

Algebra 2

Name _____

Exam Review 2nd Semester

Chapter 7

Section 7.1



1. Without graphing, determine whether the function represents exponential growth or exponential decay. Then find the y-intercept.

$y = 4.6 \left(\frac{3}{2}\right)^x$ growth (0, 4.6)

2. The initial population of bacteria in a lab is 240. The number of bacteria doubles every 2 hours. Predict the bacteria population at the end of 8 hours. SHOW WORK

$y = 240(2)^t$ t=4 ~~XXXX~~
 $y = 240(2)^4$ 3840

3. The population of Rochester was 17,500 in 1990. The population was projected to grow at a rate of about 4.5% per decade. Predict the population of the city, to the nearest thousand, in the year 2025. SHOW WORK

3.5 $y = 17,500(1 + .045)^{3.5}$
 $y = 20,414.796$ 20,000

4. Pat bought a car for \$19,500. The salesperson projected that the value of the car would decline by 18% per year for the next 5 years. What would be the value, to the nearest hundred dollars, of Pat's car after 5 years? SHOW WORK

$= 19,500(1 - .18)^5$ 7200
 7229 $A = Pe^{rt}$

5. Find the final amount of the investment if \$4000 is invested at 6.2% interest compounded monthly for 8 years. SHOW WORK

$A = P \left(1 + \frac{r}{n}\right)^{n \cdot t}$
 $4000 \left(1 + \frac{.062}{12}\right)^{12 \cdot 8}$
 \$6560.18

6. You deposit \$5500 in an account at 3.6% compounded continuously. How much do you have in the account after 10 years? SHOW WORK

$A = Pe^{r \cdot t}$ $e \approx 2.71$
 $A = 5500 e^{.036 \cdot 10}$
 \$7883.31

Section 7.3

7. Write the equation in logarithmic form.

$12^2 = 144$ $\log = \text{exponent}$
 $\log_{12} 144 = 2$

8. Write the equation in exponential form.

$\log_4 \frac{1}{64} = -3$ $4^{-3} = \frac{1}{64}$

Solve for x. **Show work.**

9. $\log_x \frac{1}{243} = -5$

$x^{-5} = \frac{1}{243}$

$x = 3$

$\frac{1}{3^5} = 3^{-5}$
 $x^{-5} = 3^{-5}$

10. $\log_{36} x = \frac{1}{2}$

$36^{1/2} = x$
 $\sqrt{36} = x$
 $x = 6$

11. $\log_2 x = 6$

$2^6 = x$

$x = 64$

12. $\log_x 8 = \frac{1}{3}$

$x^{1/3} = 8$
 $(\sqrt[3]{x})^3 = (8)^3$
 $x = 512$

Section 7.4

Write each expression as a sum or difference of logarithms. Simplify, if possible.

13. $\log_7(49y^2z)$

$\log_7 49 + 2\log_7 y + \log_7 z$

14. $\log_5 \frac{m^2}{5p}$

$2\log_5 m - \log_5 5 - \log_5 p$

Write each expression as a single logarithm. Then simplify, if possible.

15. $2\log_5 xy + \log_5 100 - \log_5 2$

$\log_5 x^2 y^2 + \log_5 100 - \log_5 2$

$\log_5 \frac{100x^2y^2}{2} = \log_5 50x^2y^2$

Best!

16. $\frac{1}{4}\log_2 81 - 2\log_2 4x$

$\log_2 \frac{\sqrt[4]{81}}{16x^2}$

$\log_2 \frac{3}{16x^2}$

Best answer

Evaluate the logarithmic expression. Round your answer to the nearest hundredth. **SHOW WORK**

17. $\log_3 62 + 1$

$\frac{\log 62}{\log 3}$

4.76

18. Calculate the pH of a lemon used to make lemonade if $[H^+]$ is about $10^{-2.3}$ moles per liter.

(Use the formula $pH = -\log_{10} [H^+]$)

$pH = -\log_{10} 10^{-2.3}$
 $-2.3 \cdot -1 = \log_{10} 10$

2.3

Section 7.5

Solve for x. Round your answer to the nearest hundredth **SHOW ALL WORK**

19. $\log_{216}(3x-7) = \frac{1}{3}$ rewrite in exp.

