

9-2

Arithmetic Sequences

Content Standard

F.1E.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Objective To define, identify, and apply arithmetic sequences

Essential Understanding In an *arithmetic sequence*, the difference between any two consecutive terms is always the same number. You can build an arithmetic sequence by adding the same number to each term.

An **arithmetic sequence** is a sequence where the difference between consecutive terms is constant. This difference is the **common difference**.

Take Note

Key Concept Arithmetic Sequence

An arithmetic sequence with a starting value a and common difference d is a sequence of the form

$$a, a + d, a + 2d, a + 3d, \dots$$

A recursive definition for this sequence has two parts:

$$a_1 = a \quad \text{initial condition}$$

$$a_n = a_{n-1} + d, \text{ for } n \geq 1 \quad \text{recursive formula}$$

An explicit definition for this sequence is a single formula:

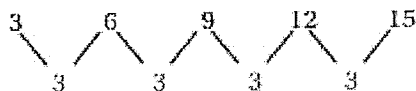
$$a_n = a + (n - 1)d, \text{ for } n \geq 1$$

Problem 1 Identifying Arithmetic Sequences

Is the sequence an arithmetic sequence?

A 3, 6, 9, 12, 15, ...

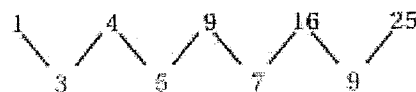
Find the differences between consecutive terms.



Each difference is 3. The sequence has a common difference. The sequence is an arithmetic sequence.

B 1, 4, 9, 16, 25, ...

Find the difference between consecutive terms.



There is no common difference. The sequence is not an arithmetic sequence.

Got It? 1. Is the sequence an arithmetic sequence?

a. 2, 4, 8, 16, ...

b. 1, 5, 9, 13, 17, ...



Problem 2 Analyzing Arithmetic Sequences

A What is the 100th term of the arithmetic sequence that begins 6, 11, ...?

The first term a is 6. The common difference d is $11 - 6 = 5$.

$$a_n = a + (n - 1)d \quad \text{Use the explicit formula.}$$

$$a_{100} = 6 + (100 - 1)5 \quad \text{Substitute 100 for } n, 6 \text{ for } a, \text{ and } 5 \text{ for } d.$$

$$a_{100} = 501 \quad \text{Simplify.}$$

The 100th term is 501.

B What are the second and third terms of the arithmetic sequence 100, \square , \square , 82, ...?

The first term a is 100. The fourth term a_4 is 82. There are 3 common differences between 100 and 82.

$$82 = 100 + 3d \quad \text{Add } 3d \text{ to move from } 100 \text{ to } 82.$$

$$-18 = 3d \quad \text{Solve for } d.$$

$$-6 = d$$

The common difference is -6 . The terms are 100, 94, 88, 82, ...

The second and third terms are 94 and 88.

- Got It?** 2. a. What is the 46th term of the arithmetic sequence that begins 3, 5, 7, ...?
 b. What are the second and third terms of this arithmetic sequence?
 80, \square , \square , 125, ...

The **arithmetic mean**, or average, of two numbers x and y is $\frac{x + y}{2}$.

In an arithmetic sequence, the middle term of any three consecutive terms is the arithmetic mean of the other two terms.



Problem 3 Using the Arithmetic Mean

GRIDDED RESPONSE

What is the missing term of the arithmetic sequence ..., 15, \square , 59, ...?

$$\text{arithmetic mean} = \frac{15 + 59}{2} = 37$$

The missing term is 37.

