5.5 A. Logarithmic Functions

Name_____

Example 1 Evaluating Logarithms to Other Bases

Without using a calculator, find each value.

a. $\log_2 16 = x$

$2^x = 16$

$2^x = 2^4$

$x = 4$

b. $\log_{\frac{1}{3}} 9 = x$

$\frac{1}{3}^x = 9$

$\frac{1}{3}^x = 3^2$

$\frac{1}{3}^x = \frac{1}{3}^{-2}$

$x = -2$

c. $\log_5(-25)$

NO SOL.

Example 2 Solving Logarithmic Equations

Solve each equation for x.

a. $\log_5 x = 3$

$5^3 = x$

$125 = x$

b. $\log_6 1 = x$

$6^x = 1$

$x = 0$

c. $\log_{\frac{1}{6}}(-3) = x$

N. S

d. $\log_6 6^1 = x$

$x = 1$

Example 3 Solving Logarithmic Equations

Solve the equation $\log_3(x - 1) = 4$.

$3^4 = x - 1$

$81 = x - 1$

$82 = x$

Example 4 Using the Laws of Logarithms

Use the Laws of Logarithms to evaluate each expression, given that $\log_7 2 = 0.3562$, $\log_7 3 = 0.5646$, and $\log_7 5 = 0.8271$.

a. $\log_7 10$

$\log_7(2 \cdot 5)$

$\log_7 2 + \log_7 5$

$.3562 + .8271$

1.1833

b. $\log_7 2.5$

$\log_7 2\frac{1}{2}$

$\log_7 \frac{5}{2}$

$\log_7 5 - \log_7 2$

$.8271 - .3562$

$.4709$

c. $\log_7 48$

$6 \cdot 8$

$2 \cdot 3 \quad 2 \cdot 2 \cdot 2$

$\log_7 2^4 \cdot 3$

$4 \log_7 2 + \log_7 3$

$4(.3562) + .5646$

1.9894

Example 5 Using the Laws of Logarithms

Simplify and write each expression as a single logarithm.

a. $\log_3(x + 2) + \log_3 y - \log_3(x^2 - 4)$

b. $3 - \log_5(125x)$

a) $\log_3 \dfrac{(x+2)y}{(x^2-4)}$ $\quad (x-2)(x+2)$

a) $\log_3 \dfrac{y}{x-2}$

b) $3 - \log_5(125x)$

$3 - \log_5 5^3 x$

$3 - 3(\log_5 5) - \log_5 x$

$3 - 3(1) - \log_5 x$

$\log_5 \dfrac{1}{x}$

Change-of-Base Formula

Scientific and graphing calculators have a LOG key and a LN key for calculating logarithms. No calculators have a key for logarithms to other bases. One way to evaluate logarithms to other bases is to use the formula below.

Change-of-Base Formula

For any positive number v,

$$\log_b v = \frac{\log v}{\log b} \quad \text{and} \quad \log_b v = \frac{\ln v}{\ln b}$$

Example 6 Evaluating Logarithms to Other Bases

Evaluate $\log_8 9$.

Nspire $\log_{\square} \square = 1.057$

$\dfrac{\log 9}{\log 8} = 1.057$

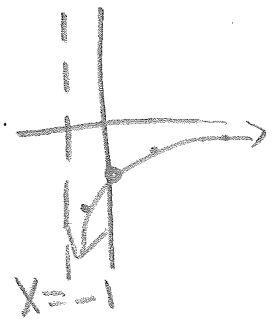
Example 7 Transforming Logarithmic Functions $\quad y = \log_2 x \quad 2^y = x$

Describe the transformation from $g(x) = \log_2 x$ to $h(x) = \log_2(x + 1) - 3$. Give the domain and range of h.

1 left down 3

x	y
1/2	-1
1	0
2	1
4	2

x	y
-1/2	-4
0	-3
1	-2
3	-1



d: $x > -1$

r: all y

Exercises 5.5.A

Note: Unless stated otherwise, all letters represent positive numbers and $b \neq 1$.

In Exercises 1–10, translate the given exponential statement into an equivalent logarithmic statement.

1. $10^{-2} = 0.01$
2. $10^3 = 1000$
3. $\sqrt[3]{10} = 10^{\frac{1}{3}}$
4. $10^{0.4771} \approx 3$
5. $10^{?k} = r$
6. $10^{[m+n]} = c$
7. $7^8 = 5{,}764{,}801$
8. $2^{-3} = \frac{1}{8}$

In Exercises 11–20, translate the given logarithmic statement into an equivalent exponential statement.

11. $\log 10{,}000 = 4$
12. $\log 0.001 = -3$
13. $\log 750 = 2.8751$
14. $\log 0.8 = -0.0969$
15. $\log_5 125 = 3$
16. $\log_8\left(\frac{1}{4}\right) = -\frac{2}{3}$
17. $\log_2\left(\frac{1}{4}\right) = -2$
18. $\log_2 \sqrt{2} = \frac{1}{2}$

In Exercises 21–28, evaluate the given expression without using a calculator.

21. $\log 10^{\sqrt{43}}$
22. $\log_{17}(17^{17})$
23. $\log 10^{\sqrt{x^2+y^2}}$
24. $\log_{3.5}(3.5^{(x^2-1)})$
25. $\log_{16} 4$
26. $\log_2 64$
27. $\log_{\sqrt{3}}(27)$
28. $\log_{\sqrt{3}}\left(\frac{1}{9}\right)$

Answers:

1. $\log_{10} .01 = -2$
2.
3.
4.
5.
6.
7.
8.
11. $10^4 = 10{,}000$
12.
13.
14.
15.
16.
17.
18.
21. $\sqrt{43}$
22.
23.
24.
25.
26.
27.
28.