$$216^{1/3} = 3x-7$$

$$x = 4.33$$

20. $\log_5(3x+10) - 3\log_5 4 = 2$

$$\log_5 \frac{3x+10}{4^3} = 2$$

$$5^2 = \frac{3x+10}{4^3}$$

$$1600 = 3x+10$$

$$x = 530$$

21. $\log_2(x+2) + \log_2(x-5) = 3$

$$\log_2(x+2)(x-5) = 3$$

$$2^3 = (x+2)(x-5)$$

$$8 = x^2 - 3x - 10$$

$$0 = x^2 - 3x - 18$$

$$(x+3)(x-6)$$

$$x = -3 \quad x = 6$$

$$x = 6$$

Solve each equation for x. Round your answer to the nearest hundredth. **SHOW ALL WORK**

22. $8^{x+5} = 120$

$$\log_8 120 = x+5$$

$$x = -2.7$$

23. $12,500(1.07^x) = 80$

$$\frac{12,500}{-12} (1.07^x) = \frac{80}{-12}$$

$$\frac{5}{5} (1.07)^x = \frac{68}{5}$$

$$1.07^x = 13.6$$

$$\log_{1.07} 13.6 = x$$

24. In 2007, a painting was evaluated at \$12,500. The assessor believed the value would increase at a rate of 5% per year. At this rate, how many years will it take for the painting to be valued at \$20,000?

$$y = 12,500(1.05)^x$$

$$\frac{20,000}{12,500} = \frac{12,500}{12,500} (1.05)^x$$

$$1.6 = 1.05^x$$

rewrite in log
 $\log_{1.05} 1.6 = x$
 use calc.
9.63 years

$$4. \frac{\frac{2x+6}{x^2-9} \cdot \frac{x^2+6x+8}{2x^2+11x+12}}{\frac{x+2}{x+3}}$$

$$\frac{2(x+3)}{(x-3)(2x+3)}$$

4. _____

$$\frac{2(x+3)}{(x-3)(x+3)} \cdot \frac{(x+3)}{(x+2)} \cdot \frac{(x+4)(x+2)}{(2x+3)(x+4)}$$

Section 8.5

$$5. \frac{7}{x-3} - \frac{x+4}{x^2-5x+6}$$

5. $\frac{6}{x-2}$

$$\frac{7(x-2)}{(x-3)(x-2)} - \frac{(x+4)}{(x-3)(x-2)}$$

$$\frac{7x-14-x-4}{(x-2)(x-3)} = \frac{6x-18}{(x-2)(x-3)} = \frac{6(x-3)}{(x-2)(x-3)}$$

$$6. \frac{x-1}{x+2} + \frac{x}{x+7} - \frac{4x-7}{x^2+9x+14}$$

6. $\frac{2x}{x+7}$

$$\frac{(x-1)(x+7)}{(x+2)(x+7)} + \frac{x(x+2)}{(x+7)(x+2)} - \frac{4x-7}{(x+2)(x+7)}$$

$$\frac{x^2+6x-7+x^2+2x-4x+7}{(x+2)(x+7)} = \frac{2x^2+4x}{(x+2)(x+7)} = \frac{2x(x+2)}{(x+7)(x+2)}$$

$$\begin{array}{r} x \cdot \frac{6+\frac{2}{x}}{1} \\ 7 \cdot \frac{1}{x} \\ 2x \cdot \frac{4-\frac{3}{2x}}{1} \end{array}$$

$$\frac{6x+2}{x} = \frac{8x-3}{2x}$$

$$\frac{(6x+2)}{x} \cdot \frac{2x}{8x-3}$$

7. _____

$$\frac{2(6x+2)}{8x-3} = \frac{12x+4}{8x-3}$$

$$8. \frac{3x}{x^2-25} + \frac{5}{5-x}$$

8. _____

$$\frac{3x}{(x-5)(x+5)} + \frac{-5}{(5-x)} = \frac{3x}{(x-5)(x+5)} + \frac{5}{(x-5)(x+5)}$$

$$\frac{8x+25}{(x-5)(x+5)}$$

Section 8.6

Solve each equation.

