

8-2

The Reciprocal Function Family

Name: _____

Date: _____ Hour: _____

- Objectives** To graph reciprocal functions
To graph translations of reciprocal functions

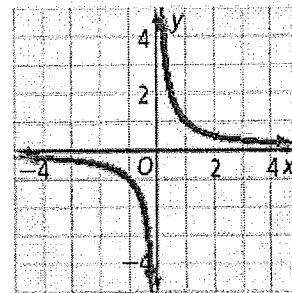
Take note

Key Concept General Form of the Reciprocal Function Family

The general form of a member of the reciprocal function family is $y = \frac{a}{x-h} + k$, where $x \neq h$.

The inverse variation functions, $y = \frac{a}{x}$, are stretches, shrinks, and reflections of the parent reciprocal function, depending on the value of a .

The graph of the parent reciprocal function $y = \frac{1}{x}$ is shown at the right.



Take note

Key Concept The Reciprocal Function Family

Parent function

$$y = \frac{1}{x}, x \neq 0$$

Stretch ($|a| > 1$)

Shrink ($0 < |a| < 1$)

Reflection ($a < 0$) across x -axis

$$y = \frac{a}{x}, x \neq 0$$

Translation (horizontal by h ; vertical by k)
with vertical asymptote $x = h$ horizontal asymptote $y = k$

$$y = \frac{1}{x-h} + k; x \neq h$$

Combined

$$y = \frac{a}{x-h} + k; x \neq h$$

$$y = \frac{1}{x}$$

The function $f(x) = \frac{1}{x}$, $x \neq 0$, is called a reciprocal Functions.

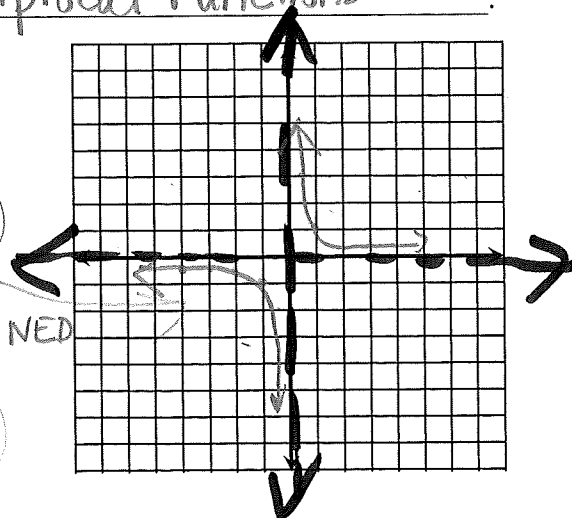
Example 1

Graph $f(x) = \frac{1}{x}$

D: all $x \neq 0$

R: all $y \neq 0$

x	y
-4	-1/4
-3	-1/3
-2	-1/2
-1	-1
0	UNDEFINED
1	1
2	1/2
3	1/3
4	1/4



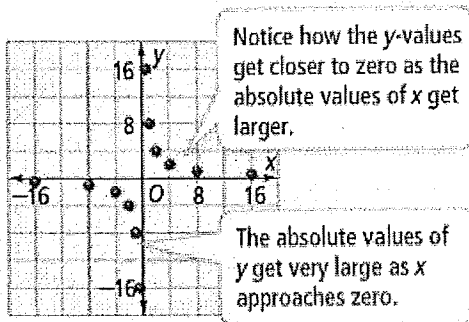
© Problem 1 Graphing an Inverse Variation Function

What is the graph of $y = \frac{a}{x}$, $x \neq 0$? Identify the x - and y -intercepts and the asymptotes of the graph. Also, state the domain and range of the function.

Step 1 Make a table of values that includes positive and negative values of x .

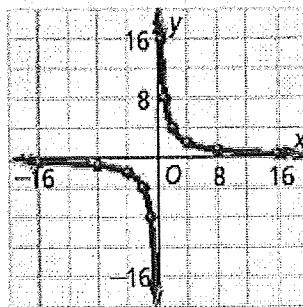
x	y	x	y
-16	$-\frac{1}{2}$	$\frac{1}{2}$	16
-8	-1	1	8
-4	-2	2	4
-2	-4	4	2
-1	-8	8	1
$-\frac{1}{2}$	-16	16	$\frac{1}{2}$

Step 2 Graph the points.



Step 3 Connect the points with a smooth curve. x cannot be zero, so there is no y -intercept. The numerator is never zero, so y is never 0. There is no x -intercept.

The x -axis is a horizontal asymptote.
The y -axis is a vertical asymptote.
Knowing the asymptotes provides you with the basic shape of the graph.
The domain is the set of all real numbers except $x = 0$.
The range is the set of all real numbers except $y = 0$.

