

## 4.4 Notes Rational Functions

Name \_\_\_\_\_

The functions in this section become much more complex than 4.3. They now have polynomials in both the numerator and denominator. These are called **rational functions**.

Our goal for this section is to learn how to graph these functions without T- tables and graphing calculators. These are a list of important values that you must locate in order to make "special" things occur in the graph.

Since these rational functions have denominators, one of the special things we are concerned about is what would give us a zero in the denominator. This is where the function is undefined. This will be a **vertical asymptote**. Also, this will help define the domain.

So... set the denominator = 0 and solve. This will give you the domain and the vertical asymptotes: VA.

EXAMPLES: Find the domain and vertical asymptotes:

a.  $f(x) = \frac{x-1}{x^2-9}$   
 $(x-3)(x+3)$   
 $x \neq 3 \quad x \neq -3$   
 $\frac{\oplus}{-3} \quad \frac{\oplus}{3}$

b.  $f(x) = \frac{5x+4}{2x^3+2x}$   
 $2x(x^2+1) \stackrel{!}{=} 0$   
 $x=0$

Domain:  $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

Domain:  $(-\infty, 0) \cup (0, \infty)$

VA:  $x = -3 \quad x = 3$   
 $x = \pm 3$

VA:  $x = 0$


Another special item to consider when analyzing rational functions is the **horizontal asymptotes**.

Compare the degree of the numerator "n" with the degree of the denominator "k"....

A. If the degree of the numerator is less than the degree of the denominator ( $n < k$ ), then the x axis or  $y=0$  is the horizontal asymptote.  $y=0$

B. If the degree of the numerator is equal to the degree of the denominator ( $n=k$ ), then the horizontal asymptote will be  $f(x) = \frac{\text{leading coefficient numerator}}{\text{leading coefficient denominator}}$

C. If the degree of the numerator is larger than the denominator, ( $n > k$ ), then there are no horizontal asymptotes...BUT 2 other types of asymptotes can exist....

A slant asymptote occurs  when the degree of the numerator is exactly one more than the degree of the denominator. For example:  $f(x) = \frac{2x^3 - x + 4}{x^2 + 1}$  or  $f(x) = \frac{x^2 - x + 4}{x + 1}$

A parabolic asymptote occurs when the degree of the numerator is exactly two more than the degree of the denominator. For example:  $f(x) = \frac{2x^4 - x + 4}{x^2 + 1}$  or  $f(x) = \frac{2x^3 - x + 4}{x + 1}$

$$x^2 + 1 \overline{) 2x^4 - x + 4}$$

$$y = 2x^2 - 2x + 1$$

$$\begin{array}{r|rrrr} -1 & 2 & 0 & -1 & 4 \\ & -2 & 2 & -1 & \\ \hline & 2 & -2 & 1 & 8 \end{array}$$

Asymptotes are found by using synthetic or long division. Ignore the remainder!

EXAMPLES: Find all horizontal, slant or parabolic asymptotes:

$$f(x) = \frac{5x^3 - 8x^2 + 4}{2x^3 + 2x}$$

$$y = \frac{5}{2} = 2.5$$

$$f(x) = \frac{x^3 + 2x - 1}{x - 1}$$

H.A. none

$$P.A. y = x^2 + 1x + 3$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 2 & -1 \\ & & 1 & 1 & \\ \hline & 1 & 1 & 3 & \end{array}$$

$$f(x) = \frac{x^2 + 3x - 5}{x + 1}$$

H.A. none

$$S.A. y = 1x + 2$$

$$\begin{array}{r|rrr} -1 & 1 & 3 & -5 \\ & & -1 & -2 \\ \hline & 1 & 2 & -7 \end{array}$$

$$f(x) = \frac{x^3 + 4}{x^4 - 7}$$

$$y = 0$$

There are a few other special parts to consider:

- The y intercept: Set  $x=0$  and solve for y.
- The x intercept(s) or roots: when  $y=0$ . Look at just what makes numerator = 0. This is the only part of a fraction that matters when we want  $y=0$ .
- Holes: Sometimes when we reduce and cancel terms, we create holes in the graph.