9. $\frac{(x+3)}{8} = \frac{3x-5}{4}$

$4(x+3) = 8(3x-5)$
 $4x+12 = 24x-40$
 $52 = 20x$

9. $x = 2.6$

10. $\frac{2}{x-2} + \frac{x}{x+4} = \frac{24}{x^2+2x-8}$

~~$\frac{2}{(x-2)(x+4)} + \frac{x}{(x-2)(x+4)} = \frac{24}{(x-2)(x+4)}$~~

$2(x+4) + x(x-2) = 24$
 $2x+8 + x^2-2x = 24$
 $x^2 = 16$ $x = \pm 4$

10. $x = 4$

$x = -4$
is extraneous

11. $\frac{4}{x+5} + \frac{4}{x^2+5x} = 3$

~~$\frac{4}{(x+5)} + \frac{4}{x(x+5)} = \frac{3}{1}$~~ $x \cdot (x+5)$

$4x + 4 = 3x(x+5)$
 $4x+4 = 3x^2+15x$

11. $\frac{1}{3}, -4$

$3x^2+11x-4=0$
 $(3x-1)(x+4)=0$
 $x = \frac{1}{3} \quad x = -4$

Section 8.2

Sketch the asymptotes and the graph of each function. Identify the domain and range.

12. $y = \frac{4}{x-1} + 3$

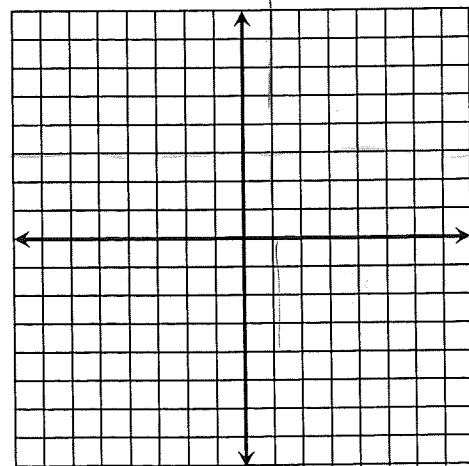
VA: $x = 1$

HA: $y = 3$

D: any $x \neq 1$

R: any $y \neq 3$

x	y
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Write an equation for the translation of $y = \frac{5}{x}$ that has the given asymptotes.

13. $x = 2$ and $y = -3$

13. $y = \frac{5}{x-2} - 3$

$$a_n = a_1 + (n-1)d$$

$$d = -2$$

2. Write an explicit formula for the sequence 8, 6, 4, 2, 0, ... then find a_{14}

$$a_n = 8 + (n-1)(-2)$$

$$a_n = -2n + 10$$

$$a_n = a_1 + (n-1)d$$

$$= 8 + (14-1)(-2) = -18$$

3. Identify the sequence as arithmetic or geometric. (Sections 9.2 & 9.3)

a. 192, 96, 48, 24, ...

b. 13, 20, 27, 34 ... $d=7$

$r = \frac{1}{2}$ geometric

arithmetic

4. Find the 50th term of the sequence 5, -2, -9, -16, ...

$$a_n = a_1 + (n-1)d$$

$$a_{50} = 5 + (50-1)(-7) = -338$$

X Find three arithmetic means between 32 and -32

$$32 \quad _ \quad _ \quad _ \quad -32$$

Section 9.3

6. Write the explicit formula for the geometric sequence. Then find the 8th term.

-4, 8, -16, ...

$$r = -2$$

$$a_n = a_1 \cdot r^{n-1}$$

$$\rightarrow a_n = -4 \cdot (-2)^{n-1}$$

$$a_n = -4(-2)^{8-1} = 512$$

7. Find the three geometric means between 486 and 6.

$$\sqrt[4]{486 \cdot 6}$$

8. Find a_{12} in the geometric sequence given $a_3 = 63$ and $a_5 = 567$.

$$\frac{7}{a_1} \quad \frac{21}{a_2} \quad \frac{63}{a_3}$$

$$a_5 = a_3 \cdot r^2$$

$$567 = 63r^2$$

$$9 = r^2$$

$$3 = r$$

$$a_n = a_1 \cdot r^{n-1}$$

$$a_{12} = 7 \cdot (3)^{12-1}$$

$$= 1,240,229$$

Sections 9.4-9.5

Find the sum of each series.

