

## 5.2 Trigonometric Applications

In this section, you will be using your knowledge of right triangle trig (SOH, CAH, TOA) to find missing sides and missing angles on right triangles. Also, recall that the Pythagorean Theorem and your knowledge about proportions is helpful in finding unknown sides of right triangles.

### Solving Right Triangles

#### Objectives

- Solve triangles using trigonometric ratios
- Solve applications using triangles

**Triangle Sum Theorem:** The sum of the measures of the angles in a triangle is  $180^\circ$ .

**Pythagorean Theorem:** In a right triangle with legs  $a$  and  $b$  and hypotenuse  $c$ ,  $a^2 + b^2 = c^2$ .

Make sure your calculator is in degree mode!

#### Example 1 Finding a Side of a Triangle

Find side  $x$  of the right triangle in Figure 6.2-1.

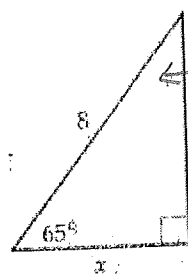


Figure 6.2-1

$$90 - 65 = 25^\circ$$

$$\cos 65^\circ = \frac{x}{8}$$

$$8 \cos 65 = x$$

$$3.38 = x$$

$$\sin 65 = \frac{y}{8}$$

$$8 \sin 65 = y$$

$$y = 7.25$$

#### Example 2 Finding an Angle of a Triangle

Find the measure of angle  $\theta$  in Figure 6.2-2.

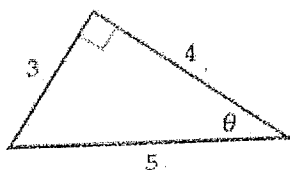


Figure 6.2-2

$$\sin \theta = \frac{3}{5}$$

$$36.87^\circ$$

$$\cos \theta = \frac{4}{5}$$

$$\tan \theta = \frac{3}{4}$$

$$\sin^{-1}$$

Figure 6.2-3d

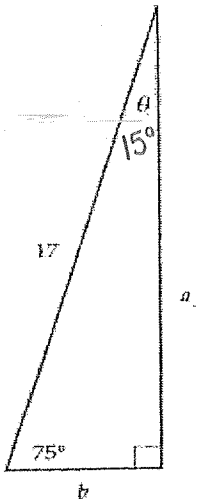


Figure 6.2-4

**Example 3** Solving a Right Triangle

Solve the right triangle in Figure 6.2-4.

$$\theta = 90 - 75 = 15^\circ$$

$$\sin 75 = \frac{a}{17} \quad \left| \quad \cos 75 = \frac{b}{17} \right.$$

$$17 \sin 75 = a$$

$$a = 16.42$$

$$b = 4.40$$

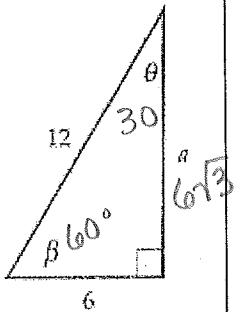


Figure 6.2-5

**Example 4** Solving a Right Triangle

Solve the right triangle in Figure 6.2-5.

$$a^2 + b^2 = 12^2$$

$$a^2 + 36 = 144$$

$$\sqrt{a^2} = \sqrt{108}$$

$$a = 6\sqrt{3}$$

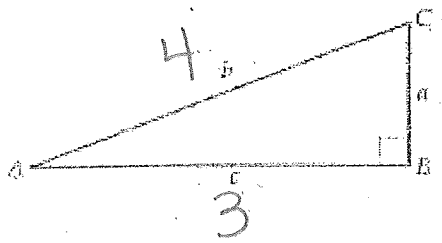
$$\sin \theta = \frac{6}{12}$$

$$\sin^{-1}(6/12) = 30^\circ$$

Example 5:

These are like 1-5 in Homework!

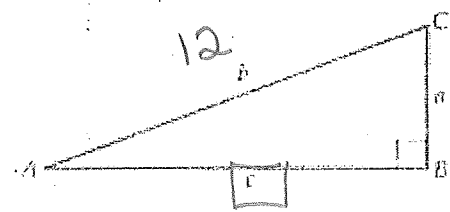
$\sin C = \frac{3}{4}$  ← opp / hyp       $b = 12$       find  $c$



$$\frac{4}{3} = \frac{12}{c}$$

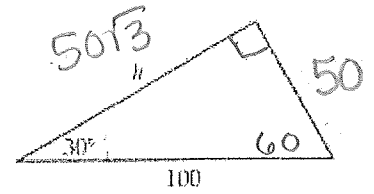
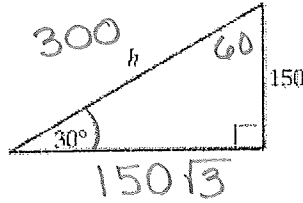
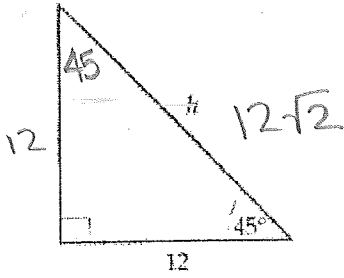
$$4c = 36$$

$$c = 9$$



Example 6: Solve for h.