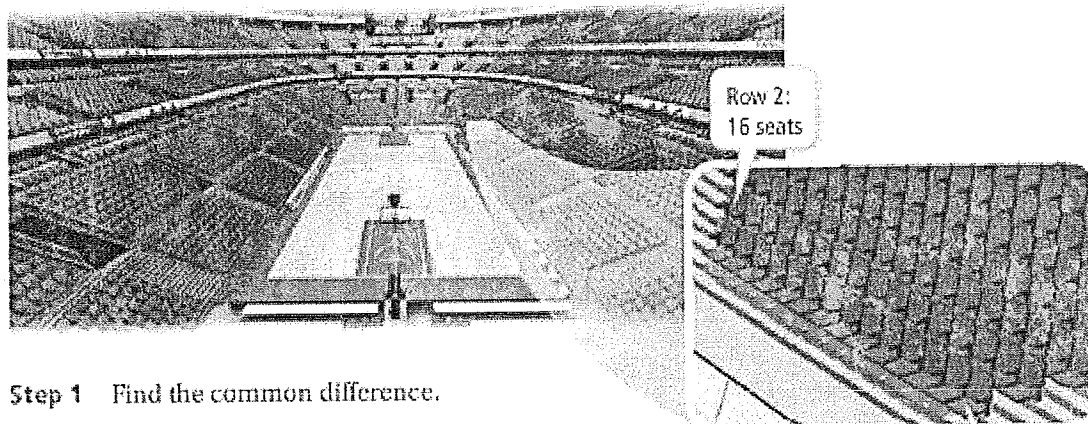


- Got It?** 3. a. The 9th and 11th terms of an arithmetic sequence are 132 and 98.
 What is the 10th term?
 b. **Reasoning** If you know the 5th and 6th terms of an arithmetic sequence, how can you find term 7 using the arithmetic mean?

37				
0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19
20	21	22	23	24
25	26	27	28	29
30	31	32	33	34
35	36	37	38	39
40	41	42	43	44
45	46	47	48	49
50	51	52	53	54
55	56	57	58	59
60	61	62	63	64
65	66	67	68	69
70	71	72	73	74
75	76	77	78	79
80	81	82	83	84
85	86	87	88	89
90	91	92	93	94
95	96	97	98	99

Problem 4 Using an Explicit Formula for an Arithmetic Sequence

Sports Arena The numbers of seats in the first 13 rows in a section of an arena form an arithmetic sequence. Rows 1 and 2 are shown in the diagram below. How many seats are in Row 13?



Step 1 Find the common difference.

$$d = a_2 - a_1 = 16 - 14 = 2$$

Step 2 Write an explicit formula for the arithmetic sequence.

$$a_n = a + (n - 1)d \quad \text{Use the explicit formula.}$$

$$\begin{aligned} a_{13} &= 14 + (13 - 1)2 && \text{Substitute 13 for } n, 14 \text{ for } a, \text{ and } 2 \text{ for } d. \\ &= 38 && \text{Simplify.} \end{aligned}$$

There are 38 seats in Row 13.

Got It? 4. The numbers of seats in the first 16 rows in a curved section of another arena form an arithmetic sequence. If there are 20 seats in Row 1 and 23 seats in Row 2, how many seats are in Row 16?

Determine whether each sequence is arithmetic. If so, identify the common difference.

7. 10, 20, 30, 40, ...

8. 1, 1, 2, 3, 5, 8, ...

9. -21, -18, -15, -12, ...

See Problem 1.

Find the 32nd term of each sequence.

13. 34, 37, 40, 43, ...

14. -9, -8.7, -8.4, ...

15. 23, 30, 37, 44, ...

See Problem 2.

Find the missing term of each arithmetic sequence.

◆ See Problem 3.

19. $-15, \square, 1, \dots$

20. $14, \square, 28, \dots$

21. $\dots, 5, \square, 21, \dots$

25. **Savings** A student deposits the same amount of money into her bank account each week. At the end of the second week she has \$30 in her account. At the end of the third week she has \$45 in her account. How much will she have in her bank account at the end of the ninth week? ◆ See Problem 4.

Find the 17th term of each sequence.

26. $a_{16} = 18, d = 5$

27. $a_{16} = 21, d = -3$

28. $a_{16} = -5, d = 12$

Find the arithmetic mean a_n of the given terms.

34. $a_{n-1} = 7, a_{n+1} = 1$

35. $a_{n-1} = 100, a_{n+1} = 140$

36. $a_{n-1} = 4, a_{n+1} = -3$

Write an explicit and a recursive formula for each sequence.