9. $-4 + (-13) + (-22) + \dots + (-76)$

$$\frac{n}{2} (a_1 + a_{12})$$

$$\frac{9}{2} (-4 + -76) = -360$$

Write the series in Summation Notation.

11. $13 + 11 + 9 + \dots + (-75)$

$$\sum_{n=1}^{45} -2n + 15$$

$$a_n = 13 + (n-1)(-2)$$

$$= -2n + 15$$

$$-75 = -2n + 15$$

$$45 = n$$

10. $1 + 4 + 16 + 64 + \dots + 65536$

$$= \frac{a_1(1-r^n)}{1-r}$$

$$= \frac{1(1-4^n)}{1-4}$$

$$a_n = a_1 \cdot r^{n-1}$$

$$65536 = 1 \cdot 4^{n-1}$$

87,381

n=9

$$-76 = -4 + (n-1)(-9)$$

$$-76 = -4 - 9n + 9$$

$$-81 = -9n$$

$$9 = n$$

Find the sum of the indicated series.

12. $\sum_{n=1}^{10} (3n+5)$ *arith.* $n = 10 - 1 + 1$
 $n = 10$
 $\frac{n}{2} (a_1 + a_{10})$
 $\frac{10}{2} (8 + 35)$
 $5(43) = 215$

13. $\sum_{n=3}^{11} (-2)(3)^{n-2}$ *geo*
 $11 - 3 = 8$
 $\frac{+1}{9}$
 $\frac{a_1(1-r^n)}{1-r}$
 $\frac{-54(1-3^9)}{1-3} = \frac{-54(1-19683)}{-2} = \frac{-54(-19682)}{-2} = -531,414$

Does the series converge or diverge. If it converges, find the sum.

14. $\frac{1}{2} + \frac{1}{16} + \frac{1}{128} + \dots$

$\frac{1}{2}, \frac{1}{16}$
 $r = \frac{1}{8}$
 $\frac{a_1}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{8}} = \frac{\frac{1}{2}}{\frac{7}{8}} = \frac{4}{14} = \frac{2}{7}$
 Converges

15. $\frac{1}{4} + \frac{3}{8} + \frac{9}{16} + \dots$

$\frac{3}{8}, \frac{1}{4}$
 $\frac{3}{8} \cdot \frac{4}{1} = \frac{12}{8} = \frac{3}{2} = r$
 $|r| > 1$
 diverges

Is the series Arithmetic or Geometric. Find the sum

16. $16 + 22 + 28 + \dots$ for 150 terms

$d=6$ *arith.*
 $\frac{n}{2} (a_1 + a_{150})$
 $\frac{150}{2} (16 + 910)$
 $69,450$
 $(= 16 + (150-1)6 = 910)$

17. $3.2 - 6.4 + 12.8 - 25.6 + \dots$ for 10 terms

geo
 $r = -2$
 $\frac{a_1(1-r^n)}{1-r}$
 $\frac{3.2(1-(-2)^{10})}{1-(-2)}$

Find the sum of the indicated series.

18. $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^{n+1}$

$a_1 = \frac{1}{16}$
 $\frac{a_1}{1-r} = \frac{\frac{1}{16}}{1-\frac{1}{4}} = \frac{\frac{1}{16}}{\frac{3}{4}} = \frac{1}{16} \cdot \frac{4}{3} = \frac{1}{12}$
 $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$
 $\frac{1}{64}$

-1091.2

19. You would like to buy a new car in 3 years. You start with \$60. Each month you plan to save 6% more than the previous month. How much will you have saved at the end of 3 years?

$n = 36$

$a_1 = 60$ $r = 1.06$
 $\frac{60(1.06^{36} - 1)}{1.06 - 1} = 7147.25$

20. An auditorium has sitting for 30 people in the 1st row, 33 in the 2nd row, 36 in the 3rd row, and so on. There are 26 rows in the auditorium. How many seats are there in the auditorium?

