Name:________________________

6.3 Angles and Radian Measure

Date:______________ Hour:__________

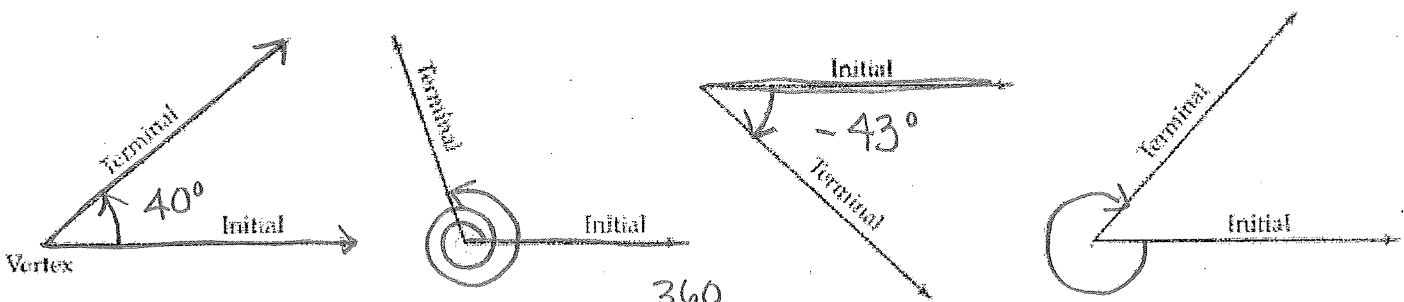
Objectives

- Use a rotating ray to extend the definition of angle measure to negative angles and angles greater than 180°
- Define radian measure and convert angle measures between degrees and radians

Extending Angle Measure

In geometry and triangle trigonometry, an angle is a static figure consisting of two rays that meet at a point. But in modern trigonometry, which will be introduced in the next section, an angle is thought of as being formed dynamically by *rotating* a ray around its endpoint, the **vertex.** The starting position of the ray is called the **initial side** and its final position after the rotation is called the **terminal side.**

The amount the ray is rotated is the measure of the angle. Counterclockwise rotations have positive measure and clockwise rotations have negative measure.



40° 830° −43° −312°

830
− 720
110

360
360
720

An angle in the coordinate plane is said to be in **standard position** if its vertex is at the origin and its initial side is on the positive x-axis.

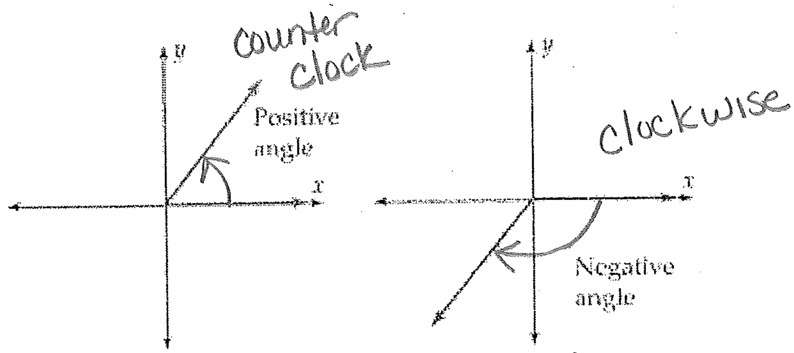


Figure 6.3-2

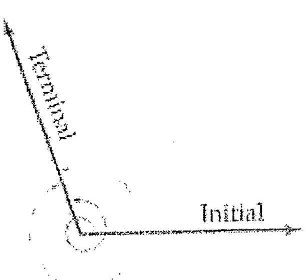


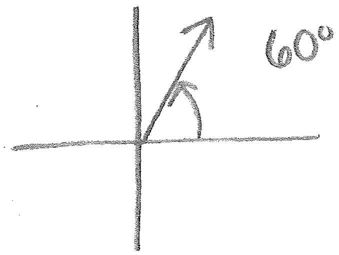
Figure 6.3-3

Angles formed by different rotations that have the same initial and terminal sides are called **coterminal.** (See Figure 6.3-3.) For example, 0° and 360° are coterminal angles.

