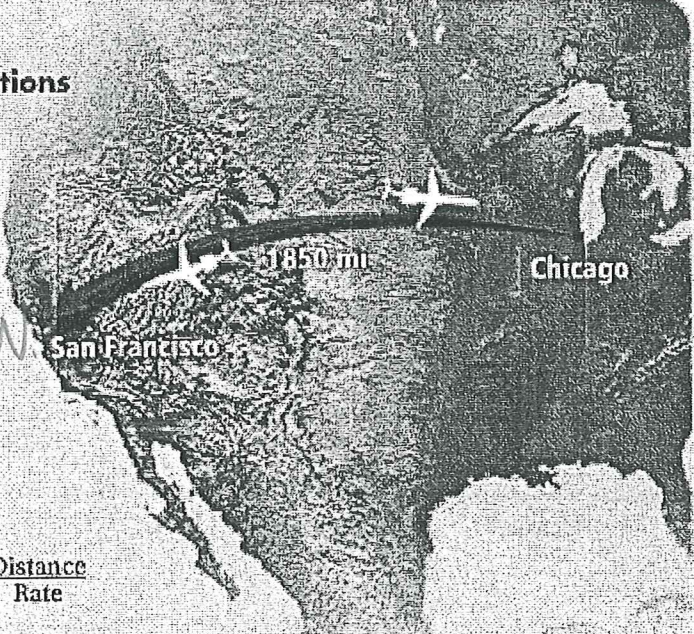
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$D = r \cdot t$   
 $\frac{D}{r} = t$

Total time = Time west to east + Time east to west

$$7.75 = \frac{1850}{480 + x} + \frac{1850}{480 - x}$$

Multiply both sides by the LCD,  $(480 + x)(480 - x)$ .

$$(480 + x)(480 - x)7.75 = (480 + x)(480 - x)\frac{1850}{480 + x} + (480 + x)(480 - x)\frac{1850}{480 - x}$$

$$7.75(480 + x)(480 - x) = 1850(480 - x) + 1850(480 + x)$$

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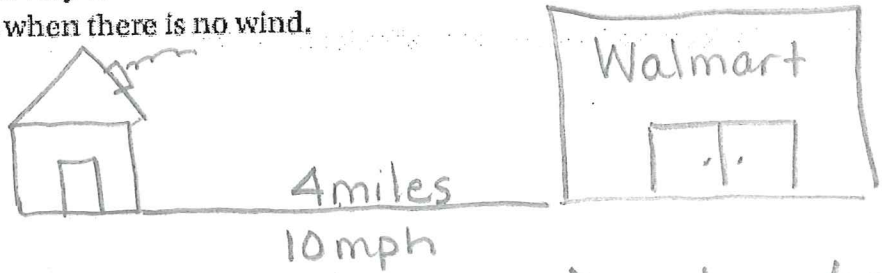
$$x = \pm 35$$

Wind speed is positive, so  $x = 35$ . The west-to-east wind speed is about 35 mi/h.

**Got It? 2. a.** You ride your bike to a store, 4 mi away, to pick up things for dinner. When there is no wind, you ride at 10 mi/h. Today your trip to the store and back took 1 hour. What was the speed of the wind today?

**b. Reasoning** Explain why there is no difference between the travel time to and from the store when there is no wind.

Distance = rate x time  
 $d = r \cdot t$   
 $\frac{d}{r} = t$



TRIP	Distance	Rate	Time
To the store <i>Tailwind</i>	4	$(R + W)$ $(10 + W)$	$t_1$
Back home <i>Headwind</i>	4	$(R - W)$ $10 - W$	$t_2$

To the store :  $t_1 = \frac{4}{10 + W}$

Back home :  $t_2 = \frac{4}{10 - W}$

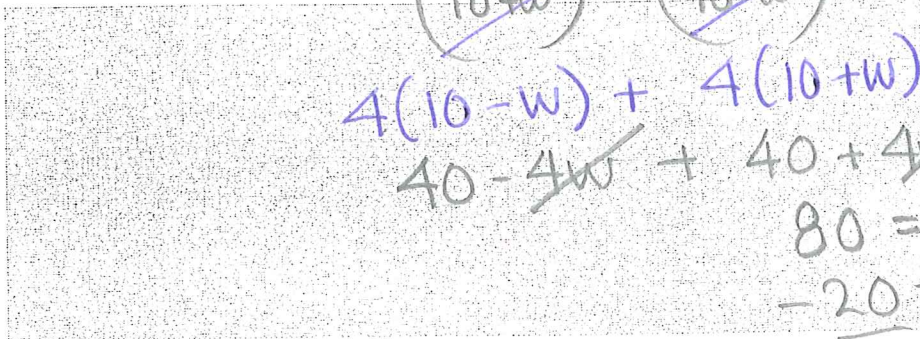
+ = 1

If... To the store + Back home = 1 hour

So.....  $t_1 + t_2 = 1$

Then.....  $\frac{4}{10 + W} + \frac{4}{10 - W} = 1$

15. Now solve the equation.



$(\frac{4}{10+W}) + (\frac{4}{10-W}) = 1$  (with  $(10+W)(10-W)$  written above)

$4(10 - W) + 4(10 + W) = (10 + W)(10 - W)$

$40 - 4W + 40 + 4W = 100 - W^2$

$80 = 100 - W^2$

$-20 = -W^2$

$\sqrt{20} = \sqrt{W^2}$

$\pm 4.5 = W$

16. The speed of the wind was about  mph.

4.5 mph

Carpenter A can build a deck in 6 hours. Carpenter B can build the same deck in 8 hours. How long will it take the carpenters working together to build one deck?

