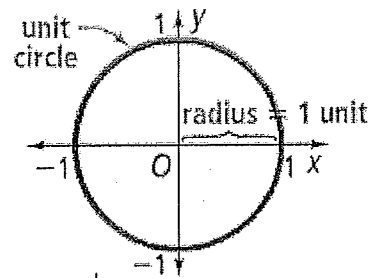
6.1 Right-Triangle Trigonometry

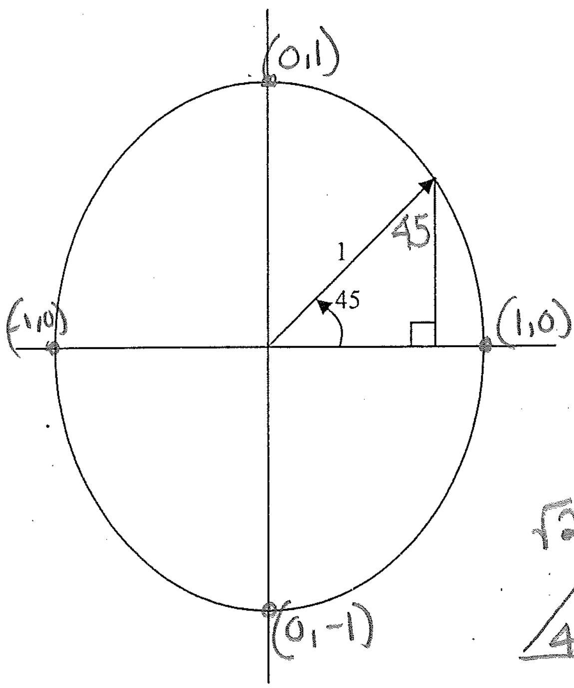
Key

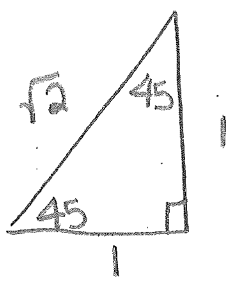
In the Unit Circle (circle with a radius of 1 unit), we are going to focus on 2 important Right Triangles:

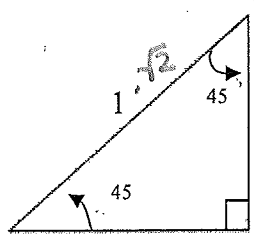
The **unit circle** has a radius of 1 unit and its center at the origin of the coordinate plane. Points on the unit circle are related to periodic functions.

You can use the symbol θ for the measure of an angle in standard position.









$x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

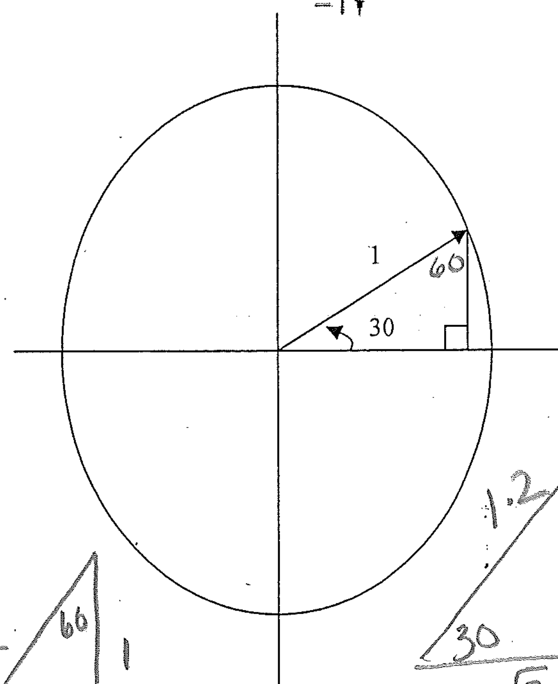
$x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

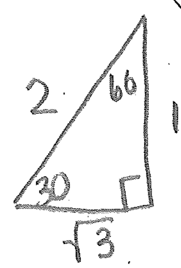
$x^2 + x^2 = 1^2$

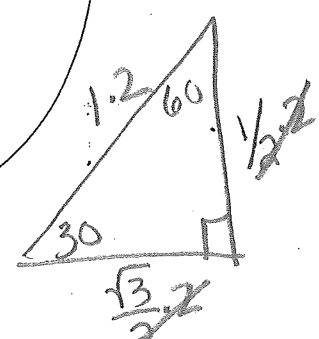
$\frac{2x^2}{2} = \frac{1}{2}$

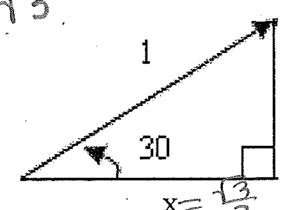
$\sqrt{x^2} = \sqrt{\frac{1}{2}}$

$x = \pm \frac{1}{\sqrt{2}}$









In geometry you learned that the side opposite the 30° angle is half of its hypotenuse.