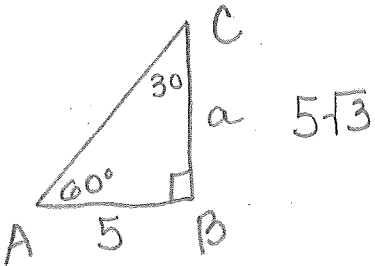
These are like 7-12.



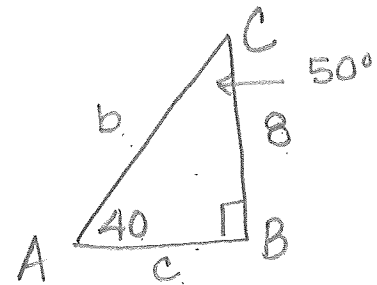
Example 7: Find the indicated value without using a calculator.

These are like 13-16.  
Hint... draw a triangle.

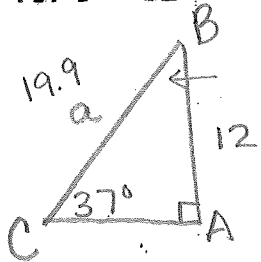
$c = 5$        $m\angle A = 60^\circ$       Find  $a$ .



In Exercises 17-24, solve the triangle with the given conditions.



18.  $c = 12$



$$m\angle C = 37^\circ$$

$$m\angle B = 53^\circ$$

$$\sin 37 = \frac{12}{a}$$

$$.6018a = 12$$

$$a = 19.94$$

$$\cos 37 = \frac{b}{19.94} \quad b = 15.9$$

20.  $a = 8$

$m\angle A = 40^\circ$

$$\tan 40 = \frac{8}{c}$$

$$c = \frac{8}{\tan 40}$$

$$c = 9.43$$

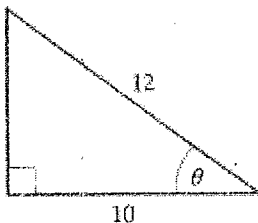
$$\sin 40 = \frac{8}{b}$$

$$b = \frac{8}{\sin 40}$$

$$\sin 40$$

$$b = 12.45$$

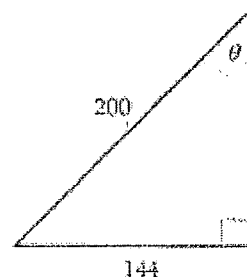
26.



$$\cos \theta = \frac{10}{12}$$

$$\cos^{-1}\left(\frac{10}{12}\right) = 33.56^\circ$$

28.

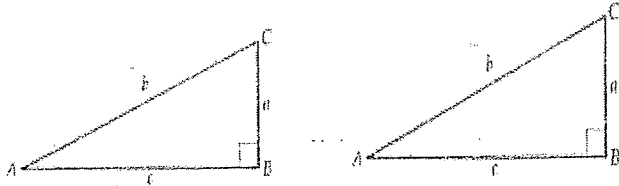
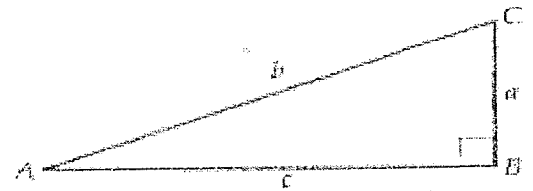


$$\sin \theta = \frac{144}{200}$$

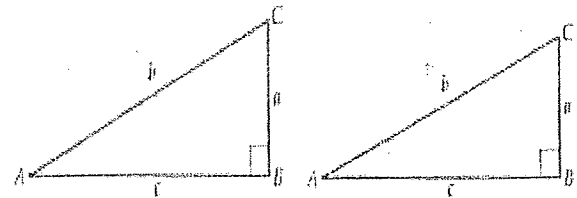
$$\sin^{-1}\left(\frac{144}{200}\right) \approx 46.1$$

## Exercises 6.2

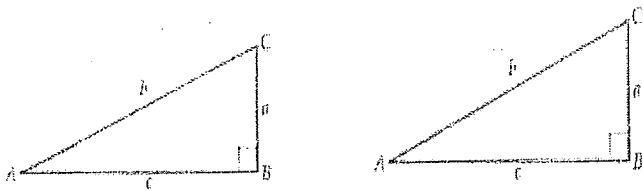
In Exercises 1–6, find side  $c$  in the figure below by using the given conditions.