41. $2, 4, 6, 8, 10, \dots$

42. $0, 6, 12, 18, 24, \dots$

43. $-5, -4, -3, -2, -1, \dots$

44. $-4, -8, -12, -16, -20, \dots$

45. $-5, -3.5, -2, -0.5, 1, \dots$

46. $-32, -20, -8, 4, 16, \dots$

9-2

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A recursive definition for this sequence has two parts:

$a_1 = a$ initial condition

$a_n = a_{n-1} + d$, for $n \geq 1$ recursive formula

An explicit definition for this sequence is a single formula:

$$a_n = a_1 + (n - 1)d, \text{ for } n \geq 1$$

Problem 1 Identifying Arithmetic Sequences

Is the sequence an arithmetic sequence?

A 3, 6, 9, 12, 15, ...

yes!

Find the differences between consecutive terms.



Each difference is 3. The sequence has a common difference. The sequence is an arithmetic sequence.

B 1, 4, 9, 16, 25, ...

Find the difference between consecutive terms.



No!

There is no common difference. The sequence is not an arithmetic sequence.

Got It? 1. Is the sequence an arithmetic sequence?

a. 2, 4, 8, 16, ...

2+4+8

No!

b. 1, 5, 9, 13, 17, ...

+4+4

recursive
 $a_1 = 1$
 $a_n = a_{n-1} + 4$
 explicit
 $a_n = 1 + (n-1)4$

Problem 2 Analyzing Arithmetic Sequences

A What is the 100th term of the arithmetic sequence that begins 6, 11, ...?

The first term a is 6. The common difference d is $11 - 6 = 5$.

$$a_n = a + (n - 1)d \quad \text{Use the explicit formula.}$$

$$a_{100} = 6 + (100 - 1)5 \quad \text{Substitute 100 for } n, 6 \text{ for } a, \text{ and } 5 \text{ for } d.$$

$$a_{100} = 501 \quad \text{Simplify.}$$

The 100th term is 501.

B What are the second and third terms of the arithmetic sequence 100, \square , \square , 82, ...?

The first term a is 100. The fourth term a_4 is 82. There are 3 common differences between 100 and 82.

$$82 = 100 + 3d \quad \text{Add } 3d \text{ to move from } 100 \text{ to } 82.$$

$$-18 = 3d \quad \text{Solve for } d.$$

$$-6 = d$$

The common difference is -6 . The terms are 100, 94, 88, 82, ...

The second and third terms are 94 and 88.

a_{46}
3, 5, 7, ... ?

Got It? 2. a. What is the 46th term of the arithmetic sequence that begins 3, 5, 7, ...?

b. What are the second and third terms of this arithmetic sequence?

80, \square , \square , 125, ...

(1, 80) (4, 125)

95, 110

$$m = \frac{80 - 125}{1 - 4} = \frac{-45}{-3} = 15$$

$$a_n = a_1 + (n-1)d$$

$$a_{46} = 3 + (46-1)(2)$$

$$a_{46} = 93$$

The arithmetic mean, or average, of two numbers x and y is $\frac{x+y}{2}$.

In an arithmetic sequence, the middle term of any three consecutive terms is the arithmetic mean of the other two terms.

Problem 3 Using the Arithmetic Mean

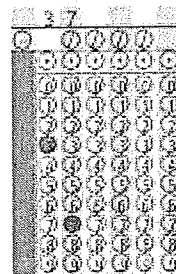
GRIDDED RESPONSE

What is the missing term of the arithmetic sequence ..., 15, \square , 59, ...?

$$\text{arithmetic mean} = \frac{15 + 59}{2} = 37$$

The missing term is 37.

$$\frac{15 + 59}{2} = \frac{74}{2} = 37$$



Got It? 3. a. The 9th and 11th terms of an arithmetic sequence are 132 and 98.