Increasing or decreasing the angle measure by adding or subtracting any multiple of 360 degrees will result in a coterminal angle

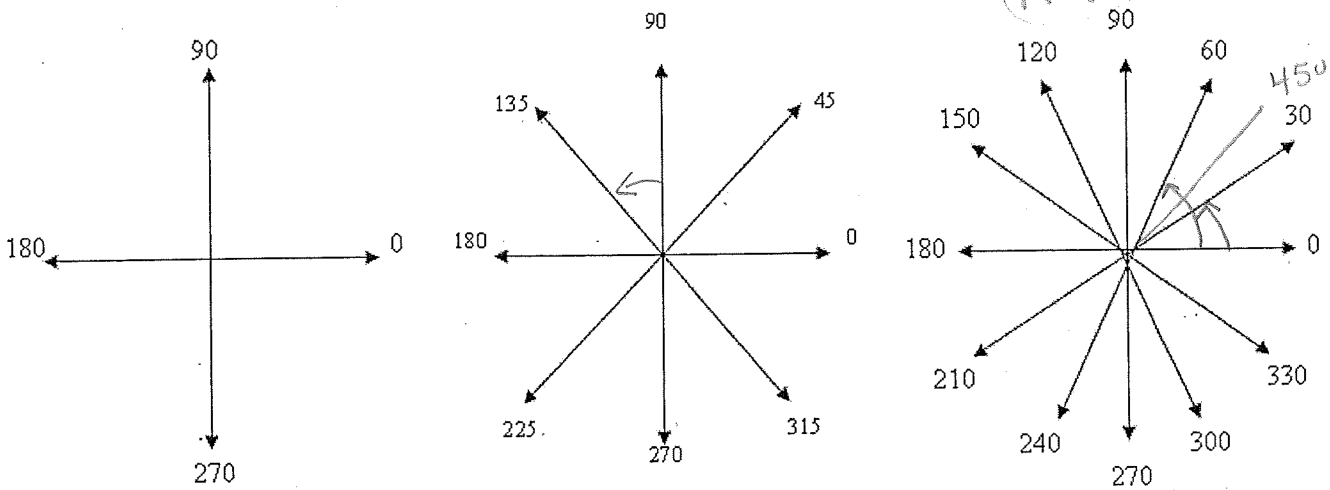
Example 1 Coterminal Angles

Find three angles coterminal with an angle of 60° in standard position.



$$\begin{array}{r} 60 \\ +360 \\ \hline 420 \\ +360 \\ \hline 780 \\ 360 \\ \hline 1140 \end{array} \qquad \begin{array}{r} 60 \\ -360 \\ \hline -300 \\ -360 \\ \hline -660° \end{array}$$

Unit Circles



Add 90° **Add 45°** **Add 30°**

Radian Measure

The angle found in Example 2 leads to another unit used in finding angle measure called a *radian*. Because it simplifies many formulas in calculus and physics, radians are used as a unit of angle measurement in mathematical and scientific applications.