In Exercises 29–36, find the missing entries in each table.

$f(x) = \log_4 x$

$4^y = x$

29.

x	0	1	2	4
$f(x) = \log_4 x$	~~?~~	? 0	1/2	1

30.

x	$\frac{1}{25}$	5	25	$\sqrt{5}$
$g(x) = \log_5 x$	?	?	?	?

31.

x	?	$\frac{1}{6}$	1	216
$h(x) = \log_6 x$	-2	?	?	?

32.

x	$\frac{10}{3}$	4	6	12
$k(x) = \log_3(x - 3)$	?	?	?	?

33.

x	0	$\frac{1}{7}$	$\sqrt{7}$	49
$f(x) = 2\log_7 x$	?	?	?	?

34.

x	?	?	100	1000
$g(x) = 3\log x$	6	3	?	?

35.

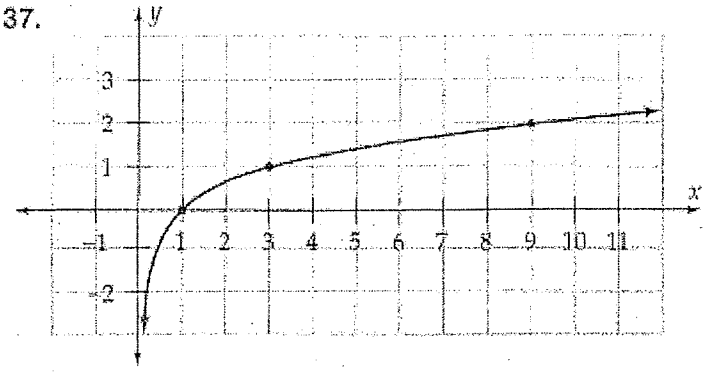
x	-2.75	-1	1	29
$h(x) = 3\log_2(x + 3)$	?	?	?	?

36.

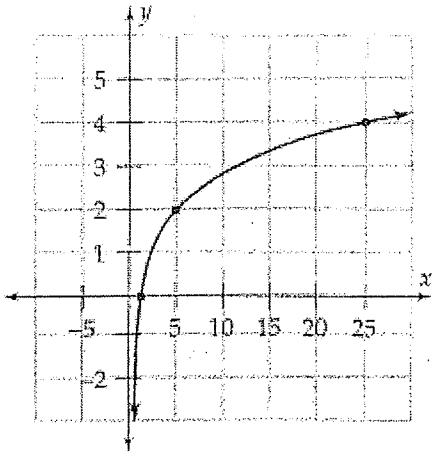
x	$\frac{1}{e}$	1	e	e^2
$k(x) = 2\ln x$	?	?	?	?

In Exercises 37–40, a graph or a table of values is given for the function $f(x) = \log_b x$. Find b.

37.



38.



In Exercises 41–46, solve each equation for x.

41. $\log_3 243 = x$

42. $\log_{81} 27 = x$

43. $\log_{27} x = \frac{1}{3}$

44. $\log_5 x = -4$

45. $\log_x 64 = 3$

46. $\log_x \left(\frac{1}{9}\right) = -\frac{2}{3}$

In Exercises 47–60, write the given expression as the logarithm of a single quantity. (See Example 5.)

47. $2\log x + 3\log y - 6\log z$

48. $5\log_8 x - 3\log_8 y + 2\log_8 z$

49. $\log x - \log(x+3) + \log(x^2-9)$

50. $\log_3(y+2) + \log_3(y-3) - \log_3 y$

51. $\frac{1}{2}\log_2(25c^2)$

52. $\frac{1}{3}\log_2(27b^6)$

53. $-2\log_4(7c)$

54. $\frac{1}{3}\log_5(x+1)$

55. $2\ln(x+1) - \ln(x+2)$

56. $\ln(z-3) + 2\ln(z+3)$

57. $\log_2(2x) - 1$

58. $2 - \log_5(25z)$

59. $2\ln(e^2 - e) - 2$

60. $4 - 4\log_5(20)$

In Exercises 61–68, use a calculator and the change-of-base formula to evaluate the logarithm.

61. $\log_2 10$

62. $\log_2 22$

63. $\log_7 5$

64. $\log_5 7$

65. $\log_{500} 1000$

66. $\log_{500} 250$

In Exercises 69–72, describe the transformation from f to g, and give the domain and range of g.

69. $f(x) = \log_5 x$ and $g(x) = \log_5(3x - 4)$

70. $f(x) = \log_7 x$ and $g(x) = -2 \cdot \log_7(x+5)$

71. $f(x) = \log_2 x$ and $g(x) = \frac{1}{3} \cdot \log_2(x-1) + 7$

72. $f(x) = \log_3 x$ and $g(x) = 3\log_4(-2x)$

Answers:

47. _____

48. _____

49. _____

50. _____

51. _____

52. _____

53. _____

54. _____

55. _____

56. _____

57. _____

58. _____

59. _____

60. _____

61. _____

62. _____

63. _____

64. _____

65. _____

66. _____

69. _____

70. _____

71. _____

72. _____