5.2

Exponential Functions

Name: Notes / H.W

Date: 2013 Hour:

Graphs of Exponential Functions

For each positive real number a, $a \neq 1$, there is an **exponential function with base a** whose domain is all real numbers and whose rule is $f(x) = a^x$. Some examples are shown below.

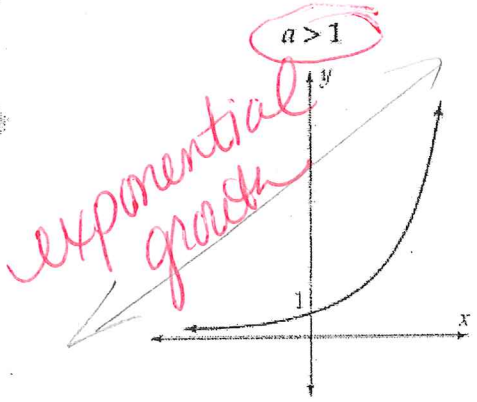
$$f(x) = 10^x \qquad g(x) = 2^x \qquad h(x) = \left(\frac{1}{2}\right)^x \qquad k(x) = \left(\frac{3}{2}\right)^x$$

The shape of the graph of an exponential function $f(x) = a^x$ depends only on the size of a, as shown in the following figures.

Graph of $f(x) = a^x$

Growth

$a > 1$

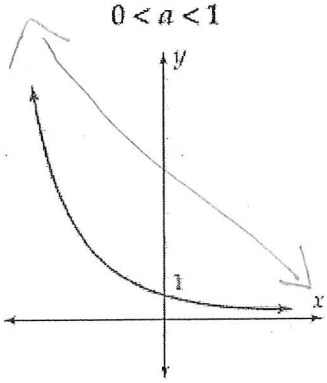


exponential growth

- graph is above x-axis
- y-intercept is 1
- $f(x)$ is increasing
- $f(x)$ approaches the negative x-axis as x approaches $-\infty$

Decay

$0 < a < 1$



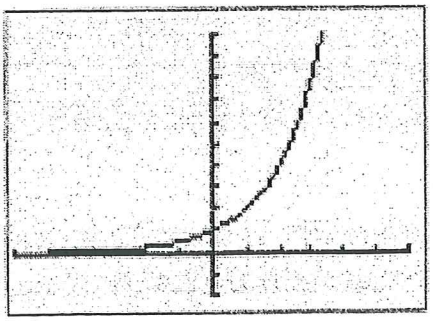
- graph is above x-axis
- y-intercept is 1
- $f(x)$ is decreasing
- $f(x)$ approaches the positive x-axis as x approaches ∞

Transformations

$Y = 2^x$

Example: $f(x) = 2^x$

$f(x) = 2^x + 2$	← Shift 2 up, horizontal asymptote $y = 2$
$f(x) = 2^x - 3$	← Shift 3 down, horizontal asymptote $y = -3$
$f(x) = 2^{x+5}$	← Shift 5 left, horizontal asymptote $y = 0$
$f(x) = 2^{x-7}$	← Shift 7 right, horizontal asymptote $y = 0$
$f(x) = 2^{x-1} + 4$	← Shift 1 right, 4 up, horizontal asymptote $y = 4$
$f(x) = -2^x$	← Reflection over x-axis (change all y's to opposite)
$f(x) = 2^{-x}$	← Reflection over y-axis (change all x's to opposite)



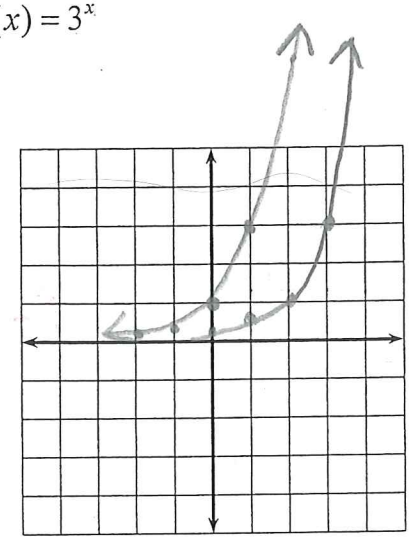
Example: Graph $f(x) = 3^{x-2}$

(take $f(x) = 3^x$ and shift 2 right)