Example of a hole:  $f(x) = \frac{x^2 - 4}{x - 2} \rightarrow \frac{(x-2)(x+2)}{x-2} \rightarrow \frac{x+2}{1}$

$x \neq 2$

Because  $\frac{x^2 - 4}{x - 2}$  and  $\frac{x+2}{1}$  are not the same, we have a problem! Because 2 made the denominator = 0 in the first function, we say a HOLE occurs at the second new function. The hole occurs at (2, ?). Plug 2 in for x and solve:  $x + 2$

$$\begin{array}{l} 2 + 2 \\ 4 \end{array}$$

So the hole is at (2, 4)

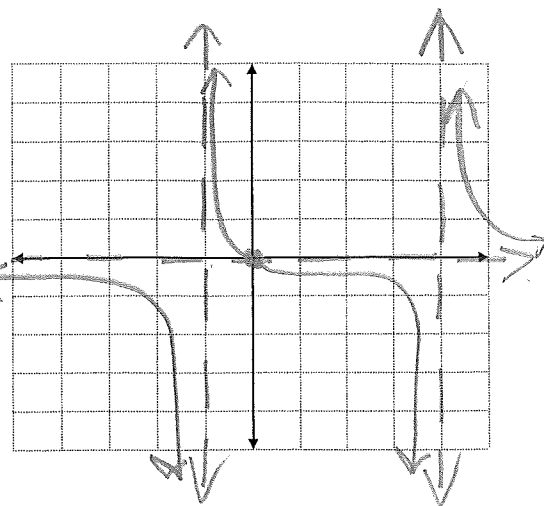
- Critical values are numbers that make both the numerator and denominator = 0.
- Factor the numerator and denominator COMPLETELY!
- Use the critical values along with all of the factors to make an exclusion chart.

1. Graph:  $f(x) = \frac{x}{x^2 - 3x - 4}$   $\xrightarrow{x=0}$   
 $(x-4)(x+1)$   
 $x=4 \quad x=-1$

CV: -1, 0, 4  
~~CV~~ Root(s):  $x =$  0  
 VA:  $x=4, x=-1$   
 HA:  $y=0$  SA: X PA: X  
 y-int. = (0,0) Hole = X

**Exclusion Chart:**

	-1	0	4	
x	-	-	+	+
x-4	-	-	-	+
x+1	-	+	+	+
f(x)	-	+	-	+



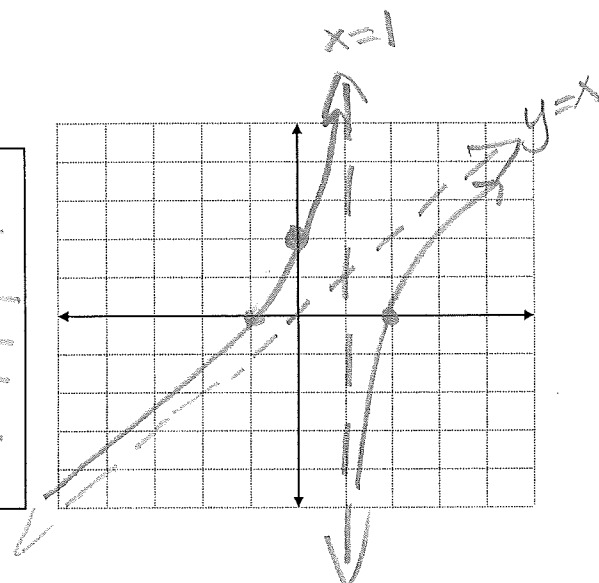
2. Graph:  $f(x) = \frac{x^2 - x - 2}{x-1} = \frac{(x-2)(x+1)}{(x-1)}$   $\xrightarrow{x=2}$   $\xrightarrow{x=-1}$   
 $\xrightarrow{x=1}$

1 | 1 -1 -2  
 1 0 X

CV: -1, 1, 2  
~~CV~~ Root(s):  $x =$  2, -1  
 VA:  $x=1$   
 HA: X SA:  $y=x$  PA: X  
 y-int. = (0,2) Hole = X

**Exclusion Chart:**

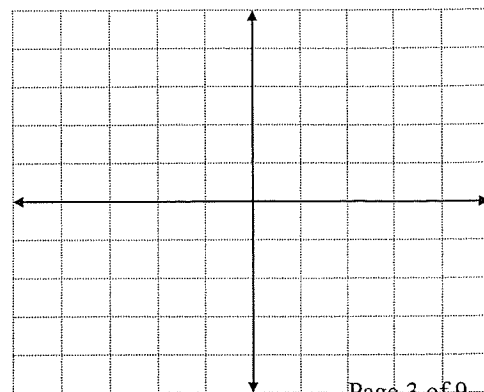
	-1	1	2	
x-2	-	-	-	+
x+1	-	+	+	+
x-1	-	-	+	+
f(x)	-	+	-	+



3. Graph:  $f(x) = \frac{x^4 - 1}{x^2}$

CV : Root(s):  $x =$  \_\_\_\_\_  
 VA: \_\_\_\_\_  
 HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_  
 y-int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**

