Algebra 2
Section 7.4 Notes
Properties of Logarithmic Functions

Name: Key

Date:_____ Hour:_____

Evaluate each logarithm.

1. $\log_4 4 = x$

$4^x = 4$

$x = 1$

2. $\log_8 \frac{1}{8} = x$

$8^x = \frac{1}{8}$ $(x=-1)$

3. $\log_2 32 = x$

$2^x = 32$

$x = 5$

Each property of exponents has a corresponding property of logarithms.

Properties of exponents:

$a^m \cdot a^n = a^{m+n}$ $\qquad (a^m)^n = a^{m \cdot n}$ $\qquad \frac{a^m}{a^n} = a^{m-n}$

Product Property of Logarithms

$\log_b(m \cdot n) = \log_b m + \log_b n \qquad m > 0 \ , \ n > 0 \ , \ b > 0 \ , \ b \neq 1$

4. $\log_2(2xy) =$ $\log_2 2 + \log_2 x + \log_2 y$

$1 + \log_2 x + \log_2 y$

Quotient Property of Logarithms

$\log_b \frac{m}{n} = \log_b m - \log_b n \qquad m > 0 \ , \ n > 0 \ , \ b > 0 \ , \ b \neq 1$

5. $\log_2 \frac{2x}{y} =$ $\log_2 2x - \log_2 y$

$\log_2 2 + \log_2 x - \log_2 y$

Power Property of Logarithms

$\log_b m^p = p \log_b m \qquad m > 0 \ , \ b > 0 \ , \ b \neq 1 \ , \ p =$ any real #

6. $\log_b(a^3) =$ $3 \log_b a$

7. $\log_5 5^2 =$ $2 \log_5 5$

$2 \cdot 1$

2

*** * ALL OF THE POWER PROPERTIES CAN BE USED IN BOTH DIRECTIONS * ***

Write each expression as a single logarithm. Then simplify, if possible.

8. $\log_3 8 + \log_3 2 =$

$\log_3 8 \cdot 2$

$\log_3 16$

$\log_3 16 = x$

$3^x = 16$

9. $\log_3 12 - \log_3 4 =$

$\log_3 \frac{12}{4}$

$\log_3 3 = 1$

10. $3\log_2 r + 5\log_2 n =$

$\log_2 r^3 + \log_2 n^5$

$\log_2 r^3 n^5$ or $\log_2 n^5 r^3$

Write each expression as a sum or difference of logarithms. Then simplify, if possible. (Expand each logarithm).

11. $\log_6 (36ac) =$ $\log_6 36 + \log_6 a + \log_6 c$

$2 + \log_6 a + \log_6 c$

$\log_6 36 = x$

$6^x = 36$

12. $\log_4 \frac{x}{64} =$ $\log_4 x - \log_4 64$

$\log_4 x - 3$

$\log_4 64 = x$

$4^x = 64$

$x = 3$

13. $\log_7 (3x)^3 =$ $3\log_7 3x$

$3(\log_7 3 + \log_7 x)$

$3\log_7 3 + 3\log_7 x$

$3x^3$

Remember:
To use the product and quotient property of logarithms the __base__ *MUST be the* __same__.

Change of Base Formula	$\log_3 16 = \frac{\log 16}{\log 3}$
For any positive real numbers $a \neq 1, b \neq 1$, and $x > 0$	$\log_b x = \frac{\log_a x}{\log_a b}$

We will let $a = 10$ in the change of base formula, so that we can use the log key on our calculator.

$\log_{10} 3 = \log 3$

Evaluate each logarithmic expression to the nearest hundredth.

14. $\log_7 56 = X$

$7^x = 56$

15. $9 + \log_3 29$

$\downarrow$

$\frac{\log 29}{\log 3}$

Logarithmic functions define many measurement scales in the sciences, including the pH, decibel, and Richter scale.

The function $y = -\log_{10} x$ is used in chemistry to measure pH levels. The pH of a solution describes its acidity. Substances that are more acidic have a lower pH, while substances that are less acidic, or basic, have a higher pH. The pH of a substance is defined as $\text{pH} = -\log_{10}[H^+]$ where $[H^+]$ is the hydrogen ion concentration of a solution in moles per liter.

Examples:

16. The pH of a carbonated soda is 3. What is the $[H^+]$ for this soda?

$3 = -\log_{10} H^+$

$-\log_{10} H^+ = 3$

$\log_{10} H^+ = -3$

$10^{-3} = H^+$

or →

$10^3 = X^{-1}$

$1000 = X^{-1}$

$1000 = \frac{1}{X}$

$1000X = 1$

$X = \frac{1}{1000}$

17. Find the pH for a solution with a $[H^+] = 3 \times 10^{-2}$.

$\frac{1}{10^3} = H^+$

$\frac{1}{1000} = H^+$

$ph = -1 \log_{10} (3 \times 10^{-2})$

$ph = -1 \log_{10} .03$ ← use a calc.

$ph = 1.5229$

HOMEWORK

Write each expression as a single logarithm.

1. $\log 7 + \log 2$

2. $\log_2 9 - \log_2 3$

3. $\log 8 - 2\log 6 + \log 3$

Expand each logarithm.

4. $\log x^3y^5$

5. $\log_7 49xyz$

6. $\log \frac{a^2b^3}{c^4}$

7. $\log_4 5\sqrt{x}$

Use the Change of Base Formula to evaluate each expression.

8. $\log_2 9$

9. $\log_5 10$

10. $\log_4 7$

11. **The concentration of hydrogen ions in a batch of homemade ketchup is 10^{-4}. What is the pH level of the ketchup?** (Use $\text{pH} = -\log_{10}\left[H^+\right]$)