

An exponential equation is any equation that contains the form b^{cx} , such as $a = b^{cx}$ where the exponent includes a variable.

A logarithmic equation is an equation that includes one or more logarithms involving a variable.

You can use logarithms to solve exponential equations. You can use exponents to solve logarithmic equations.

Solve.

1. $16^{3x} = 8$

$$\begin{aligned} (2^4)^{3x} &= 2^3 \\ 2^{12x} &= 2^3 \\ \frac{12x}{12} &= \frac{3}{12} \\ x &= \frac{1}{4} \end{aligned}$$

2. $27^{2x} = \frac{1}{9}$

$$\begin{aligned} (3^3)^{2x} &= \frac{1}{3^2} \\ 3^{6x} &= 3^{-2} \\ 6x &= -2 \\ x &= -\frac{1}{3} \end{aligned}$$

When we can't get the bases to be the same we can use the following property.

If $x = y$ and $x > 0$ and $y > 0$, then $\log x = \log y$

Solve each equation for x . Round your answer to the nearest ten-thousandth if necessary.

3. $15^{3x} = 285$

$$\log_{15} 285 = 3x$$

4. $8 + 3^x = 10$

$$\begin{aligned} \frac{8}{-8} \quad \frac{-8}{-8} \\ 3^x &= 2 \\ \log_3 2 &= x \\ x &= .631 \end{aligned}$$

To solve an equation that contains logarithms: isolate the logarithms on one side, use the logarithm properties to combine the logs, and then change to exponential form and solve.

5. $\log(5x+2) = 2$

$$\begin{aligned} 10^2 &= 5x+2 \\ 100 &= 5x+2 \\ 98 &= 5x \\ x &= 19.6 \end{aligned}$$

6. $\log(2x^2) - \log 5 = 1$

$$\begin{aligned} \log_{10} \frac{2x^2}{5} &= 1 \\ 10 &= \frac{2x^2}{5} \\ 50 &= 2x^2 \\ \sqrt{25} &= \sqrt{x^2} \\ \pm 5 &= x \end{aligned}$$

7. $\log(x-3) + \log x = 1$

Condense

$$\begin{aligned} \log_{10} (x-3)(x) &= 1 \\ 10 &= (x-3)x \\ 10 &= x^2 - 3x \\ 0 &= x^2 - 3x - 10 \\ 0 &= (x-5)(x+2) \\ x &= 5 \quad x \neq -2 \end{aligned}$$

8. Your MP3 player has about 126,000,000 bytes of memory. Each month you plan to use 5% of the memory remaining. How many months will it take you to use $\frac{1}{4}$ of the memory?

$$\begin{aligned} 126,000,000 (1 - .05)^t &= 31,500,000 \\ 126,000,000 (.95)^t &= 31,500,000 \\ .95^t &= .25 \end{aligned}$$

$$\begin{aligned} \log_{.95} .25 &= t \\ t &= 27.03 \end{aligned}$$

HOMEWORK**Solve each equation.**

1. $4^{3x} = 64$

2. $2^{5x+1} = 32$

Solve each equation. Round to the nearest ten-thousandth.

3. $2^x = 3$

4. $8 + 10^x = 1008$

5. $9^{2y} = 66$

6. The equation $y = 6.72(1.014)^x$ models the world population y , in billions of people, x years after the year 2000. Find the year in which the world population is about 8 billion.

Solve each equation. Check your answers.

7. $2 \log x = -1$

8. $\log x + 4 = 8$

9. $3 \log x = 1.5$

Solve each equation.

10. $\log 2x + \log x = 11$

11. $\log 5 - \log 2x = 1$

12. As a town gets smaller, the population of its high school decreases by 6% each year. The senior class has 160 students now. In how many years will it have about 100 students? Write an equation. Then solve the equation without graphing.