Algebra 2

Notes Section 6.6
Function Operations

Name Key
Date _____ Hour _____

Take note **Key Concepts Function Operations**

Addition	$(f+g)(x) = f(x) + g(x)$
Subtraction	$(f-g)(x) = f(x) - g(x)$
Multiplication	$(f \cdot g)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

The domains of the sum, difference, product, and quotient functions consist of the x-values that are in the domains of *both* f and g. Also, the domain of the quotient function does not contain any x-value for which $g(x) = 0$.

Example 1 Let $f(x) = 2x^2+8$ and $g(x) = x-3$. Perform each function operation and then find the domain of the result.

a. $(f+g)(x)$

$2x^2+8 + x-3$
$2x^2+x+5$
d: arn

b. $(f-g)(x)$

$(2x^2+8)-(x-3)$
$2x^2-x+11$
arn

c $(g-f)(x)$

$(x-3)-(2x^2+8)$
$-2x^2+x-11$
arn

Example 2 Let $f(x) = 3x^2-11x-4$ and $g(x) = 3x+1$. Perform each function operation and then find the domain of the result.

a. $(f \cdot g)(x)$

$(3x^2-11x-4)(3x+1)$
$9x^3+3x^2-33x^2-11x-12x-4$
$9x^3-30x^2-23x-4$
d: arn

b. $\left(\frac{f}{g}\right)(x)$

$\frac{3x^2-11x-4}{3x+1}$
arn $x \neq -1/3$
$\frac{(3x+1)(x-4)}{(3x+1)}$

c. $2f(x)-g(x)-3$

$2(3x^2-11x-4)-(3x+1)-3$
$6x^2-22x-8-3x-1-3$
$6x^2-25x-12$
arn

Example 3 Evaluate the function f for the given value. $f(x) = x^2-3x+5$ for $f(-6)$

$(-6)^2-3(-6)+5$
$36+18+5$
$36+23$
59

take note

Key Concept Composition of Functions

The composition of function g with function f is written as $g \circ f$ and is defined as $(g \circ f)(x) = g(f(x))$. The domain of $g \circ f$ consists of the x-values in the domain of f for which $f(x)$ is in the domain of g.

$(g \circ f)(x) = g(f(x))$ 1. Evaluate $f(x)$ first.
2. Then use $f(x)$ as the input for g.

Function composition is not commutative since $f(g(x))$ does not always equal $g(f(x))$.

Example 4 Let $f(x) = x - 5$ and $g(x) = x^2$. Find each value or expression.

a. $(f \circ g)(x)$

$f(x) = x - 5 \quad g(x) = x^2$

$x^2 - 5$

b. $(f \circ g)(-3)$ = $f(g(-3))$

$(-3)^2 = 9$

$f(9) = 9 - 5 = 4$

c. $(g \circ f)(-3)$ $g(f(-3))$

$f(-3) = -3 - 5 = -8$

$g(-8) = 64$

Example 5 Find $f(g(x))$ and $g(f(x))$ for $f(x) = 2x^2 - 1$ and $g(x) = 3x$

a. $f(g(x))$ =

$f(x) = 2x^2 - 1 \quad g(x) = 3x$

$2(3x)^2 - 1$

$(3x)(3x)$

$2(9x^2)$

$18x^2 - 1$

b. $g(f(x))$ =

$g(x) = 3x \quad f(x) = 2x^2 - 1$

$3(2x^2 - 1)$

$6x^2 - 3$

Example 6 You have a coupon good for \$5 off the price of any large pizza. You also get a 10% discount on any pizza if you show your student ID. How much more would you pay for a large pizza if the cashier applies the coupon first?

Coupon first

\$ of pizza = x

$.90(x - 5)$

$.90(10 - 5)$

$.90(5)$

\$ 4.50

vs

discount first

$.90x - 5$

$.90(10) - 5$

$9 - 5$

\$ 4

Cheaper

Name _____ Class _____ Date _____

6-6 Practice

Form K

Function Operations

Let $f(x) = 4x + 8$ and $g(x) = 2x - 12$. Perform each function operation and then find the domain of the result.

1. $(f + g)(x)$

   $f(x) + g(x)$

2. $(f - g)(x)$

3. $(f \cdot g)(x)$

4. $\left(\frac{f}{g}\right)(x)$

Let $f(x) = x + 2$ and $g(x) = \sqrt{x} - 1$. Perform each function operation and then find the domain of the result.

5. $(f + g)(x)$

6. $(f \cdot g)(x)$

7. $\left(\frac{f}{g}\right)(x)$

8. $\left(\frac{g}{f}\right)(x)$

Let $f(x) = x - 2$ and $g(x) = x^2$. Find each value. To start, use the definition of composing functions to find a function rule.

9. $(g \circ f)(4)$

   $f(4) = 4 - 2 = 2$

10. $(f \circ g)(-1)$

11. $(g \circ f)(-3)$

Let $f(x) = \sqrt{x}$ and $g(x) = (x + 2)^2$. Find each value.

12. $(f \circ g)(-5)$

13. $(f \circ g)(0)$

14. $(g \circ f)(4)$

Name _____ Class _____ Date _____

6-6 Practice (continued) Form K

Function Operations

15. A car dealer offers a 15% discount off the list price x of any car on the lot. At the same time, the manufacturer offers a \$1000 rebate for each purchase of a car.
    a. Write a function $f(x)$ to represent the price after discount.
    b. Write a function $g(x)$ to represent the price after the \$1000 rebate.
    c. Suppose the list price of a car is \$18,000. Use a composite function to find the price of the car if the discount is applied before the rebate.
    d. Suppose the list price of a car is \$18,000. Use a composite function to find the price of the car if the discount is applied after the rebate.
    e. **Reasoning** Between parts (c) and (d), will the dealer want to apply the discount before or after the rebate? Why?

16. **Error Analysis** $f(x) = 2\sqrt{x}$ and $g(x) = 3x - 6$. Your friend gives a domain for $\left(\frac{f}{g}\right)(x)$ as $x \geq 0$. Is this correct? If not, what is the correct domain?

Let $f(x) = 2x^2 - 3$ and $g(x) = \frac{x+1}{2}$. Find each value.

17. $f(g(2))$

18. $g(f(-3))$

19. $(f \circ f)(-1)$

20. **Reasoning** A local bookstore has a sale on all their paperbacks giving a 10% discount. You also received a coupon in the mail for \$4 off your purchase. If you buy 2 paperbacks at \$8 each, is it less expensive for you to apply the discount before the coupon or after the coupon? How much will you save?