$x^2 + \left(\frac{1}{2}\right)^2 = 1^2$

$x^2 + \frac{1}{4} = 1$

$\sqrt{x^2} = \sqrt{\frac{3}{4}}$

$x = \frac{\sqrt{3}}{2}$

Trigonometric Ratios

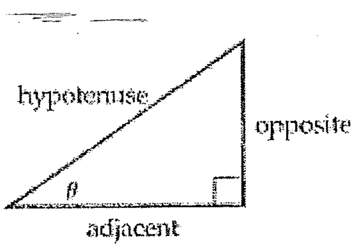


Figure 6.1-6

For a given acute angle θ in a right triangle:

The *sine* of θ, written as $\sin\theta$, is the ratio

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

The *cosine* of θ, written as $\cos\theta$, is the ratio

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

The *tangent* of θ, written as $\tan\theta$, is the ratio

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

In addition, the reciprocal of each ratio above is also a trigonometric ratio.

cosecant of θ

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\sin\theta}$$

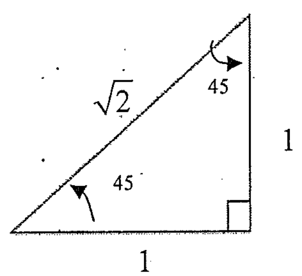
secant of θ

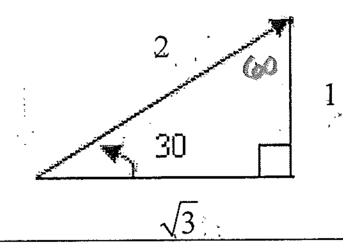
$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{1}{\cos\theta}$$

cotangent of θ

$$\cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{1}{\tan\theta}$$

NOTE The Greek letter θ (theta) is commonly used to label the measure of an angle in trigonometry.





sin 45 = $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	sin 30 = $\frac{1}{2}$	sin 60 = $\frac{\sqrt{3}}{1}$
cos 45 = $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	cos 30 = $\frac{\sqrt{3}}{2}$	cos 60 = $\frac{1}{2}$
tan 45 = $\frac{1}{1} = 1$	tan 30 = $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	tan 60 = $\frac{\sqrt{3}}{1}$
csc 45 = $\frac{\sqrt{2}}{1}$	csc 30 = $\frac{2}{1}$	csc 60 = $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
sec 45 = $\frac{\sqrt{2}}{1}$	sec 30 = $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	sec 60 = $\frac{2}{1}$
cot 45 = $\frac{1}{1}$	cot 30 = $\frac{\sqrt{3}}{1}$	cot 60 = $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

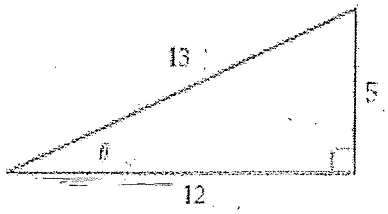


Figure 6.1-7

Example 2 Evaluating Trigonometric Ratios

Evaluate the six trigonometric ratios of the angle θ shown in Figure 6.1-7.

Solution

The opposite side has length 5, the adjacent side has length 12, and the hypotenuse has length 13.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{13} \approx 0.3846 \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{13}{5} = 2.6$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{13} \approx 0.9231 \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{13}{12} \approx 1.0833$$

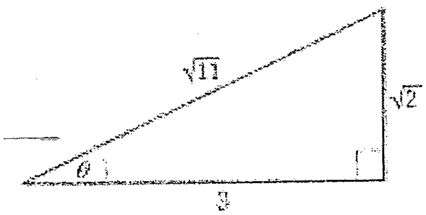
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{5}{12} \approx 0.4167 \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{12}{5} = 2.4$$

When using the Calculator to evaluate a trig ratio, make sure your calculator is in degree mode for this section.

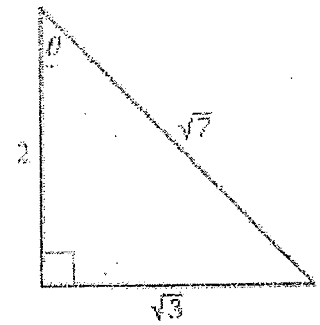
cos 68 =	**cot 39 =**	**csc 25 =**
Keystrokes for graphing calculator: cos 68 = .3746	Keystrokes for graphing calculator: 1 ÷ tan 39 = 1.235	Keystrokes for graphing calculator: 1 ÷ sin 25 = 2.366
Keystrokes for Ti-30 XA: 68 Cos = .3746	Keystrokes for Ti-30 XA: 1 ÷ 39 tan = 1.235	Keystrokes for Ti-30 XA: 1 ÷ 25 Sin

In Exercises 9–14, find the six trigonometric ratios for θ.

9.



11.



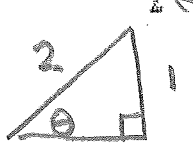
In Exercises 15–20, use a calculator in degree mode to find the following. Round your answers to four decimal places.

15. $\sin 32°$ 16. $\cos 68°$ 17. $\tan 6°$

18. $\csc 25°$ 19. $\sec 47°$ 20. $\cot 39°$

In Exercises 21–26, use the exact values of the trigonometric ratios for the special angles to find a value of θ that is a solution of the given equation. (See Example 5.)

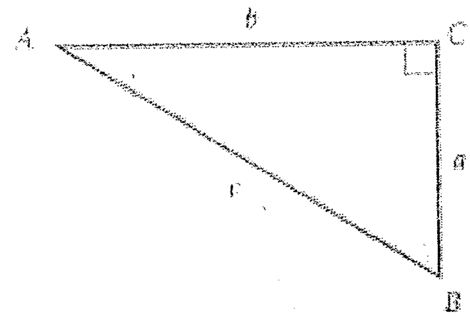
21. $\sin\theta = \frac{1}{2}$ ← opp, ← hyp 22. $\tan\theta = 1$ 23. $\csc\theta = \sqrt{2}$



$\theta = 30°$

24. $\cot\theta = \sqrt{3}$ 25. $\cos\theta = \frac{1}{2}$ 26. $\sec\theta = 2$

In Exercises 27–32, refer to the figure below. Find the exact value of the trigonometric ratio for the given values of a, b, and c.



27. $a = 4, b = 2, \tan B =$ ___?___

28. $a = 5, c = 7, \sin A =$ ___?___

29. $b = 3, c = 8, \cos A =$ ___?___

30. $a = 12, b = 15, \cot A =$ ___?___

31. $a = 7, c = 16, \sec B =$ ___?___

32. $b = 2, c = 3, \csc B =$ ___?___