4. Graph:  $f(x) = \frac{x^3 + 8}{x + 1}$

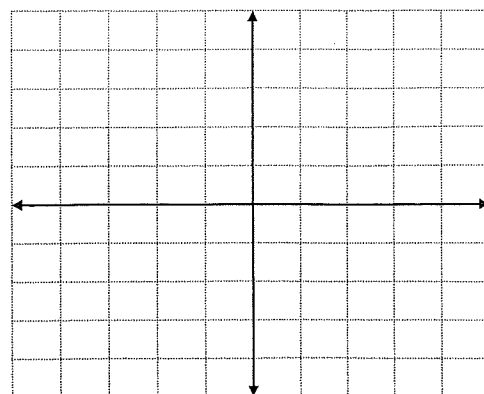
CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

y-int. = \_\_\_\_\_ Hole = \_\_\_\_\_

Exclusion Chart:



CV: 0, -5, 1, 2, -2, 3, -3

5. Graph:  $f(x) = \frac{x^3 + 4x^2 - 5x}{(x^2 - 4)(x^2 - 9)}$

$\frac{x(x+5)(x-1)}{(x-2)(x+2)(x-3)(x+3)}$

CV : Root(s):  $x =$  0, -5, 1

VA:  $x = \pm$  \_\_\_\_\_

Only one of these!

HA: \_\_\_\_\_

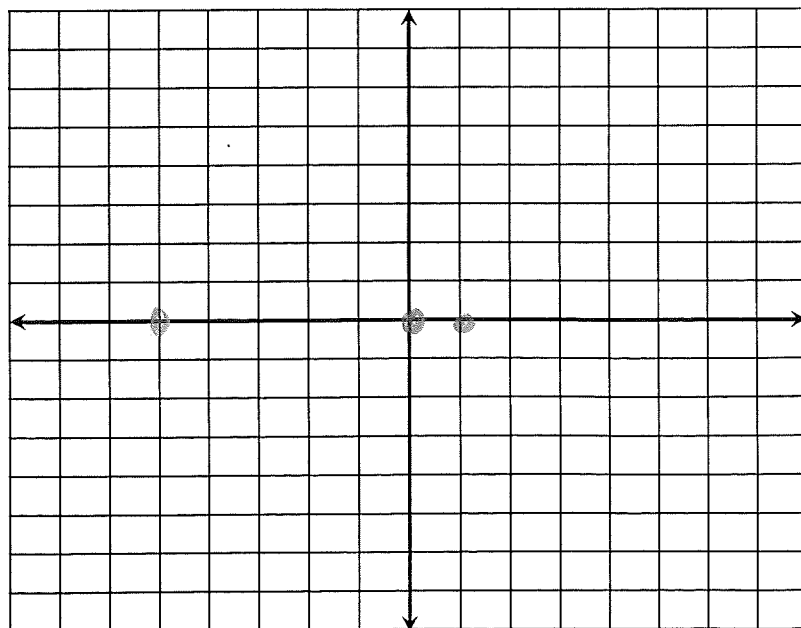
SA: \_\_\_\_\_

PA: \_\_\_\_\_

y-int. = \_\_\_\_\_

Hole = \_\_\_\_\_

	-5	-3	-2	0	1	2	3
x	-	-	-	-	+	+	+
x+5	-	+	+	+	+	+	+
x-1	-	-	-	-	-	+	+
x-2	-	-	-	-	-	-	+
x+2	-	-	-	+	+	+	+
x-3	-	-	-	-	-	-	-
x+3	-	-	+	+	+	+	+
f(x)	-	+	-	+	-	+	-





## Precalculus

### Section 4.4

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Hour: \_\_\_\_\_

23. Graph:  $f(x) = \frac{1}{x+5}$

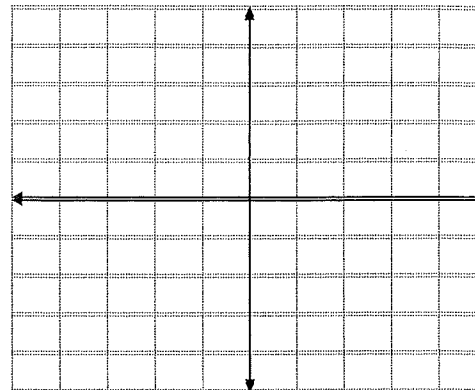
CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

$y$ -int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**



25. Graph:  $f(x) = \frac{-3}{2x+5}$

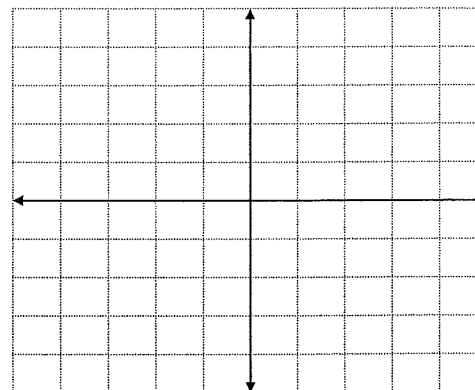
CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