Definition of Radian Measure

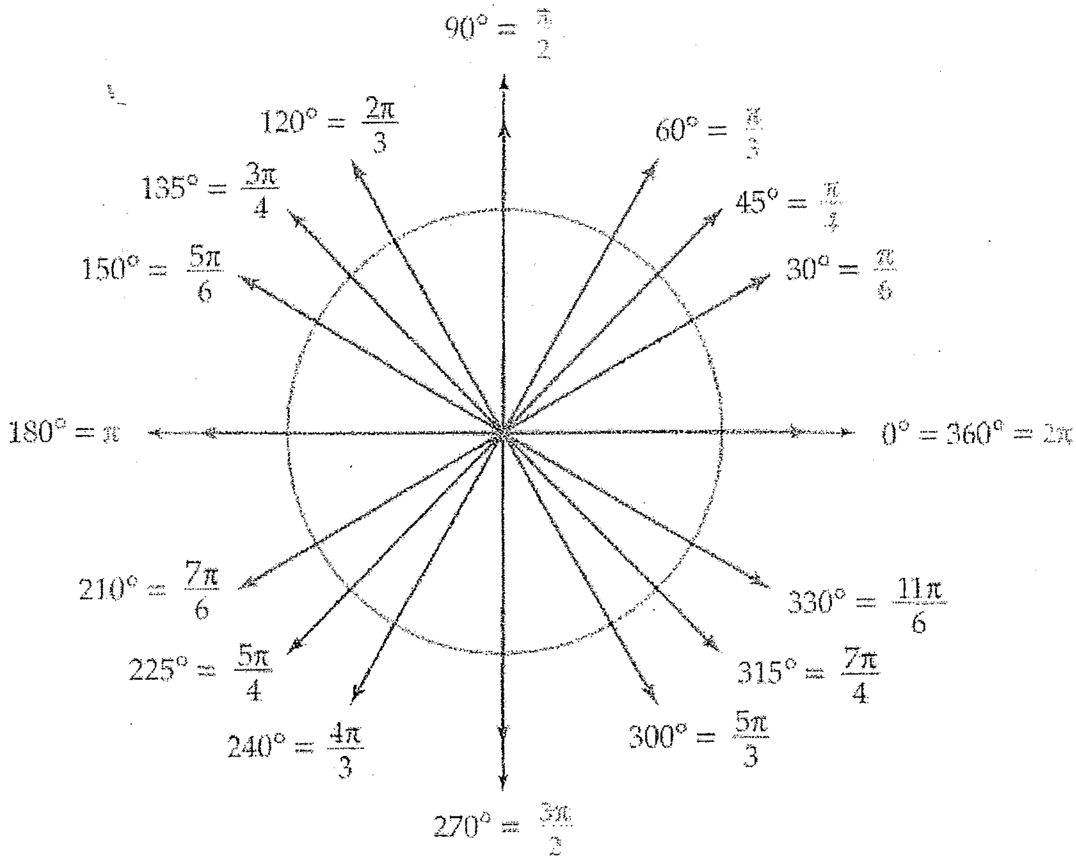
The radian measure of an angle is the distance traveled along the unit circle in a counterclockwise direction by the point P, as it moves from its starting position on the initial side to its final position on the terminal side of the angle.

$$1 \text{ radian} = \left(\frac{180}{\pi}\right)^{\circ} \approx 57.3^{\circ}$$

If the vertex of an angle is the center of a circle of radius r, then an angle of 1 radian intercepts an arc of length r.

Movement along the unit circle is counterclockwise for positive measure and clockwise for negative measure.

3



NOTE Radian measurements are usually given in terms of π; however, it is useful to know the decimal equivalents for common measurements when using a calculator.

$\pi \approx 3.14$ $\quad 2\pi \approx 6.28$

$\frac{\pi}{2} \approx 1.57$ $\quad \frac{\pi}{4} \approx 0.79$

$\frac{\pi}{6} \approx 0.52$ $\quad \frac{\pi}{3} \approx 1.05$

Converting Between Degrees and Radians

π radians $= 180°$. $\qquad$ 1 radian $= \left(\frac{180}{\pi}\right)^\circ \approx 57.3°$, $\qquad$ $\frac{\pi}{180}$ radians $= 1°$

Radian/Degree Conversion

To convert radians to degrees, multiply by $\frac{180}{\pi}$.

To convert degrees to radians, multiply by $\frac{\pi}{180}$.