2 right

First graph $f(x) = 3^x$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9



x	y
0	1/9
1	1/3
2	1
3	3

Example: Graph $f(x) = e^x + 2$

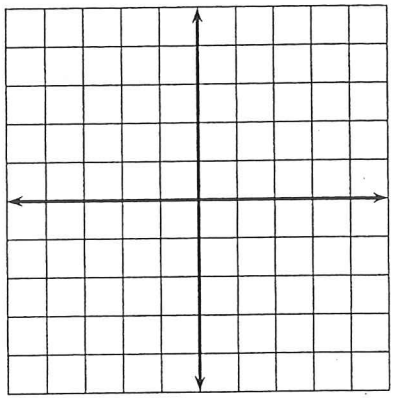
$e \approx 2.718\ldots$

(take $f(x) = e^x$ and shift 2 up)

First graph $f(x) = e^x$

add 2 to y value

x	y



Example: Describe the transformation $f(x) = -3(2^{\frac{1}{2}x-1}) + 4$

d e a b c f

$f(x) = -3(2^{\frac{1}{2}x-1}) + 4$

Rewrite the equation:

$y = -3\left(2^{\frac{1}{2}(x-2)}\right) + 4$

c a b d

Parent function $f(x) = 2^x$

-1

$\frac{1}{2}$

List the transformation:

a) Horizontal stretch by 2 (mult the x)

b) Right 2 (add 2 to x value) by 2)

mult the y value by -3

c) r.o. x axis

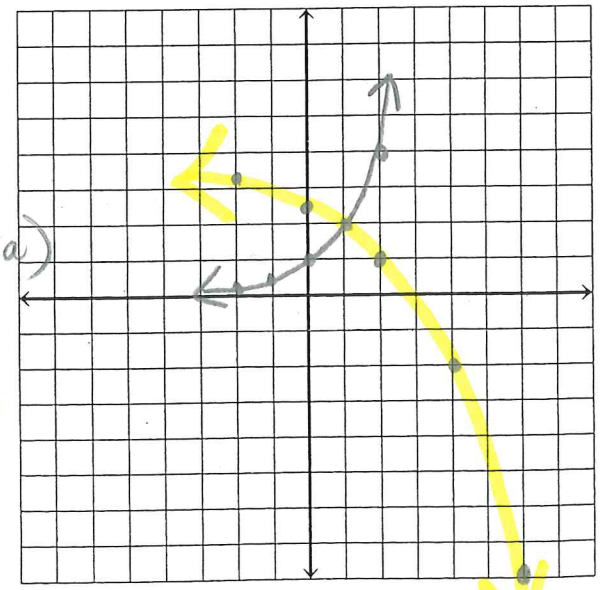
d) V. S by 3

e) V. Shift 4 up (add 4 to y val)

f)

a b c

$y=2^x$ Type:		mult x by 2		add 2 to x		mult -3		y+4	
x	y	x	y	x	y	x	y	x	y
-2	1/4	-4	1/4	-2	1/4	-2	-3/4	-2	3.25
-1	1/2	-2	1/2	0	1/2	0	-3/2	0	2.5
0	1	0	1	2	1	2	-3	2	1
1	2	2	2	4	2	4	-6	4	-2
2	4	4	4	6	4	6	-12	6	-8



$-3/4 + 4$

$-1.5 + 4$

$-3 + 4$

Exercises 5.2

In Exercises 1–6, list the transformations needed to transform the graph of $h(x) = 2^x$ into the graph of the given function. (Section 3.4 may be helpful.)

1. $f(x) = 2^x - 5$
2. $g(x) = -(2^x)$
3. $k(x) = 3(2^x)$
4. $g(x) = 2^{x-1}$
5. $f(x) = 2^{x+2} - 5$
6. $g(x) = -5(2^{x-1}) + 7$

In Exercises 7–13, list the transformations needed to transform the graph of $h(x) = 3^x$ into the graph of the given function. (Section 3.4 may be helpful.)