$30 + 33 + 36 + \dots$

26th row

$a_{26} = 30 + (26-1)(3)$

$30 + 75$
 $= 105$

$\frac{n}{2} (a_1 + a_{26})$
 $\frac{26}{2} (30 + 105)$

$13(135) = 1755$ seats

Chapter 13
Section 13.2

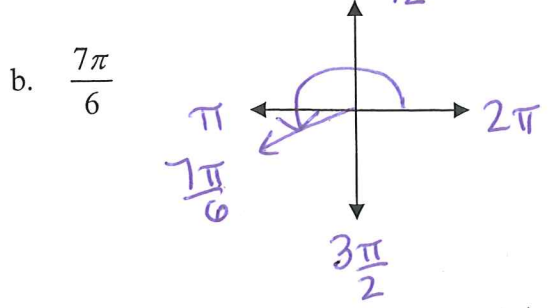
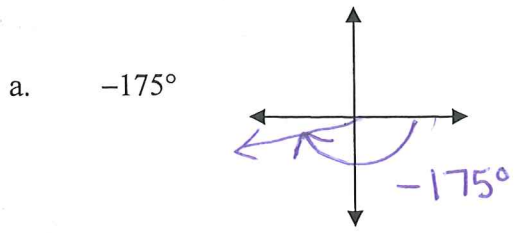
0, 360

For each angle, find all the coterminal angles such that $-360^\circ < \theta < 360^\circ$

1. $-545^\circ + 360 = -185$ 2. $862^\circ - 360 = 502$ 3. 142 4. -218

$-185, 175$ $+ 360 = 175$ $-360 = -360$ -360

3. Sketch each angle in standard position.



Section 13.3

4. Convert 144° to radians.

$$\frac{144^\circ}{1} \cdot \frac{\pi}{180^\circ} = \frac{4\pi}{5}$$

5. Convert $\frac{21}{6}\pi$ to degrees.

$$\frac{21\pi}{6} \cdot \frac{180}{\pi} = 630^\circ$$

A circle has a diameter of 64 inches. For each central angle measure, find the length, in inches to the nearest tenth, of the arc intersected by the angle. **SHOW WORK**

