{ (,) }

Notes

6-7 Inverse Relations and Functions

Use the relation {(4, 5), (6, 7), (12, 20), (8, 3), (2, 7)}. Write the domain and range of the relation.

Domain : d: {2, 4, 6, 8, 12}

Range : r: {3, 5, 7, 20}

Complete the diagram below. Use relation r {(0, 1), (2, 3), (4, 1), (8, 3)}.

Relation r

Domain	Range
0, 2, 4, 8	1, 3

Inverse of Relation r

Domain	Range
1, 3	0, 2, 4, 8

Problem 1 Finding the Inverse of a Relation

Got It? What are the graphs of t and its inverse?

4. Complete the table of values for the inverse of relation t.

Relation t

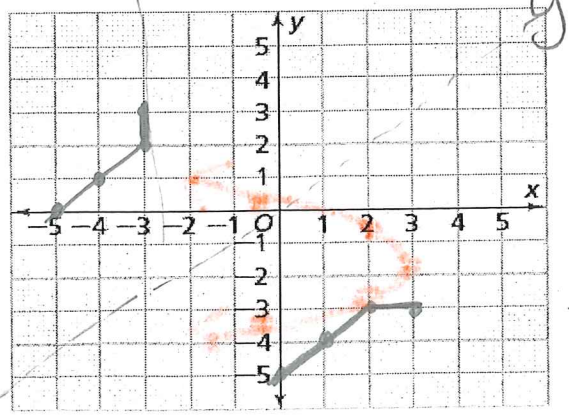
x	y
0	−5
1	−4
2	−3
3	−3

Inverse of Relation t

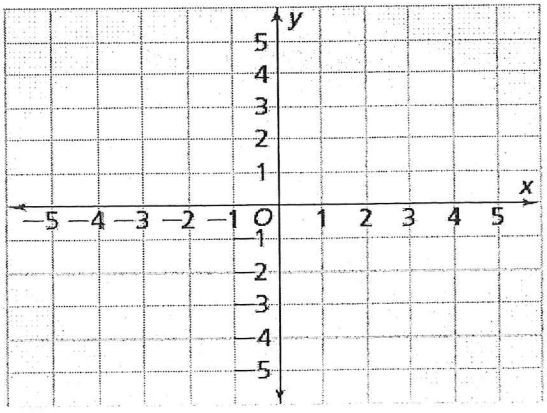
x	y
−5	0
−4	1
−3	2
−3	3

5. Plot the points from the Relation t table and from the Inverse of Relation t table.

Relation t



Inverse of Relation t



Problem 2 Finding an Equation for the Inverse

Got It? What is the inverse of $y = 2x + 8$?

6. Switch the x and y values in the function.

function $y = 2x + 8$

inverse $x = 2y + 8$

Solve the inverse equation for y.

$x = 2y + 8$

$\frac{x}{2} - \frac{8}{2} = \frac{2y}{2}$

$\frac{1}{2}x - 4 = y$

$y = \frac{1}{2}x - 4$

Steps for finding the inverse of a function:

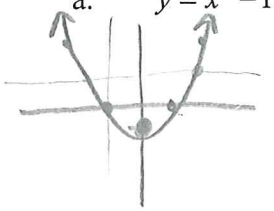
1) **Switch the x and y**

2) **Then solve for y**

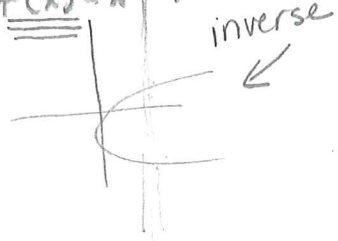
Note: if the inverse is also a function (does it pass the vertical line test), then use this special notation: $f^{-1}(x)$.

