Algebra 2
Section 7.1 Notes
Exponential Functions

Name: ______________________
Date: ____________ Hour: __________

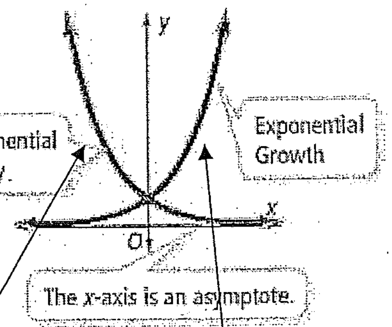
An exponential function is a function in the general form: $y = ab^x$,
$a \neq 0,\ b > 0,\ b \neq 1$ ex. $y = 3(4)^x$ $y = 1.2^x$

a b

Two types of exponential behavior are *exponential growth* and *exponential decay*.

For **exponential growth**, as the value of x increases, the value of y increases. For **exponential decay**, as the value of x increases, the value of y decreases, approaching zero.

The exponential functions shown here are *asymptotic* to the x-axis. An **asymptote** is a line that a graph approaches as x or y increases in absolute value.



Concept Summary Exponential Functions

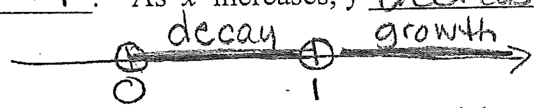
For the function $y = ab^x$,
- if $a > 0$ and $b > 1$, the function represents exponential growth.
- if $a > 0$ and $0 < b < 1$, the function represents exponential decay.

In either case, the y-intercept is $(0, a)$, the domain is all real numbers, the asymptote is $y = 0$, and the range is $y > 0$.

Two types of exponential functions are growth and decay

Exponential growth is represented by a function of the form $y = ab^x$ where $b > 1$. As x increases, y increase

Exponential Decay is represented by a function of the form $y = ab^x$ where $0 < b < 1$. As x increases, y decreases



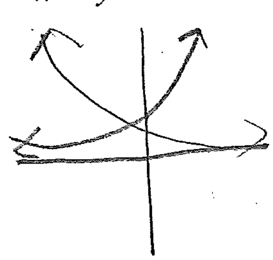
Tell whether each function represents exponential growth or decay. Then find the y-intercept.

1. $y = 14(0.3)^x$ decay (0,14)

2. $y = \frac{1}{2}(1.03)^x$ growth (0,1/2)

3. $y = 2000(1.4)^x$ growth (0,2000)

4. $y = 45^{-x}$ decay (0,1)



growth

5. Bacteria reproduce, or grow in number, by dividing. If we have 1 bacteria and it doubles every hour, how many bacteria would we have after 25 hours. If we have 100 bacteria and double every hour, how many would we have after 25 hours?

$\boxed{y = 1 \cdot 2^x}$

$y = 2^{25}$

hour	# bacteria	
0	1	$*2$ $\quad 2^0$
1	2	2^1
2	4	$*2$ $\quad 2^2$
3	8	$*2 \rightarrow 2^3$
25	2^{25}	

33,554,432

hour	# bacteria	
0	100	$*2$
1	200	$*2$
2	400	

$y = 100(2)^{25}$

3,355,443,20

Example 5 is an example of growth.

An equation for exponential growth is $\boxed{y = a(b)^x}$ where $a =$ y intercept / initial amount
$x =$ time periods
$b =$ base

b is the base or growth factor.
decay

Predict the population of bacteria for each situation and time period.

6. 225 bacteria that triple every hour for 7 hours

$y = 225 \cdot 3^x$

$y = 225 \cdot 3^7$

492,075 bacteria

7. 340 bacteria that double every half hour for 6 hours

$y = 340 \cdot 2^x$

$y = 340 \cdot 2^{12}$

1,392,640 bacteria

When something increases or decreases by a **percent** it is an exponential growth or decay. The growth or decay factor will be $1 + r$ / $1 - r$ where $r =$ percent in decimal

So the equation becomes: $y = a(1 \pm r)^t$ where $a =$ initial amount / y int.
$r =$ rate
$t =$ time #of time

$60\% = r = .6$

$1.25\% = r = .0125$

10 years 1+r

8. The population of the United States was 248,718,301 in 1990 and was projected to grow at a rate of about 8% per decade. Predict the population, to the nearest hundred thousand, ~~for 2025~~.

.08

1990 > 1 dec.
2000 > 1 dec
2010 > 1 dec
2020
30 30

$t=3.5$

$y=248{,}718{,}301(1+.08)^x$

$y=248{,}718{,}301(1.08)^x$

$y=248{,}718{,}30(1.08)^{3.5}$

325,604,866 people

9. The rate at which caffeine is eliminated from the bloodstream of an adult is about 15% per hour. An adult drinks a caffeinated soda, and the caffeine in his or her bloodstream reaches a peak of 30 milligrams. Predict the amount, to the nearest tenth of a milligram, of caffeine remaining 1 hour after the peak and 4 hours after the peak.

$(1-.15)=.85$

$y=30(.85)^x$

$y=30(.85)^1$
$=25.5$ mg

$y=30(.85)^4$
$=15.7$ mg

10. A new computer that sells for \$1350 depreciates 14% per year. What is its estimated value after 5 years? Round to the nearest ten dollars.

$y=1350(1-.14)^x$

$y=1350(.86)^x$

$y=1350(.86)^5$

\$635.08

\$640

11. For the given annual rate of change, find the corresponding growth or decay factor.

+500%	+250%	-50%	-25%
$1+5=6$	$1+2.5$ 3.5	$1-.5$ $.5$	$1-.25$ $.75$

HOMEWORK!!!!!

Without graphing, determine whether the function represents exponential growth or decay. Then find the y-intercept.

1. $f(x) = 4\left(\frac{5}{6}\right)^x$
2. $y = 12\left(\frac{17}{10}\right)^x$
3. $f(x) = 2^{-x}$

4. Suppose you deposit \$2000 in a savings account that pays interest at an annual rate of 4%. If no money is added or withdrawn from the account, answer the following questions.
   a. How much will be in the account after 3 years?
   b. How much will be in the account after 18 years?
   c. How many years will it take for the account to contain \$2500?
   d. How many years will it take for the account to contain \$3000?

Write an exponential function to model each situation. find each amount after the specified time.

5. A population of 120,000 grows 1.2% per year for 15 years.

6. A population of 1,860,000 decreases 1.5% each year for 12 years.