1.  $\cos A = \frac{12}{13}$        $b = 39$



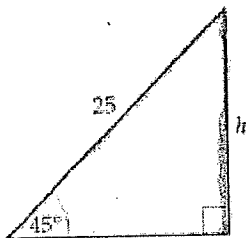
5.  $\cot A = 6$        $a = 14$



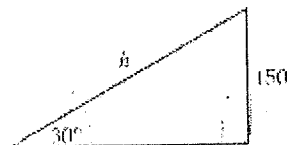
3.  $\tan A = \frac{5}{12}$        $a = 15$

In Exercises 7–12, find the exact value of  $h$  without using a calculator.

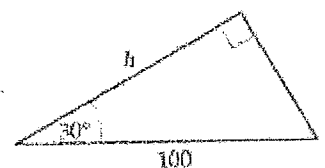
7.



9.

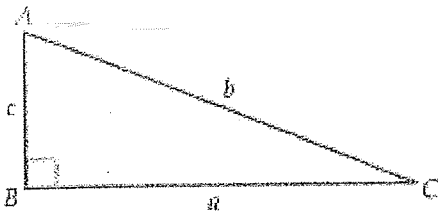


11.



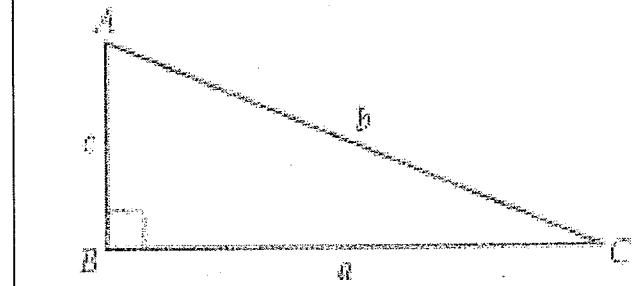
Use the figure below for Exercises 13-24.

In Exercises 13-16, find the indicated value without using a calculator.



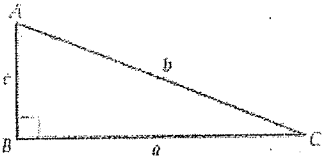
13.  $a = 4$        $m\angle A = 60^\circ$       Find  $c$ .

15.  $c = 10$        $m\angle A = 30^\circ$       Find  $a$ .

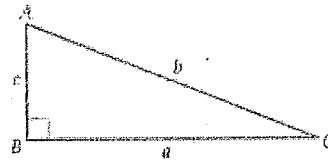


In Exercises 17-24, solve the triangle with the given conditions.

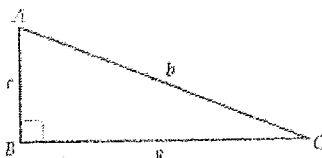
17.  $b = 10$        $m\angle C = 50^\circ$



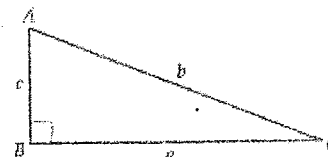
19.  $a = 6$        $m\angle A = 14^\circ$



21.  $c = 5$        $m\angle A = 65^\circ$

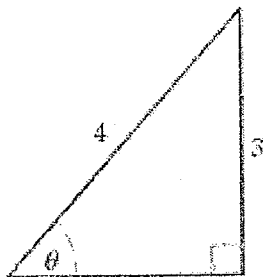


23.  $b = 3.5$        $m\angle A = 72^\circ$

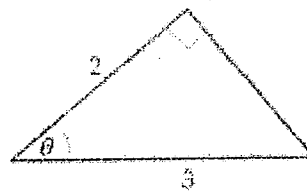


In Exercises 25–28, find angle  $\theta$ .

25. \_\_\_\_\_



27.



In Exercises 29–36, use the figure for Exercises 13–24 to find angles  $A$  and  $C$  under the given conditions.

Hint..... Draw a triangle!

29.  $a = 4$  and  $c = 6$

31.  $a = 7$  and  $b = 10$

33.  $b = 18$  and  $c = 12$

35.  $a = 2.5$  and  $c = 1.4$

29.

31.

33.

35.