

6-4

Rational Exponents

Content Standards

Reviews N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Also reviews N.RN.1

Objective To simplify expressions with rational exponents

Essential Understanding You can write a radical expression in an equivalent form using a fractional (rational) exponent instead of a radical sign.

In general, $\sqrt[n]{x} = x^{\frac{1}{n}}$ for any positive integer n . Like the radical form, the exponent form indicates the principal root.

$$\sqrt{36} = 36^{\frac{1}{2}}$$

$$\sqrt[3]{64} = 64^{\frac{1}{3}}$$

$$\sqrt[4]{16} = 16^{\frac{1}{4}}$$

Problem 1 Simplifying Expressions with Rational Exponents

What is the simplified form of each expression?

A $216^{\frac{1}{3}} \rightarrow \frac{\sqrt[3]{216}}{\sqrt[3]{6 \cdot 6 \cdot 6}}$
6

B $7^{\frac{1}{2}} \cdot 7^{\frac{1}{2}} = 7^{\frac{1}{2} + \frac{1}{2}} = 7^1 = 7$

C $5^{\frac{1}{5}} \cdot 125^{\frac{1}{5}} = \sqrt[5]{5} \cdot \sqrt[5]{125} = 5$
5.5.5

Got It? 1. What is the simplified form of each expression?

a. $64^{\frac{1}{2}} = \sqrt{64} = 8$

b. $11^{\frac{1}{2}} \cdot 11^{\frac{1}{2}} = 11$

c. $3^{\frac{1}{3}} \cdot 12^{\frac{1}{3}} = 3^{\frac{1}{3}} \cdot 12^{\frac{1}{3}}$

$\frac{\sqrt[3]{3} \cdot \sqrt[3]{12}}{\sqrt[3]{3 \cdot 2 \cdot 2 \cdot 3}}$
6

If $\sqrt[n]{x} = x^{\frac{1}{n}}$, it follows from the Laws of Exponents that for all real numbers $\sqrt[n]{x^m} = (x^m)^{\frac{1}{n}} = (x^{\frac{1}{n}})^m = (\sqrt[n]{x})^m$. This leads to the definition of a rational exponent.

Take note

Key Concept Rational Exponent

If the n th root of a is a real number, m is an integer, and $\frac{m}{n}$ is in lowest terms, then

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ and } a^{-\frac{m}{n}} = \sqrt[n]{a^{-m}} = (\sqrt[n]{a})^{-m} \quad \text{If } m \text{ is negative, } a \neq 0.$$

Problem 2 Converting Between Exponential and Radical Forms

A What are $x^{\frac{3}{7}}$ and $y^{-3.5}$ in radical form?

$x^{\frac{3}{7}} = \sqrt[7]{x^3}$

$y^{-3.5} = \frac{1}{y^{3.5}} = \frac{1}{y^{\frac{7}{2}}} = \frac{1}{\sqrt{y^7}} = \frac{1}{y^3 \sqrt{y}}$
(y y y y y y y)

$3.5 \rightarrow 3\frac{1}{2} = \frac{7}{2}$

$\frac{1}{y^{\frac{7}{2}}} = \frac{1}{\sqrt{y^7}} = \frac{1}{y^3 \sqrt{y}}$

B What are $\sqrt{a^5}$ and $(\sqrt[5]{b})^3$ in exponential form?

$$a^{5/2} \quad b^{3/5}$$

Got It? 2. a. What are the expressions $w^{-5/8}$ and $w^{0.2}$ in radical form?

$$w^{-5/8} = w^{2/10} = w^{1/5} \quad w^{-5/8} = \frac{1}{w^{5/8}} = \frac{1}{\sqrt[8]{w^5}} \quad \frac{\sqrt[8]{w^3}}{\sqrt[8]{w^3}} = \frac{\sqrt[8]{w^3}}{w}$$

b. What are the expressions $\sqrt[4]{x^3}$ and $(\sqrt[5]{y})^4$ in exponential form?

$$x^{3/4} \quad y^{4/5}$$

$$\frac{x^{3/4}}{x^{4/3}} = x^{3/4 - 4/3} = x^{-7/12} = \frac{1}{x^{7/12}} = \frac{1}{\sqrt[12]{x^7}}$$

$$y^{5/4} = \frac{y^{1/4}}{y^{1/4}}$$

All the properties of integer exponents apply to rational exponents.

Take note

Properties of Rational Exponents

Let m and n represent rational numbers. Assume that no denominator equals 0.

