

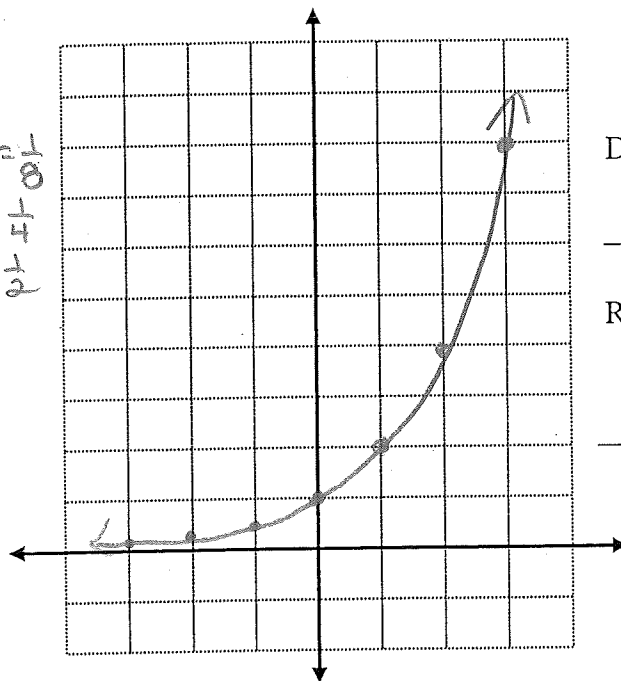
Graphs of Exponential Functions

- Use a calculator to complete the following table for the function  $y = 2^x$ .
- Plot the points on the grid provided and connect them into a smooth "curve."

$y = 1 \cdot 2^x$

$y = 2^x$

x	y
-3	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	4
3	8



Domain of  $y = 2^x$ :

all real numbers

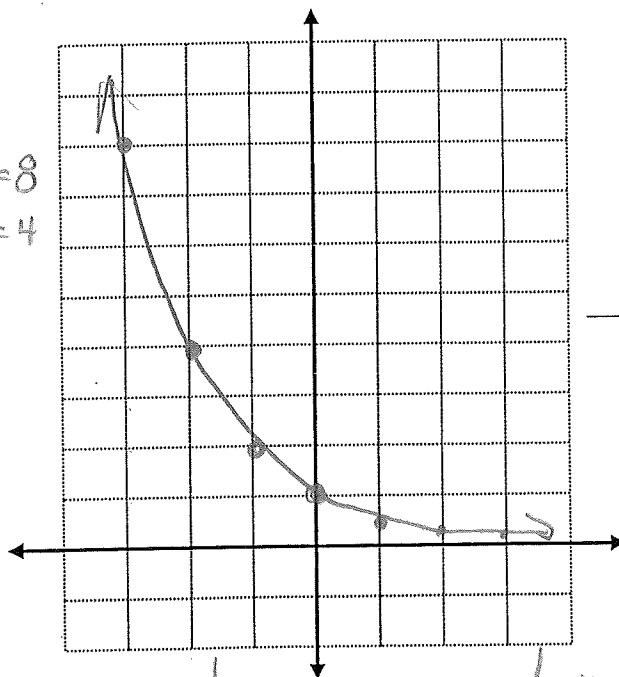
Range of  $y = 2^x$ :

$y > 0$

- Use a calculator to complete the following table for the function  $y = (\frac{1}{2})^x$ .
- Plot the points on the grid provided and connect them into a smooth "curve."

$y = (\frac{1}{2})^x$

x	y
-3	$(\frac{1}{2})^{-3} = 2^3 = 8$
-2	$(\frac{1}{2})^{-2} = 2^2 = 4$
-1	$(\frac{1}{2})^{-1} = 2$
0	$(\frac{1}{2})^0 = 1$
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$



$y = 1 \cdot (\frac{1}{2})^x$

Domain of  $y = (\frac{1}{2})^x$ :

all real numbers

Range of  $y = (\frac{1}{2})^x$ :

$y > 0$

How are these two graphs related?

domain all real numbers /  $y > 0$

5. Where do both curves cross the y-axis?

(0, 1)

Where do they cross the x-axis?

they don't

**Exponential growth** happens when a quantity INCREASES by a fixed rate each time period. The first example showed exponential growth, which is why the graph went **upward**.

$$y = ab^x \quad \text{if } b > 1$$

**Exponential decay** happens when a quantity DECREASES by a fixed rate each time period. The second example showed exponential decay, which is why the graph went **downward**.

$$0 < b < 1 \quad y = ab^x$$

The following example shows how to determine whether a function is linear, quadratic, or exponential given a table of values for the function.

6. Determine if each table represents a linear, quadratic, or exponential function.

a. quadratic    b. exponential    c. linear    d. exponential

x	y
10	4
20	14
30	29
40	49

+10  
+15  
+20

x	y
0	5
5	10
10	20
15	40
20	80

$$y = ab^x$$

$$y = 5 \cdot 2^x$$

x	y
0	3
2	7
4	11
6	15
8	19

+4  
+4  
+4  
+4

$$y = mx + b$$

$$y = 2x + 3$$

x	y
0	5
3	20
6	80
9	320
12	1280

\*4  
\*4  
\*4  
\*4

$$y = 5 \cdot 4^x$$

IF the x values have a constant change then for each type of function the following is true:

Quadratic: The y values have a common second difference.

Linear: The y values have a common first difference.

Exponential: The y values have a common ratio.

The following example shows how to determine whether a function is linear, quadratic, or exponential given the equation of the function.