2. Find the inverse of each function. Is the inverse a function? NOT

a. $y = x^2 - 1$



$f(x) = x^2 - 1$



$y = x^2 - 1$

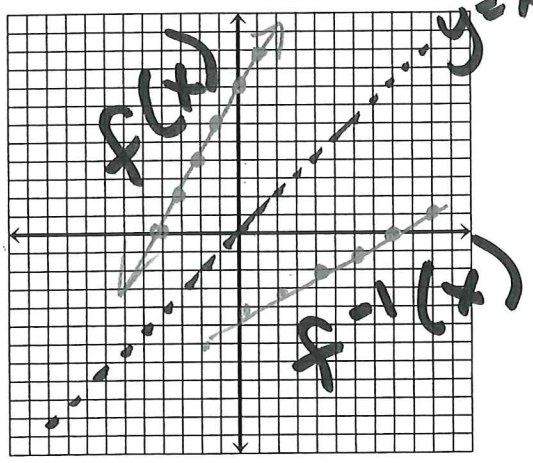
$x = y^2 - 1$ resolve for y

$\sqrt{x+1} = y^2$

$\pm\sqrt{x+1} = y$ $y = \pm\sqrt{x+1}$

Problem 3 Graphing a Relation and Its Inverse

Got It? What are the graphs of $y = 2x + 8$ and its inverse?



$f(x)$

x	y
0	8
1	10
-1	6
-2	4

$f^{-1}(x)$

x	y
8	0
10	1
6	-1
4	-2

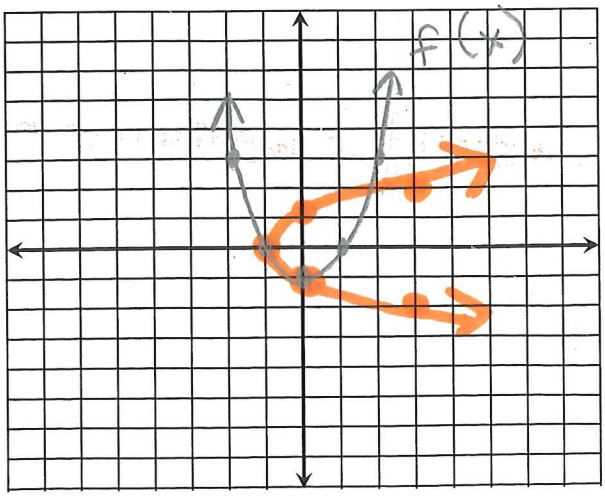
b. $y = 2(x-3)^2 + 2$

$f(x)$

x	y
-2	3
-1	0
0	-1
1	0
2	3

inverse

x	y
3	-2
0	-1
-1	0
0	1
3	2



$(-\infty, \infty)$

d: all

r: $[-1, \infty)$

d: $[-1, \infty)$

r: $(-\infty, \infty)$

Determining Whether an Inverse is a Function

A one-to-one function is a function in which every element in the range of the function corresponds with exactly one element in the domain.

Example of a one-to-one function: {(0,1), (5,2), (6,4)}
Domain: { 0, 5, 6 }
Range: { 1, 2, 4 }

Each element is the domain { 0, 5, 6 } corresponds with a unique element in the range. Therefore this function is a one-to-one function.

The two functions below only differ by 1 number. However, that small difference is all that was necessary to make function #1 not be a one-to-one function.

#1

Function

{(2, 9) , (4, 5) , (11, 5) }

(9,2) (5,4) (5,11)

#2

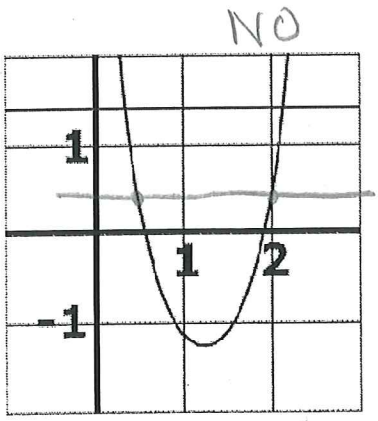
1-to-1 Function

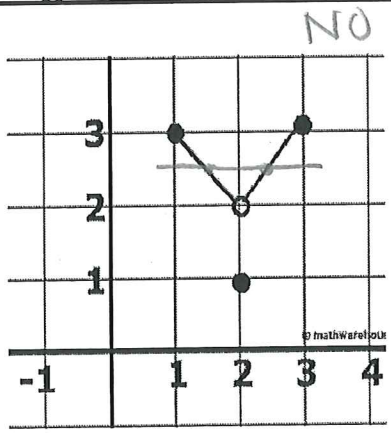
{(2, 9) , (4, 5) , (11, 6)}

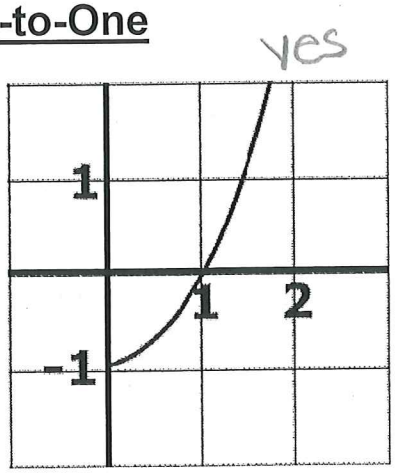
yes!