6. $\frac{8\pi}{3}$ radians

7. 540°

8. A 3.2 m pendulum swings through an angle of 64° . What distance does the tip of the pendulum travel?

Section 13.2

9. Find each trigonometric function value. Give **exact** answers. (Use a unit circle.)

a. $\sin(-240^\circ) = \frac{\sqrt{3}}{2}$

b. $\sec\left(\frac{5\pi}{3}\right) = \frac{2}{1}$

c. $\tan(-90^\circ) = \text{undefined}$

d. $\cot(690^\circ) = \frac{\sqrt{3}}{-1} \text{ or } -\sqrt{3}$

$\frac{-360}{330}$

10. Find each trigonometric function value. Give **exact** answers. (Use a unit circle.)

e. $\cos 2\pi = \underline{1}$

f. $\sin\left(\frac{7\pi}{6}\right) = \underline{-\frac{1}{2}}$

g. $\cos(-45^\circ) = \underline{\frac{1}{\sqrt{2}}}$
 -45°

h. $\csc\left(\frac{4\pi}{3}\right) = \underline{\frac{2}{-\sqrt{3}}}$

hyp opp

Sections 13.4-13.5

11. Identify the amplitude, if it exists, and the period of each function.

a. $y = 2\cos(\theta + 60^\circ) + 3$
 $2\cos kx + 3$

b. $y = -\frac{1}{2}\sin 5(\theta - 30^\circ) + 4$

amplitude: $\underline{2}$

amplitude: $\underline{\frac{1}{2}}$

period: $\underline{2\pi}$

period: $\underline{\frac{2\pi}{5}}$

Increment $\frac{2\pi}{5} \cdot \frac{1}{2} = \frac{\pi}{5}$

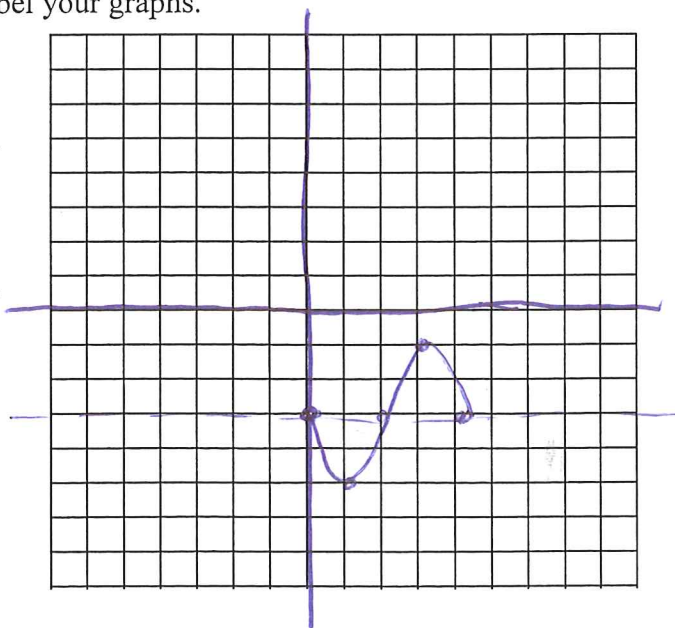
12. Find the period, amplitude, maximum, and minimum value of each function. Then **graph** at least one period of the function. Label your graphs.

$y = -2\sin(\theta - 180^\circ) - 3$
 period $\underline{2\pi}$

amplitude $\underline{2}$

maximum _____

minimum _____



13. Find the period, amplitude, maximum, and minimum value of each function. Then **graph** at least one period of the function. Label your graphs.

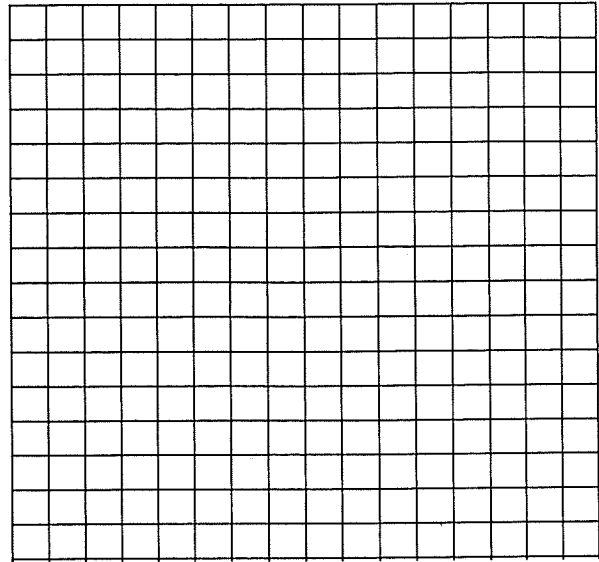
$$y = \frac{1}{2} \cos(\theta + 60^\circ) + 1$$

period _____

amplitude _____

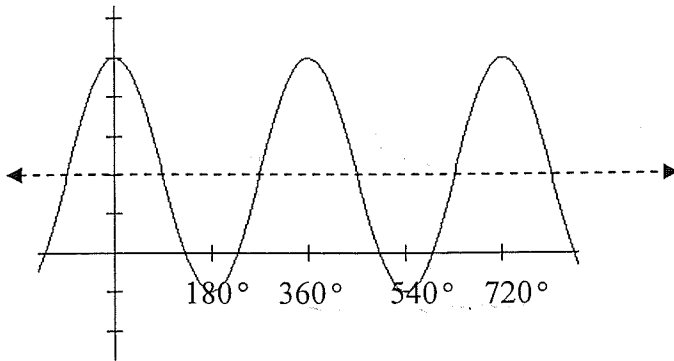
maximum _____

minimum _____



14. Each graph shown is the graph of an equation using either sine or cosine. Write an equation to match. The equations may have amplitude changes and/or vertical shifts. All scales on the y -axis go by 1.

a. _____



b. _____