7. $f(x) = 3^x + 4$
8. $g(x) = 3^{-x}$
9. $k(x) = \frac{1}{4}(3^x)$
10. $g(x) = 3^{0.4x}$
11. $f(x) = 3^{2-x}$
12. $f(x) = 8 + 5(3^x)$
13. $g(x) = 4(3^{-0.15x})$

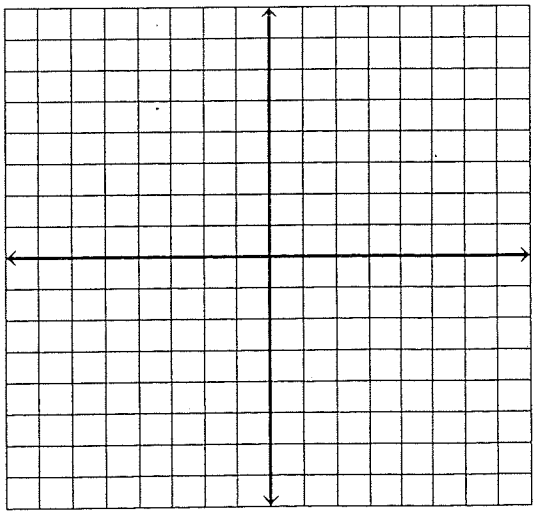
1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____
11. _____
12. _____
13. _____

Sketch a complete graph:

For Exercises 14-18, complete the table, and graph each function on the grid provided.

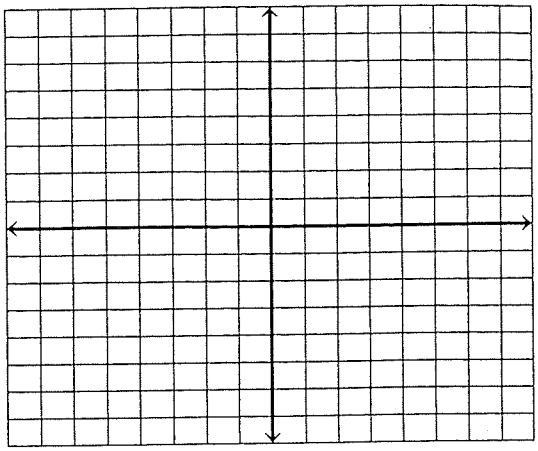
14. $f(x) = -3(2)^{\frac{1}{3}x-2}$

	Type:				
x	y				



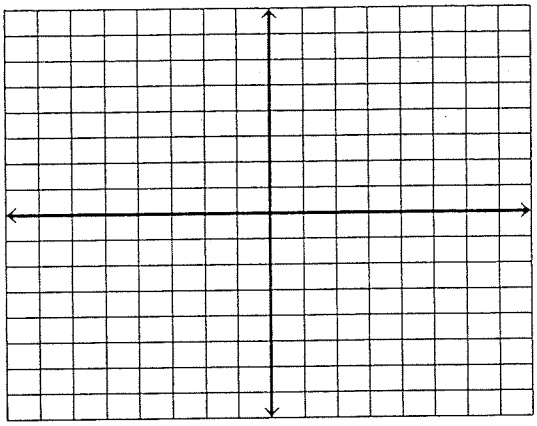
15. $f(x) = -3(2)^{-x+1} + 2$

Type:					
x	y				



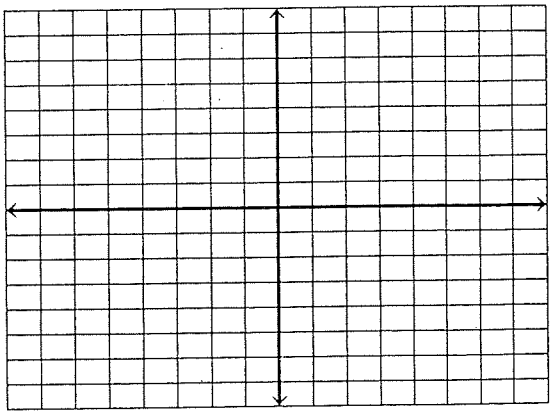
16. $y = -2(3)^{x-4} + 3$

Type:					
x	y				



17. $f(x) = -(2^{x-6}) - 3$

Type:					
x	y				



18. $f(x) = 3(\frac{1}{2}^{x+4}) - 1$

Type:					
x	y