- In the first function below, since the number 5 in the **range** corresponds with both 4 and 11 in the **domain**, this **function** is **not** one-to-one.
- On the other hand, **function** #2 is a one to one function because each element in the **domain** has 1 and only 1 corresponding element in the **range**.

Determining Whether a Graph is One-to-One







A function *f* is one-to-one if and only if no horizontal line intersects the graph of *f* more than once

If *f* passed the Horizontal Line Test, then we use f^{-1} to denote the inverse function.

take note

Key Concept Composition of Inverse Functions

If f and f^{-1} are inverse functions, then

$(f^{-1} \circ f)(x) = x$ and $(f \circ f^{-1})(x) = x$ for x in the domains of f and f^{-1}, respectively.

Problem 6 Composing Inverse Functions

Got It? Let $g(x) = \frac{4}{x+2}$. What is $g^{-1}(x)$?

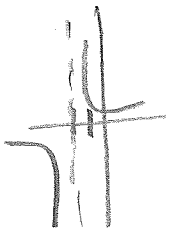
$y = \frac{4}{x+2}$

$x = \frac{4}{y+2}$

$x(y+2) = 4$

$y+2 = \frac{4}{x}$

$y = \frac{4}{x} - 2$



$g(x) = \begin{cases} D: (-\infty,-2) \cup (-2,\infty) \\ R: (-\infty,0) \cup (0,\infty) \end{cases}$

$g(x)^{-1} = \begin{cases} D: (-\infty,0) \cup (0,\infty) \\ R: (-\infty,-2) \cup (-2,\infty) \end{cases}$

Is $g^{-1}(x)$ a function? Yes

1. Find the inverse of the functions and state whether or not the inverse is a function.

a. $f(x) = 3x - 4$

$y = 3x - 4$

$x = 3y - 4$

$\frac{x}{3} + \frac{4}{3} = \frac{3y}{3}$

$y = \frac{1}{3}x + \frac{4}{3}$

$f^{-1}(x) = \frac{1}{3}x + \frac{4}{3}$

$f(x) = \begin{cases} D: (\text{arn}) \\ R: (\text{arn}) \end{cases}$

$f(x)^{-1} = \begin{cases} D: \text{arn} \\ R: \text{arn} \end{cases}$

Is $f^{-1}(x)$ a function? Yes

b. $f(x) = x^2 - 7$

$f(x) = \begin{cases} D: \\ R: \end{cases}$

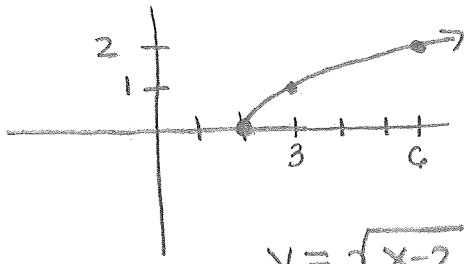
$f(x)^{-1} = \begin{cases} D: \\ R: \end{cases}$

Is $f^{-1}(x)$ a function? _____

2. For each function, find the inverse and the domain and range of the function and its inverse. Determine whether the inverse is a function.