Example 3 Converting From Radians to Degrees

Convert the following radian measurements to degrees.

a. $\frac{\cancel{\pi}}{5} \cdot \frac{180}{\cancel{\pi}}$

36°

b. $\frac{4\cancel{\pi}}{\cancel{9}} \cdot \frac{\cancel{180}^{20}}{\cancel{\pi}}$

80°

c. $6\cancel{\pi} \cdot \frac{180}{\cancel{\pi}}$

1080

Example 4 Converting From Degrees to Radians

Convert the following degree measurements to radians.

a. $75°$ $\frac{75°}{1} \cdot \frac{\pi}{180°}$

$\frac{5\pi}{12}$

b. $220°$

$\frac{220°}{1} \cdot \frac{\pi}{180°}$

$\frac{11\pi}{9}$

c. $400°$

$400° \cdot \frac{\pi}{180}$

$\frac{20}{9}\pi$

$\pm 360° = 2\pi$

$\pm 2\pi$

> **Increasing or decreasing the radian measure of an angle by an integer multiple of 2π results in a coterminal angle.**

Example 5: Find one positive and one negative coterminal angle to $-\frac{\pi}{3}$ ← radians

$-\frac{\pi}{3} + \frac{2\pi \cdot 3}{1 \cdot 3}$

$-\frac{\pi}{3} + \frac{6\pi}{3} = \frac{5\pi}{3}$ positive

$-\frac{\pi}{3} - \frac{2\pi \cdot 3}{1 \cdot 3}$

$-\frac{\pi}{3} - \frac{6\pi}{3} = -\frac{7\pi}{3}$ negative

Example 6: State the radian measure of an angle between 0 and 2π that is coterminal to $-\frac{3\pi}{4}$

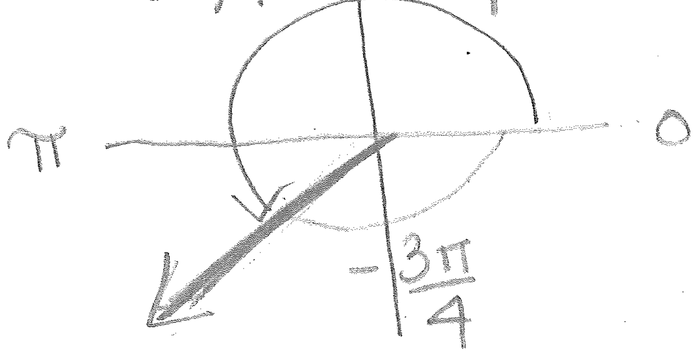
$5\pi/4 = -\frac{3\pi}{4}$

$+2\pi$

$-\frac{3\pi}{4} + \frac{2\pi \cdot 4}{1 \cdot 4}$

$-\frac{3\pi}{4} + \frac{8\pi}{4}$

$\frac{5\pi}{4}$



Name________________________________
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Exercises 6.3

In Exercises 1–10, find the degree and radian measure of the angle in standard position formed by rotating the terminal side by the given amount.

1. $\frac{1}{9}$ of a circle $\frac{1}{9} \cdot \cancel{360}^{40} = \underline{40^\circ}$ $\frac{1}{9} \cdot 2\pi = \underline{2\pi/9}$

3. $\frac{1}{18}$ of a circle $\frac{1}{18} \cdot 360 = \underline{20^\circ}$ $\frac{1}{18} \cdot 2\pi = \underline{\frac{\pi}{9}}$

5. $\frac{1}{36}$ of a circle

7. $\frac{2}{3}$ of a circle

9. $\frac{4}{5}$ of a circle

In Exercises 11–22, convert the given radian measure to degrees.

11. $\frac{\pi}{5}$

13. $-\frac{\pi}{10}$

15. $\frac{3\pi}{4}$

17. $\frac{\pi}{45}$

19. $-\frac{5\pi}{12}$

21. $\frac{27\pi}{5}$

In Exercises 23–34, convert the given degree measure to radians. Write your answer in terms of π.

23. 6°

25. -12°

27. 75°

29. 135°

31. -225°

33. 930°

In Exercises 35–42, state the radian measure of an angle in standard position between 0 and 2π that is coterminal with the given angle in standard position.

35. $-\frac{\pi}{3}$

37. $\frac{19\pi}{4}$

39. $-\frac{7\pi}{5}$

41. 7

In Exercises 43–46, find the radian measure of four angles in standard position that are coterminal with the given angle in standard position.

43. $\frac{\pi}{4}$

45. $-\frac{\pi}{6}$