$y$ -int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**



27. Graph:  $f(x) = \frac{3x}{x-1}$

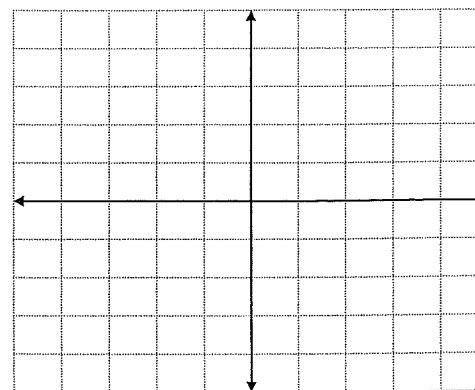
CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

$y$ -int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**



29. Graph:  $f(x) = \frac{2-x}{x-3}$

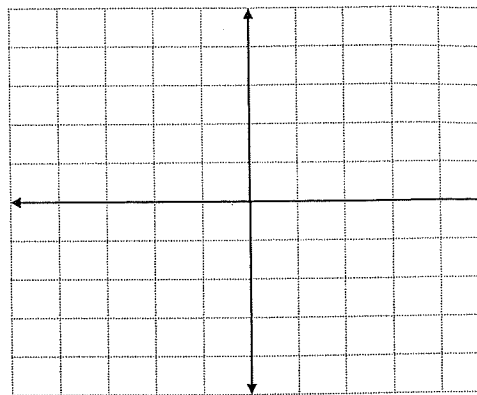
CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

y-int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**



31. Graph:  $f(x) = \frac{1}{x(x+1)^2}$

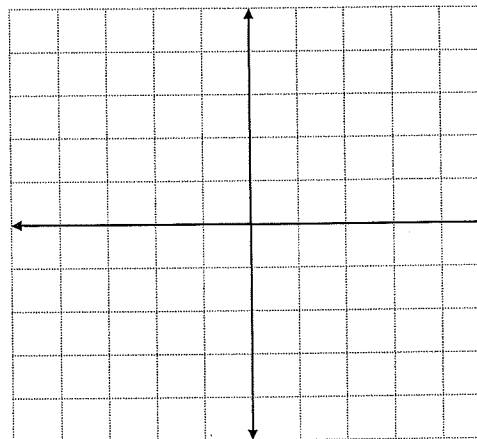
CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

y-int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**



33. Graph:  $f(x) = \frac{x-3}{x^2+x-2}$

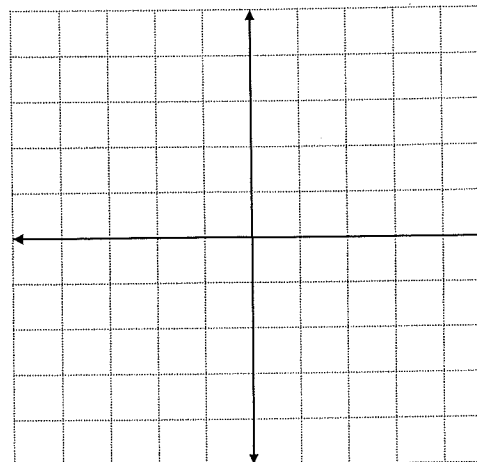
CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

y-int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**



35. Graph:  $f(x) = \frac{(x^2 + 6x + 5)(x + 5)}{(x + 5)^3(x - 1)}$

CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

y-int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**

37. Graph:  $f(x) = \frac{-4x^2 + 1}{x^2}$

CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

y-int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**

39. Graph:  $f(x) = \frac{x^2 + 2x}{x^2 - 4x - 5}$

CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

y-int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**

41. Graph:  $f(x) = \frac{(x+3)(x-3)}{(x-5)(x+4)(x+3)}$

CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

y-int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**

43. Graph:  $f(x) = \frac{x^2 - x - 6}{x - 2}$

CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

y-int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**

45. Graph:  $f(x) = \frac{4x^2 + 4x - 3}{2x - 5}$

CV : Root(s):  $x =$  \_\_\_\_\_

VA: \_\_\_\_\_

HA: \_\_\_\_\_ SA: \_\_\_\_\_ PA: \_\_\_\_\_

y-int. = \_\_\_\_\_ Hole = \_\_\_\_\_

**Exclusion Chart:**

47. Graph:  $f(x) = \frac{(x-2)(x^2 + 2x + 4)}{x^2 + x - 2}$