What is the 10th term?

b. Reasoning If you know the 5th and 6th terms of an arithmetic sequence, how can you find term 7 using the arithmetic mean?

$$132 \quad a_{10} \quad 98$$

$$\frac{132 + 98}{2} = \frac{230}{2} = 115$$

a_1 a_2
14, 16

Problem 4 Using an Explicit Formula for an Arithmetic Sequence

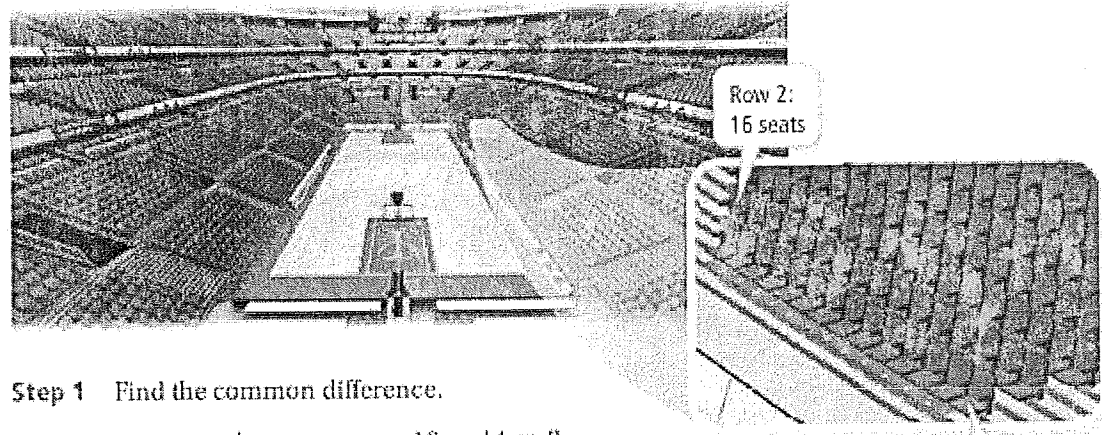
Sports Arena The numbers of seats in the first 13 rows in a section of an arena form an arithmetic sequence. Rows 1 and 2 are shown in the diagram below. How many seats are in Row 13?

$$a_{13} = a_1 + (n-1)d$$

$$= 14 + (13-1)2$$

$$= 14 + 24$$

$$a_{13} = 38$$



Step 1 Find the common difference.

$$d = a_2 - a_1 = 16 - 14 = 2$$

Step 2 Write an explicit formula for the arithmetic sequence.

$$a_n = a + (n - 1)d \quad \text{Use the explicit formula.}$$

$$a_{13} = 14 + (13 - 1)2 \quad \text{Substitute 13 for } n, 14 \text{ for } a, \text{ and } 2 \text{ for } d.$$

$$= 38 \quad \text{Simplify.}$$

There are 38 seats in Row 13.

Got It? 4. The numbers of seats in the first 16 rows in a curved section of another arena form an arithmetic sequence. If there are 20 seats in Row 1 and 23 seats in Row 2, how many seats are in Row 16?

20, 23 a_{16}

$$a_{16} = a_1 + (n-1)d$$

$$= 20 + (16-1)(3)$$

$$= 20 + (15)3$$

$$= 65$$

Determine whether each sequence is arithmetic. If so, identify the common difference.

7. 10, 20, 30, 40, ...
+10 +10
yes!
 $d=10$

8. 1, 1, 2, 3, 5, 8, ...
No!

9. -21, -18, -15, -12, ...
+3 +3 +3
yes
 $d=3$

See Problem 1.

$$a_1 = -21$$

$$a_n = a_{n-1} + 3$$

$$a_n = -21 + (n-1)(3)$$

$$= -21 + 3n - 3$$

$$a_n = 3n - 24$$

Find the 32nd term of each sequence.

13. 34, 37, 40, 43, ...

$$a_{32} = a_1 + (n-1)d$$

$$a_{32} = 34 + (32-1)3$$

$$a_{32} = 34 + (31)(3)$$

$$= 34 + 93$$

$$= 127$$

14. -9, -8.7, -8.4, ...

$$d = .3$$

$$a_{32} = a_1 + (n-1)d$$

$$= -9 + (32-1) \cdot 3$$

$$= .3$$

15. 23, 30, 37, 44, ...

$$a_n = a_1 + (n-1)d$$

$$a_n = 23 + (32-1)7$$

$$= 23 + (31)7$$

$$= 240$$

See Problem 2.

Find the missing term of each arithmetic sequence.