Carpenter A: Can do  $\frac{1}{6}x$  of the job in 1 hour working alone.

Carpenter B: Can do  $\frac{1}{8}x$  of the job in 1 hour working alone.

So working together  $\frac{1}{6}x + \frac{1}{8}x = 1$

$$\begin{aligned} \frac{1}{6}x + \frac{1}{8}x &= 1 \\ 24\left(\frac{1}{6}x\right) + 24\left(\frac{1}{8}x\right) &= 24(1) \\ 4x + 3x &= 24 \\ 7x &= 24 \\ \frac{7x}{7} &= \frac{24}{7} \\ x &\approx 3.4 \text{ hrs} \end{aligned}$$

One delivery driver can complete a route in 6 hours. Another driver can complete the same route in 5 hours. If the drivers work together, about how many hours will they take to complete the route?

Delivery Driver A: Can do  $\frac{1}{6}x$  of the job in 1 hour working alone.

Delivery Driver B: Can do  $\frac{1}{5}x$  of the job in 1 hour working alone.

So working together  $\frac{1}{6}x + \frac{1}{5}x = 1$

$$\begin{aligned} \frac{1}{6}x + \frac{1}{5}x &= 1 \\ 30\left(\frac{1}{6}x\right) + 30\left(\frac{1}{5}x\right) &= 30(1) \\ 5x + 6x &= 30 \\ 11x &= 30 \\ \frac{11x}{11} &= \frac{30}{11} \\ x &\approx 2.7 \text{ hrs} \end{aligned}$$

**Problem 3** Using a Graphing Calculator to Solve a Rational Equation

What are the solutions of the rational equation? Use a graphing calculator to solve.

$$\frac{2}{x+2} + \frac{x}{x-2} = 1$$

$$\begin{aligned} (x+2)(x-2)\left(\frac{2}{x+2}\right) + (x+2)(x-2)\left(\frac{x}{x-2}\right) &= (x+2)(x-2)(1) \\ 2x - 4 + x^2 + 2x &= x^2 - 4 \\ 4x &= 0 \quad x=0 \end{aligned}$$

**Got It?** 3. What are the solutions of the rational equation  $\frac{x+2}{1-2x} = 5$ ?

Use a graphing calculator to solve.

$$\begin{aligned} \frac{x+2}{1-2x} &= \frac{5}{1} \\ x+2 &= 5 - 10x \\ 11x &= 3 \\ x &= \frac{3}{11} \approx .27 \end{aligned}$$

4. **Flight** If the speed of an airplane is 350 mi/h with a tail wind of 40 mi/h, what is the speed of the plane in still air?

$$\begin{array}{r} r+w = 350 \\ r+40 = 350 \\ \underline{-40 \quad -40} \\ r = 310 \text{ mph} \end{array}$$

20. **Transportation** The speed  $s$  of an airplane is given by  $s = \frac{d}{t}$ , where  $d$  represents the distance and  $t$  is the time. **See Problem 2.**

- a. A plane flies 700 miles from New York to Chicago at a speed of 360 mi/h. Find the time for the trip.
- b. On the return trip from Chicago to New York, a tail wind helps the plane move faster. The total flying time for the round trip is 3.5 h. Find the speed  $x$  of the tail wind.

a)  $D = r \cdot t$   
 $\frac{700}{360} = \frac{360}{360} t$   
 $1.94 = t$

New York  $\rightarrow$  Chicago  
 1.94 hrs.  
 Time  $3.5 - 1.94 = 1.56$   
 $D = r \cdot t$   
 $700 = (360 + w)(1.56)$   
 $w = 89 \text{ mph}$

37. **Storage** One pump can fill a tank with oil in 4 hours. A second pump can fill the same tank in 3 hours. If both pumps are used at the same time, how long will they take to fill the tank?

$$12 \left[ \frac{1}{4}x + \frac{1}{3}x = 1 \right]$$

$$3x + 4x = 12$$

$$7x = 12$$

$$x = 1.7 \text{ hrs}$$

38. **Teamwork** You can stuff envelopes twice as fast as your friend. Together, you can stuff 6750 envelopes in 4.5 hours. How long would it take each of you working alone to complete the job?

$$x = 500$$

$$\frac{6750}{x} = 13.5$$

$$\frac{6750}{2x} = 6.75$$

$$2x(4.5) + x(4.5) = 6750$$

$$9x + 4.5x = 6750 \quad x = 500$$

$$\frac{6750}{x} + \frac{6750}{2x} = 4.5$$

$$6.75 \text{ hr.} \quad x = 500$$

$$13.5 \text{ hrs.}$$