Property	Example	Property	Example
$a^m \cdot a^n = a^{m+n}$	$8^{1/3} \cdot 8^{2/3} = 8^{1/3+2/3} = 8^1 = 8$	$a^{-m} = \frac{1}{a^m}$	$9^{-1/2} = \frac{1}{9^{1/2}} = \frac{1}{3}$
$(a^m)^n = a^{mn}$	$(5^{1/2})^4 = 5^{1/2 \cdot 4} = 5^2 = 25$	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{7^{3/2}}{7^{1/2}} = 7^{3/2-1/2} = 7^1 = 7$
$(ab)^m = a^m b^m$	$(4 \cdot 5)^{1/2} = 4^{1/2} \cdot 5^{1/2} = 2 \cdot 5^{1/2}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{27}\right)^{1/3} = \frac{5^{1/3}}{27^{1/3}} = \frac{5^{1/3}}{3}$



Problem 4 Combining Radical Expressions

What is $\frac{\sqrt{x^3}}{\sqrt{x^2}}$ in simplest form?

$$\frac{x^{3/4}}{x^{1/4}} = x^{3/4-1/4} = x^{2/4} = x^{1/2} = \sqrt{x}$$



Got It? 4. What is each product or quotient in simplest form?

a. $\sqrt{3}(\sqrt[4]{3})$

$$3^{1/2} \cdot 3^{1/4} = 3^{3/4} = \sqrt[4]{3^3} = \sqrt[4]{27}$$

b. $\frac{\sqrt{x^3}}{\sqrt[3]{x^2}}$

$$\frac{x^{3/2}}{x^{2/3}} = x^{3/2-2/3} = x^{9/6-4/6} = x^{5/6} = \sqrt[6]{x^5}$$

c. $\sqrt{7}(\sqrt[3]{7})$

$$7^{1/2} \cdot 7^{1/3} = 7^{3/6+2/6} = 7^{5/6} = \sqrt[6]{7^5}$$

Problem 5 Simplifying Numbers With Rational Exponents

What is each number in simplest form?

A $16^{-2.5}$

Method 1

$$\begin{aligned} 16^{-2.5} &= 16^{-\frac{5}{2}} \\ &= (2^4)^{-\frac{5}{2}} \\ &= 2^{4 \cdot -\frac{5}{2}} \\ &= 2^{-10} \\ &= \frac{1}{2^{10}} = \frac{1}{1024} \end{aligned}$$

Method 2

$$\begin{aligned} 16^{-2.5} &= 16^{-\frac{5}{2}} \\ &= \frac{1}{16^{\frac{5}{2}}} \\ &= \frac{1}{(\sqrt{16})^5} \\ &= \frac{1}{4^5} \\ &= \frac{1}{1024} \end{aligned}$$

$$\begin{aligned} 16^{-2.5} &= \frac{1}{16^{2.5}} \\ &= \frac{1}{16^2 \cdot 16^{1/2}} \\ &= \frac{1}{16^2 \cdot 4} \\ &= \frac{1}{1024} \end{aligned}$$

B $(-32)^{\frac{4}{5}} = 16$

Method 1

$$\begin{aligned} &\sqrt[5]{(-32)^4} \\ &\sqrt[5]{-32 \cdot -32 \cdot -32 \cdot -32} \\ &\quad \quad \quad \begin{matrix} 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\ 2 \cdot 2 \cdot 2 \cdot 2 \end{matrix} \end{aligned}$$

Method 2

Got It? 5. What is each number in simplest form?

a. $32^{-\frac{3}{5}}$

$$\frac{1}{32^{3/5}}$$

$$\sqrt[5]{\frac{1}{32^3}} = \frac{1}{8}$$

b. $16^{\frac{3}{4}}$

$$\sqrt[4]{16^3}$$

$$\sqrt[4]{16 \cdot 16 \cdot 16} = 8$$

c. $9^{-3.5}$

$$\begin{aligned} \frac{1}{9^{3.5}} &= \frac{1}{9^3 \cdot 9^{1/2}} \\ &= \frac{1}{729 \cdot 3} \\ &= \frac{1}{2187} \end{aligned}$$

$$\frac{32 \cdot 32 \cdot 32}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$$

Problem 6 Writing Expressions in Simplest Form

What is each expression in simplest form?

A $(-8x\sqrt{xy})^{\frac{2}{3}}$

B $(16y^{-8})^{-\frac{3}{4}}$

Simplify each expression.