a. $f(x) = \sqrt{x-2}$

$y = \sqrt{x-2}$



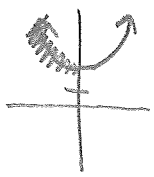
$y = \sqrt{x-2}$

$(x)^2 = (\sqrt{y-2})^2$

$x^2 = y - 2$

$x^2 + 2 = y$

$y = x^2 + 2$



$f(x)$ D: $\{x \geq 2\}$

R: $\{y \geq 0\}$

$f^{-1}(x) = x^2 + 2$

$f(x)^{-1}$ D: $\{x \geq 0\}$

$f^{-1}(x)$

R: $\{y \geq 2\}$

Is $f(x)^{-1}$ a function? Yes

b. $f(x)=(x-2)^2$

$f(x)$ D: {_____}

R: {_____}

$f(x)^{-1}$ D: {_____}

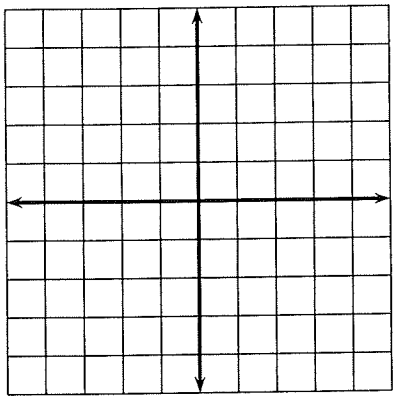
R: {_____}

Is $f(x)^{-1}$ a function? _____

Find the inverse of each relation. Graph the given relation and its inverse.

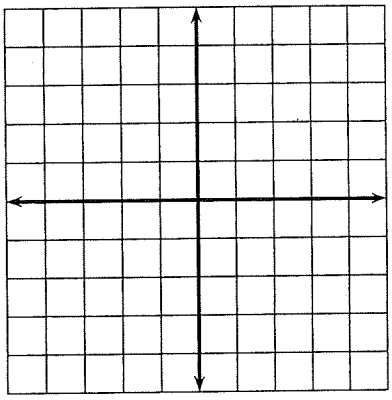
9.

x	y
1	0
2	1
3	2
4	3



11.

x	y
−3	2
−2	2
−1	2
0	2



Find the inverse of each function. Is the inverse a function?

13. $y = 2x - 1$

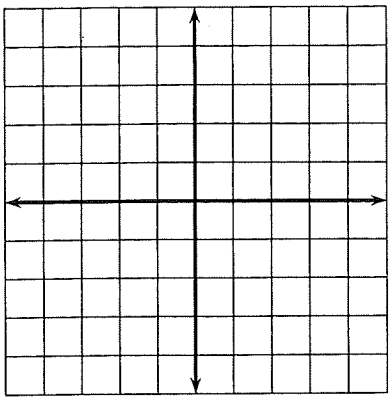
15. $y = 5 - 2x^2$

17. $y = 3x^2 - 5$

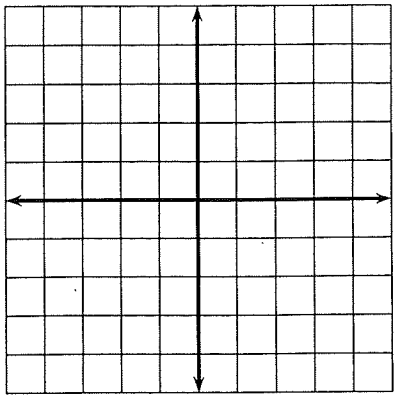
19. $y = (3x - 4)^2$

Graph each relation and its inverse.

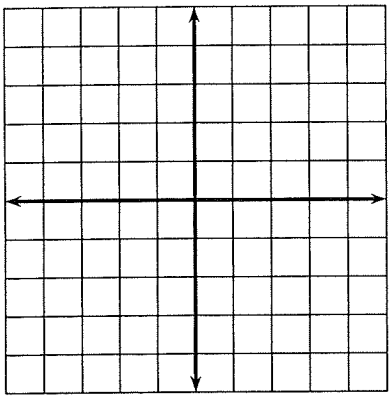
21. $y = 2x - 3$



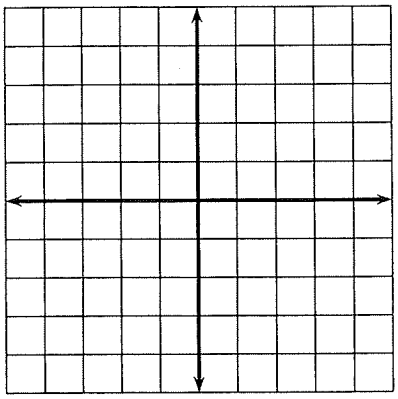
23. $y = -x$



25. $y = -x^2$



27. $y = (x - 1)^2$



For each function, find the inverse and the domain and range of the function and its inverse. Determine whether the inverse is a function.

30. $f(x) = 3x + 4$

31. $f(x) = \sqrt{x - 5}$

32. $f(x) = \sqrt{x + 7}$

34. $f(x) = 2x^2 + 2$

35. $f(x) = -x^2 + 1$