Type of Function	General Equation	Example
Linear Function	$y = mx + b$ $y' = x'$	$y = 3x + 5$ $y = -2x - 6$
Quadratic Function	$y = x^2$ $y = x^2$	$y = 3x^2$ $y = (x - 3)^2 + 5$
Exponential Function	$y = ab^x$ $y = b^x$	$y = 5(7)^x$ $y = 400(2)^{-x}$

**Examples**

Identify each function as linear, quadratic, or exponential.

7.  $y = 0.3x^2 + 6$  quadratic      8.  $y = 15x - 8$  linear  
 9.  $y = 2 - 3^x$  exponential      10.  $y = x(x - 8)$  quadratic

The growth in the value of investments earning **compound** interest is modeled by an exponential

function.      Compound Interest       $A(t) = P \left( 1 + \frac{r}{n} \right)^{nt}$

$n$  = number of times a year interest is earned  
 $A$  = Amount in account at time  $t$ .  
 $P$  = Dollars invested.

$r$  = Interest rate. (Percentage expressed as a decimal)

$n$  = Compounding period.

$t$  = Time in years.

- $n = 1$  annual
- $n = 2$  semi/biannual
- $n = 4$  quarterly
- $n = 12$  monthly
- $n = 365$  daily

semi annual  $n = 1/2$

11. Find the amount of a \$100 investment after 10 years at 5% interest compounded annually, semiannually, quarterly, and daily.

a. Annually  $n=1$

$$A = P \left( 1 + \frac{r}{n} \right)^{n \cdot t}$$

$$A = 100 \left( 1 + \frac{.05}{1} \right)^{1 \cdot 10}$$

$$= 162.89$$

b. Semiannually  $n=2$

$$100 \left( 1 + \frac{.05}{2} \right)^{2 \cdot 10}$$

$$= \$163.86$$

c. Quarterly  $n=4$

$$100 \left( 1 + \frac{.05}{4} \right)^{4 \cdot 10}$$

$$164.36$$

d. Daily  $n=365$

$$100 \left( 1 + \frac{.05}{365} \right)^{365 \cdot 10}$$

$$\$164.87$$

Interest can also be compounded continuously. The formula for **continuously** compounded interest uses the number  $e$ .

$$A(t) = P \cdot e^{r \cdot t}$$

12. Suppose you won a contest at the start of 5<sup>th</sup> grade that deposited \$3000 in an account that pays 5% annual interest compounded continuously. How much will you have after 4 years?

$e \approx 2.718...$

$$A = P e^{r \cdot t}$$

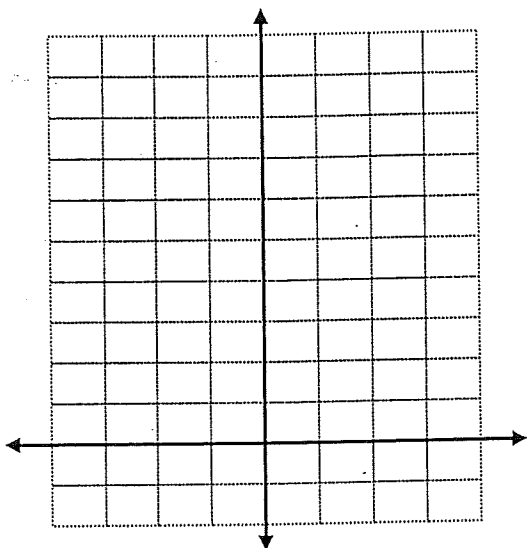
$$= 3000 e^{(.05)(4)}$$

$$\$3664.21$$

# HOMWORK!

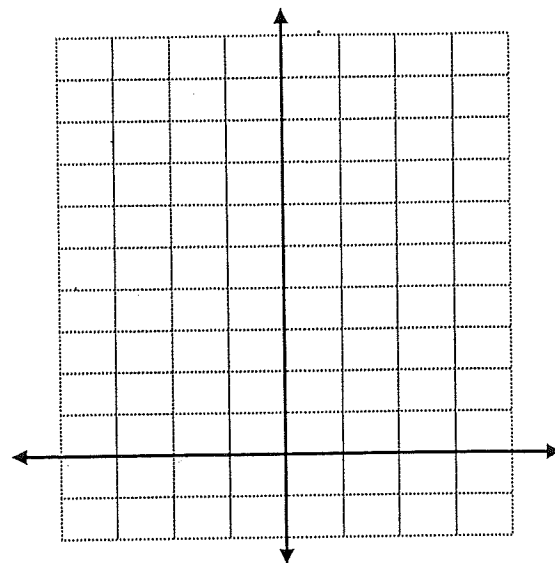
1.  $y = 3^x$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



2.  $y = 2\left(\frac{1}{5}\right)^x$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



3. Determine if each table represents a linear, quadratic, or exponential function.

a. \_\_\_\_\_ b. \_\_\_\_\_ c. \_\_\_\_\_ d. \_\_\_\_\_

x	y
4	25
8	14
12	3
16	-8

x	y
1	6
2	18
3	54
4	162

x	y
12	75
14	43
16	20
18	6

x	y
3	160
6	80
9	40
12	20

Identify each function as linear, quadratic, or exponential.

4.  $g(x) = 10x + 3$  \_\_\_\_\_

5.  $f(x) = (44 - x)x$  \_\_\_\_\_

6.  $f(x) = 12(12.5)^x$  \_\_\_\_\_

7.  $h(x) = 0.5^x - 3.5$  \_\_\_\_\_

8. The final amount for \$5000 invested for 25 years at 10% annual interest compounded semiannually is \$57,337.

a. What is the effect of doubling the amount invested?

b. What is the effect of doubling the annual interest rate?

c. What is the effect of doubling the investment period?

d. Which of the above has the greatest effect on the final amount of the investment?