10. $36^{\frac{1}{2}} = \sqrt{36} = 6$

13. $10^{\frac{1}{2}} \cdot 10^{\frac{1}{2}} = 10$

16. $2^{\frac{1}{2}} \cdot 32^{\frac{1}{2}} = 8$

$$\frac{\sqrt{2} \cdot \sqrt{32}}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$$

11. $27^{\frac{1}{3}} = 3$

14. $(-3)^{\frac{1}{3}} \cdot (-3)^{\frac{1}{3}} \cdot (-3)^{\frac{1}{3}} = -3$

17. $3^{\frac{1}{3}} \cdot 9^{\frac{1}{3}} = 3$

12. $49^{\frac{1}{2}} = 7$

15. $7^{\frac{1}{2}} \cdot 21^{\frac{1}{2}} = 7 \cdot \sqrt{21} = 7\sqrt{3}$

18. $3^{\frac{1}{3}} \cdot 27^{\frac{1}{3}} = 3$

$$\sqrt[3]{3} \cdot \sqrt[3]{27} = 3$$

See Problem 1.

Write each expression in radical form.

19. $x^{\frac{1}{6}}$ $\sqrt[6]{x}$

20. $x^{\frac{1}{5}}$ $\sqrt[5]{x}$

21. $x^{\frac{2}{3}}$ $\sqrt[3]{x^2}$

22. $y^{\frac{2}{5}}$ $\sqrt[5]{y^2}$

23. $y^{\frac{1}{9}}$ $\frac{1}{y^{\frac{8}{9}}} = \frac{1}{y^{\frac{8}{9}}}$

24. $t^{\frac{3}{4}}$ $\frac{1}{t^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{t}}$

25. $x^{-1.5}$ $x^{\frac{3}{2} = 1\frac{1}{2}}$
 $\frac{1}{x^{\frac{3}{2}}} = \frac{1}{x\sqrt{x}}$

26. $y^{1.2}$ $y^{\frac{6}{5}}$
 $y^{\frac{6}{5}} = \sqrt[5]{y^6}$

See Problem 2.

Write each expression in exponential form.

27. $\sqrt{-10}$ N.P

28. $\sqrt[3]{7x^3}$ $(7x^3)^{\frac{1}{3}}$

29. $\sqrt{(7x)^3}$ $(7x)^{\frac{3}{2}}$

30. $(\sqrt{7x})^3$ $(7x)^{\frac{3}{2}}$

31. $\sqrt[3]{a^2}$ $a^{\frac{2}{3}}$

32. $(\sqrt[3]{a})^2$ $a^{\frac{2}{3}}$

33. $\sqrt[4]{c^2}$ $a^{\frac{1}{2}}$

34. $\sqrt[3]{(5xy)^6}$

$(5xy)^{\frac{6}{3}} = (5xy)^2$

Find each product or quotient.

39. $(\sqrt[4]{6})(\sqrt[3]{6})$

40. $\frac{\sqrt[9]{y^3}}{\sqrt[3]{y^9}}$

41. $\sqrt{5} \cdot \sqrt[5]{5}$

42. $\sqrt[7]{7} \cdot \sqrt[3]{7}$

43. $\frac{\sqrt[6]{4}}{\sqrt[3]{4}}$

44. $\sqrt[4]{18} \cdot \sqrt{12}$

45. $\frac{\sqrt{6}}{\sqrt[3]{36}}$

46. $\frac{\sqrt{x^4y}}{\sqrt[4]{x^2y^8}}$

See Problem 4.

Simplify each number.

47. $8^{\frac{2}{3}}$

48. $64^{\frac{1}{3}} 64^{\frac{1}{3}}$

49. $(-8)^{\frac{2}{3}}$

50. $(-32)^{\frac{1}{5}}$

See Problem 5.

Write each expression in simplest form.

55. $(x^3)^{-3} x^{-2} = \frac{1}{x^2}$

56. $(x^{\frac{4}{7}})^7 = x^{-4} = \frac{1}{x^4}$

57. $(3x^{\frac{2}{3}})^{-1}$

58. $5(x^{\frac{1}{3}})^{-1} \frac{5}{x^{\frac{2}{3}}} = \frac{5}{\sqrt[3]{x^2}}$

59. $(-27x^{-9})^{\frac{1}{3}}$

60. $(-32y^{15})^{\frac{1}{5}}$

61. $(x^{\frac{1}{3}} y^{\frac{2}{3}})^{-6}$

62. $(x^{\frac{2}{3}} y^{\frac{1}{6}})^{-12}$

63. $(\frac{x^3}{x-1})^{-4}$

64. $(\frac{x^2}{x-11})^{\frac{1}{3}}$

65. $(\frac{x^{\frac{1}{2}}}{y^{\frac{1}{4}}})^{12}$

66. $(\frac{x^{\frac{2}{3}}}{y^{\frac{1}{2}}})^{15}$

$\frac{1}{3x^{\frac{2}{3}}} = \frac{1}{3^{\frac{2}{3}}x^{\frac{2}{3}}}$
 $3^{-1} \cdot x^{-\frac{2}{3}}$

See Problem 6.