◆ See Problem 3.

19. $-15, \square, 1, \dots$

$$\frac{-15+1}{2} = \frac{-14}{2} = -7$$

20. $14, \square, 28, \dots$

$$\frac{14+28}{2} = \frac{42}{2} = 21$$

21. $\dots, 5, \square, 21, \dots$

$$\frac{5+21}{2} = \frac{26}{2} = 13$$

25. Savings A student deposits the same amount of money into her bank account each week. At the end of the second week she has \$30 in her account. At the end of the third week she has \$45 in her account. How much will she have in her bank account at the end of the ninth week? ◆ See Problem 4.

$$\begin{array}{ccc} a_1 & , & a_2 & a_3 & & a_9 \\ 15 & , & 30 & , & 45 & \end{array}$$

$$\begin{aligned} a_9 &= a_1 + (n-1)d \\ &= 15 + (9-1)15 \\ &= 15 + 8(15) \\ &= 135 \end{aligned}$$

Find the 17th term of each sequence.

26. $a_{16} = 18, d = 5$

$$\begin{array}{cc} a_{16} & a_{17} \\ 18 & \rightarrow 23 \\ +5 & \end{array}$$

27. $a_{16} = 21, d = -3$

$$\begin{array}{l} a_{16} = 21 \\ a_{17} = 18 \end{array}$$

28. $a_{18} = -5, d = 12$

$$\begin{array}{l} a_{18} = -5 \\ a_{17} = -17 \end{array}$$

Find the arithmetic mean a_n of the given terms.

34. $a_{n-1} = 7, a_{n+1} = 1$

$$\begin{array}{ccc} a_{n-1} & , & a_n & , & a_{n+1} \\ 7 & , & a_n & , & 1 \\ \frac{7+1}{2} & = & 4 \end{array}$$

35. $a_{n-1} = 100, a_{n+1} = 140$

$$\begin{array}{ccc} 100 & , & a_n & , & 140 \\ \frac{100+140}{2} & = & 120 \end{array}$$

36. $a_{n-1} = 4, a_{n+1} = -3$

$$\begin{array}{ccc} 4 & , & a_n & , & -3 \\ \frac{4+(-3)}{2} & = & \frac{1}{2} \end{array}$$

Write an explicit and a recursive formula for each sequence.

41. $2, 4, 6, 8, 10, \dots$

$$\begin{array}{l} R = a_1 = 2 \\ a_n = a_{n-1} + 2 \end{array}$$

$$\begin{array}{l} E = a_n = a_1 + (n-1)d \\ a_n = 2 + (n-1)2 \\ a_n = 2n \end{array}$$

42. $0, 6, 12, 18, 24, \dots$

$$\begin{array}{l} a_1 = 0 \\ a_n = a_{n-1} + 6 \end{array}$$

$$\begin{array}{l} a_n = 0 + (n-1)6 \\ a_n = 6n - 6 \end{array}$$

43. $-5, -4, -3, -2, -1, \dots$

$$\begin{array}{l} a_1 = -5 \\ a_n = a_{n-1} + 1 \end{array}$$

$$\begin{array}{l} a_n = -5 + (n-1)(1) \\ a_n = 1n - 6 \end{array}$$

44. $-4, -8, -12, -16, -20, \dots$

$$\begin{array}{l} a_1 = -4 \\ a_n = a_{n-1} - 4 \end{array}$$

$$\begin{array}{l} a_n = -4 + (n-1)(-4) \\ = -4 - 4n + 4 \\ a_n = -4n \end{array}$$

45. $-5, -3.5, -2, -0.5, 1, \dots$

$$\begin{array}{l} a_1 = -5 \\ a_n = a_{n-1} + 1.5 \end{array}$$

$$a_n = -5 + (n-1)(1.5)$$

46. $-32, -20, -8, 4, 16, \dots$

$$\begin{array}{l} d = 12 \\ a_1 = -32 \\ a_n = a_{n-1} + 12 \end{array}$$

$$\begin{array}{l} a_n = -32 + (n-1)(12) \\ = -32 + 12n - 12 \\ a_n = 12n - 44 \end{array}$$