Key

5.5 Properties and Laws of Logarithms

Basic Properties of Logarithms

Common logarithms	Natural logarithms
1. $\log v$ is defined only when $v > 0$	1. $\ln v$ is defined only when $v > 0$
2. $\log 1 = 0$ and $\log 10 = 1$	2. $\ln 1 = 0$ and $\ln e = 1$
3. $\log 10^k = k$ for every real number k	3. $\ln e^k = k$ for every real number k
4. $10^{\log v} = v$ for every $v > 0$	4. $e^{\ln v} = v$ for every $v > 0$

Product Law of Logarithms

For all $v, w > 0$,

$$\log(vw) = \log v + \log w$$

$$\ln(vw) = \ln v + \ln w.$$

Example 2 Using the Product Law of Logarithms

Use the Product Law of Logarithms to evaluate each logarithm.

a. Given that $\log 3 = 0.4771$ and $\log 11 = 1.0414$, find $\log 33$.

b. Given that $\ln 7 = 1.9459$ and $\ln 9 = 2.1972$, find $\ln 63$.

a. $\log 33$
$\log 3 \cdot 11$
$\log 3 + \log 11$
$.4771 + 1.0414$
1.5185

b. $\ln 63$
$\ln 7.9$
$\ln 7 + \ln 9$
$1.9459 + 2.1972$
4.1431

Quotient Law of Logarithms

For all $v, w > 0$,

$$\log\left(\frac{v}{w}\right) = \log v - \log w$$

$$\ln\left(\frac{v}{w}\right) = \ln v - \ln w.$$

Example 3 Using the Quotient Law of Logarithms

Use the Quotient Law of Logarithms to evaluate each logarithm.

a. Given that $\log 28 = 1.4472$ and $\log 7 = 0.8451$, find $\log 4$.

b. Given that $\ln 18 = 2.8904$ and $\ln 6 = 1.7918$, find $\ln 3$.

a. $\log 4$
$\log \frac{28}{7}$
$\log 28 - \log 7$
$1.4472 - .8451$
$.6021$

b. $\ln 3$
$\ln \frac{18}{6}$
$\ln 8 - \ln 6$
$2.8904 - 1.7918$
1.0986

Power Law of Logarithms

For all k and $v > 0$,

$$\log v^k = k \log v,$$

$$\ln v^k = k \ln v.$$

Example 4 Using the Power Law of Logarithms

Use the Power Law of Logarithms to evaluate each logarithm.

a. Given that $\log 6 = 0.7782$, find $\log \sqrt{6}$.

b. Given that $\ln 50 = 3.9120$, find $\ln \sqrt[3]{50}$.

a.	b. $\ln \sqrt[3]{50}$
$\log \sqrt{6}$	$\ln 50^{1/3}$
$\log 6^{1/2}$	$\ln 50^{1/3}$
$\frac{1}{2} \log 6$	$\frac{1}{3} \ln 50$
$\frac{1}{2}(.7782)$	$\frac{1}{3}(3.9120)$
.3891	1.304

Example 5 Simplifying Expressions

Write $\ln 3x + 4 \ln x - \ln 3xy$ as a single logarithm.

$\ln 3x + \ln x^4 - \ln 3xy$

$\ln \frac{3x^5}{3xy}$

$\ln \frac{x^4}{y}$

Example 6 Simplifying Expressions

Simplify $\ln\left(\frac{\sqrt{x}}{x}\right) + \ln(\sqrt[4]{ex^2})$.

$\ln \frac{\sqrt{x}}{x} \cdot \frac{\sqrt[4]{ex^2}}{1}$

$\ln \frac{x^{1/2} \cdot e^{1/4} \cdot x^{2/4}}{x}$

$\ln \sqrt[4]{e}$

A). $\ln 2x + 2\ln x - \ln 3y$

$\ln \frac{2x \cdot x^2}{3y}$

$\ln \frac{2x^3}{3y}$

B). $\ln e^2 x + \ln ey - 3$

$2\ln e + \ln x + \ln e + \ln y - 3$

$2 + \ln x + 1 + \ln y - 3$

$\ln xy$

Given → $\log 7 \approx .8451$, $\log 3 \approx .4771$, $\log 5 \approx .6990$, $\log 2 \approx .3010$

using the product, quotient and power laws, find….

a. $\log 12$

$\log 2 \cdot 2 \cdot 3$

$\log 2 + \log 2 + \log 3$

$.3010 + .3010 + .4771$

b. $\log \frac{3}{14}$

$\frac{3}{2 \cdot 7}$

$\log 3 - (\log 2 + \log 7)$

$.4771 - (.3010 + .8451)$

$-.669$

c. $\log 8$

$\log 2^3$

$3 \log 2$

$3(.3010)$

$.9030$

Example: Given $u = \ln x$ and $v = \ln y$, write the expression in terms of u+ v

$\ln x^3 y^2$

$3 \ln x + 2 \ln y$

$3u + 2v$

Example: $\log(2x) = 3$

$\log_{10}(2x) = 3$

$10^3 = 2x$

$1000 = 2x$

$500 = x$

Example: $5 + \ln(x-1) = 8$

$-5 \quad -5$

$\ln(x-1) = 3$

$\downarrow$

$\log_e(x-1) = 3$

$e^3 = x - 1$

$20.09 = x - 1$

$21.09 = x$

Exercises 5.5

In Exercises 1–4, solve each equation by using the basic properties of logarithms.

1. $\log(x - 3) = 2$

3. $\ln(x + 4) = -1$

In Exercises 5–10, use laws of logarithms and the values given below to evaluate each logarithmic expression.

$\log 7 = 0.8451$ $\log 5 = 0.6990$
$\log 3 = 0.4771$ $\log 2 = 0.3010$

5. $\log 8$

7. $\log\left(\frac{5}{7}\right)$

9. $\log 0.6$

In Exercises 11–20, write the given expression as a single logarithm.

11. $\ln x^2 + 3 \ln y$

13. $\log(x^2 - 9) - \log(x + 3)$

15. $2 \ln x - 3(\ln x^2 + \ln x)$

17. $3 \ln(e^3 - e) - 3$

19. $\log 10x + \log 20y - 1$

In Exercises 21–26, let $u = \ln x$ and $v = \ln y$. Write the given expression in terms of u and v. For example,
$\ln x^3y = \ln x^3 + \ln y = 3 \ln x + \ln y = 3u + v$.

21. $\ln(x^2y^5)$

23. $\ln(\sqrt{x} \cdot y^2)$

25. $\ln\left(\sqrt[3]{x^2\sqrt{y}}\right)$