5.4 Common and Natural Logarithmic Functions

$2^3 = 8$

$2^? = 10$ ← How will we solve this problem?

$2^4 = 16$

Exponential Form	Logarithmic Form
$2^3 = 8$	$\log_2 8 = 3$
$2^4 = 16$	$\log_2 16 = 4$

If the base is not written in the problem, it is understood that it is base 10.

Example 1 Evaluating Common Logarithms

Without using a calculator, find each value.

a. $\log 1000$ b. $\log 1 = x$ c. $\log \sqrt{10}$ d. $\log(-3)$

$\log_{10} 1000$ $10^x = 1$ $10^x = \sqrt{10}$ N.S

$10^x = 1000$ $x = 0$ $10^x = 10^{1/2}$

$x = 3$ $x = 1/2$

Example 2 Using Equivalent Statements

Solve each equation by using an equivalent statement.

a. $\log_2 x = 2$ b. $10^x = 29$

$10^2 = x$ $\log_{10} 29 = x$

$100 = x$ 1.4624

When the base and the number are the same like, $\log_3 3^1$, $\log_4 4^1$, $\log_7 7^1$, $\log_{12} 12^1$, etc. the answer will always be 1.

Example 3 Evaluating Natural Logarithms

Use a calculator to find each value.

a. $\ln 0.15$ b. $\ln 186$ c. $\ln(-5)$

$\log_e .15$ 5.23 N.S

-1.89712

Example 4: Solve for x. $\log_2 \frac{1}{8} = x$

use calc

-3

$2^x = \frac{1}{8}$

$2^x = \frac{1}{2^3}$

$2^x = 2^{-3}$

$x = -3$

When we change from base 10 log (common logs) to base e (natural logs), we use "ln" instead of "log".

$\ln = \log_e$

Example 5: Solve a. $\ln e^3$ b. $\ln e^{x^2+2y}$

$\log_e e^3$ $\log_e e^{(x^2+2y)}$

3 x^2+2y

1st way	2nd way
c. $e^{\ln 34.17}$	$e^{\ln 34.17} = x$
34.17	$\log_e x = \ln 34.17$
	$\ln x = \ln 34.17$
	$x = 34.17$

Exercises 5.4

Unless stated otherwise, all letters represent positive numbers.

In Exercises 1–4, find the value of each logarithm.

1. $\log 10{,}000$

2. $\log 0.001$

3. $\log \dfrac{\sqrt{10}}{1000}$

4. $\log \sqrt[3]{0.01}$

In Exercises 5–14, translate the given logarithmic statement into an equivalent exponential statement.

5. $\log 1000 = 3$

6. $\log 0.001 = -3$

7. $\log 750 = 2.8751$

8. $\log 0.8 = -0.0969$

9. $\ln 3 = 1.0986$

10. $\ln 10 = 2.3026$

11. $\ln 0.01 = -4.6052$

12. $\ln s = r$

13. $\ln(x^2 + 2y) = z + w$

14. $\log(a + c) = d$

In Exercises 15–24, translate the given exponential statement into an equivalent logarithmic statement.

15. $10^{-2} = 0.01$

16. $10^3 = 1000$

17. $10^{0.4771} = 3$

18. $10^{7k} = r$

19. $e^{3.25} = 25.79$

20. $e^{-4} = 0.0183$

21. $e^{\frac{12}{7}} = 5.5527$

22. $e^{k} = t$

23. $e^{\frac{2}{r}} = w$

24. $e^{4mw} = m$

In Exercises 25–36, evaluate the given expression without using a calculator.

25. $\log 10^{\sqrt{43}}$

26. $\log 10^{\sqrt{x^2+y^2}}$

27. $\ln e^{15}$

28. $\ln e^{3.78}$

29. $\ln \sqrt{e}$

30. $\ln \sqrt[5]{e}$

31. $e^{\ln 931}$

32. $e^{\ln 34.17}$

33. $\ln e^{r+5}$

34. $\ln e^{x^2+2y}$

35. $e^{\ln x^2}$

36. $e^{\ln \sqrt{x+}}$