

9-3

Geometric Sequences

Content Standard

Prepares for A.SSE.4 Derive the formula for the sum of a geometric series (when the common ratio is not 1), and use the formula to solve problems.

Objective To define, identify, and apply geometric sequences

You build a *geometric sequence* by multiplying each term by a constant.

Essential Understanding In a *geometric sequence*, the ratio of any term to its preceding term is a constant value.



Key Concept Geometric Sequence

A **geometric sequence** with a starting value a and a **common ratio** r is a sequence of the form

$$a, ar, ar^2, ar^3, \dots$$

A recursive definition for the sequence has two parts:

$$a_1 = a \quad \text{initial condition}$$

$$a_n = a_{n-1} \cdot r, \text{ for } n > 1 \quad \text{recursive formula}$$

An explicit definition for this sequence is a single formula:

$$a_n = a_1 \cdot r^{n-1}, \text{ for } n \geq 1$$



Problem 1 Identifying Geometric Sequences

Is the sequence geometric? If it is, what are a_1 and r ?

A 3, 6, 12, 24, 48, ...

Find the ratios between consecutive terms.

$$\begin{array}{ccccccccc} 3 & & 6 & & 12 & & 24 & & 48 \\ & \diagdown & / & \diagdown & / & \diagdown & / & \diagdown & / \\ & \frac{6}{3} & & \frac{12}{6} & & \frac{24}{12} & & \frac{48}{24} & \\ & = & & = & & = & & = & \\ & & & & & & & & 2 \end{array}$$

The common ratio is 2. The sequence is geometric with $a_1 = 3$ and $r = 2$.

How do I find the ratios between consecutive terms?
Divide the second term by the first term, then the third term by the second term, and so on.

B 3, 6, 9, 12, 15, ...

Find the ratio between consecutive terms.

$$\begin{array}{ccccccccc} 3 & & 6 & & 9 & & 12 & & 15 \\ & \diagdown & / & \diagdown & / & \diagdown & / & \diagdown & / \\ & \frac{6}{3} & & \frac{9}{6} & & \frac{12}{9} & & \frac{15}{12} & \\ & \neq & & \neq & & \neq & & \neq & \end{array}$$

The ratios are different. With no common ratio, the sequence is not geometric.

C $3^5, 3^{10}, 3^{15}, 3^{20}, \dots$

Use the properties of exponents to simplify the ratios of successive terms.

$$\begin{array}{ccccccccc} 3^5 & & 3^{10} & & 3^{15} & & 3^{20} \\ & \diagdown & / & \diagdown & / & \diagdown & / \\ & \frac{3^{10}}{3^5} & & \frac{3^{15}}{3^{10}} & & \frac{3^{20}}{3^{15}} & \\ & = & & = & & = & \\ & & & & & & 3^5 \end{array}$$

The common ratio is 3^5 . The sequence is geometric with $a_1 = 3^5$ and $r = 3^5$.



Got It? 1. Is the sequence geometric? If it is, what are a_1 and r ?

a. 2, 4, 8, 16, ...

b. 1, 5, 9, 13, 17, ...

c. $2^3, 2^7, 2^{11}, 2^{15}, \dots$



Problem 2 Analyzing Geometric Sequences

What are the indicated terms of the geometric sequence?

A the 10th term of the geometric sequence 4, 12, 36, ...

The first term a_1 is 4. The common ratio r is $12 \div 4 = 3$.

$$a_n = a_1 r^{n-1} \quad \text{Use the explicit formula.}$$

$$a_{10} = 4 \cdot 3^{10-1} \quad \text{Substitute 10 for } n, 4 \text{ for } a_1, \text{ and } 3 \text{ for } r.$$

$$a_{10} = 78,732 \quad \text{Simplify.}$$

The 10th term is 78,732.

B the second and third terms of the geometric sequence 2, \square , \square , -54, ...

The first term a_1 is 2. The fourth term a_4 is -54.

$$a_n = a_1 r^{n-1} \quad \text{Use the explicit formula.}$$

$$a_4 = 2r^{4-1} \quad \text{Substitute 2 for } a_1 \text{ and 4 for } n.$$

$$-54 = 2r^3 \quad \text{Substitute } -54 \text{ for } a_4. \text{ Simplify.}$$

$$-27 = r^3 \quad \text{Solve for } r.$$

$$-3 = r$$

The common ratio is -3. Begin with 2 and multiply by -3.

2, -6, 18, -54, ...

The second and third terms are -6 and 18.

What do you need to find the second term given the first term?
You need the common ratio.



Got It? 2. What is the 2nd term of the geometric sequence 3, \square , 12, ...?

In a geometric sequence, the square of the middle term of any three consecutive terms is equal to the product of the other two terms. For example, examine the sequence 2, -6, 18, -54, . . .

$$\begin{array}{c} (-6)^2 = 2 \cdot 18 = 36 \\ \underbrace{\hspace{2cm}} \\ 2, -6, 18, -54, \dots \\ \underbrace{\hspace{2cm}} \\ 18^2 = (-6)(-54) = 324 \end{array}$$

In an arithmetic sequence, recall that the middle term of any three consecutive terms is the arithmetic mean of the other two terms.

The **geometric mean** of two positive numbers x and y is \sqrt{xy} .

Note that the geometric mean is positive by definition. While there are two possible values for the missing term in the geometric sequence 3, \square , 12, . . . , there is only one geometric mean. The geometric mean is one possible value to fill in the geometric sequence. The opposite of the geometric mean is the other.

Problem 4 Using the Geometric Mean

Multiple Choice What are the possible values of the missing term of the geometric sequence?


48, \square , 3, . . .

- (A) ± 4 (B) ± 9 (C) ± 12 (D) ± 20

Find the geometric mean of 48 and 3.


$$\begin{aligned} \sqrt{48 \cdot 3} &= \sqrt{144} \\ &= 12 \end{aligned}$$

The possible values for the missing term are ± 12 . The correct answer choice is C.

 **Got It?** 4. The 9th and 11th terms of a geometric sequence are 45 and 80. What are possible values for the 10th term?

9.3 Algebra 2
Homework

Name _____

Determine whether each sequence is geometric. If so, find the common ratio.  See Problem 1.

7. 1, 2, 4, 8, ...

8. 1, 2, 3, 4, ...


9. 1, -2, 4, -8, ...

13. 18, -6, 2, $-\frac{2}{3}$, ...

14. 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, ...

15. 10, 15, 22.5, 33.75, ...

Find the eighth term of each geometric sequence.


 See Problem 2.


19. 3, 9, 27, ...


20. -3, 6, -12, ...

21. 10, 5, 2.5, ...

Find the missing term of each geometric sequence. It could be the geometric mean or its opposite.

 See Problem 4.

26. 6, , 911.25, ...

27. 9180, , 256, ...

28. $\frac{2}{5}$, , $\frac{8}{45}$, ...

Write an explicit formula for each sequence. Then generate the first five terms.

32. $a_1 = 1, r = 0.5$

33. $a_1 = 100, r = -20$

34. $a_1 = 7, r = 1$

Identify each sequence as *arithmetic*, *geometric*, or *neither*. Then find the next two terms.

38. 45, 90, 180, 360, ...

39. 25, 50, 75, 100, ...

40. 3, -3, 3, -3, ...