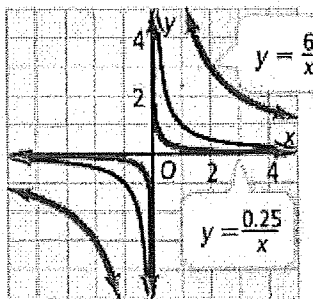


© Problem 2 Identifying Reciprocal Function Transformations

For each given value of a , how do the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x}$ compare? What is the effect of a on the graph?

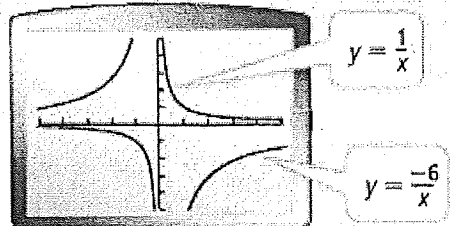
A $a = 6$

The graph (in red) of $y = \frac{6}{x}$ is a stretch of the graph of $y = \frac{1}{x}$ (in black) by the factor 6.



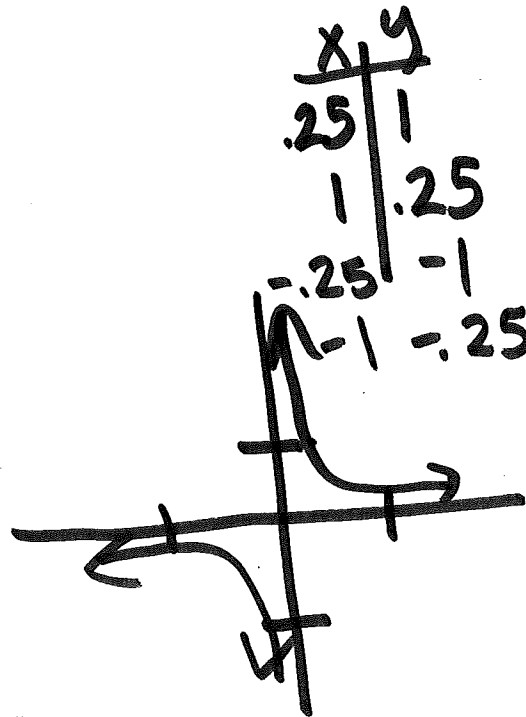
B $a = 0.25$

The graph (in blue) of $y = \frac{0.25}{x}$ is a shrink of the graph of $y = \frac{1}{x}$ (in black) by the factor $\frac{1}{4}$.



C $a = -6$

The graph of $y = \frac{-6}{x}$ is the stretch by the factor 6 in part A followed by a reflection across the x -axis.



Problem 3 Graphing a Translation

What is the graph of $y = \frac{1}{x+1} - 2$? Identify the domain and range.

Step 1 Draw the asymptotes (red). $x = -1$

For, $y = \frac{1}{x+1} - 2$, $h = -1$ and $k = -2$.

The vertical asymptote is $x = -1$.

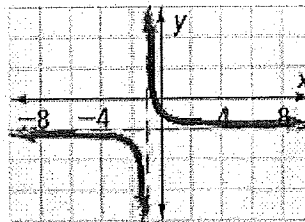
The horizontal asymptote is $y = -2$.

Step 2 Translate the graph of $y = \frac{1}{x}$.

The graph of $y = \frac{1}{x}$ contains the points (1, 1) and (-1, -1). Translate these points 1 unit to the left and 2 units down to (0, -1) and (-2, -3), respectively.

Draw the branches through these points (blue).

The domain is the set of all real numbers except $x = -1$. The range is the set of all real numbers except $y = -2$.



Problem 4 Writing the Equation of a Transformation

Multiple Choice This graph of a function is a translation of the graph of $y = \frac{2}{x}$. What is an equation for the function?

A. $y = \frac{2}{x+3} + 4$

C. $y = \frac{2}{x-3} + 4$

B. $y = \frac{2}{x+3} - 4$

D. $y = \frac{2}{x-3} - 4$

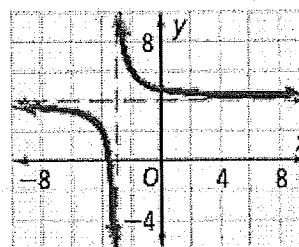
The asymptotes are $x = -3$ and $y = 4$. Thus $h = -3$ and $k = 4$.

$y = \frac{a}{x-h} + k$ Use the general form.

$y = \frac{2}{x-(-3)} + 4$ Substitute for a , h , and k .

$y = \frac{2}{x+3} + 4$ Simplify.

The correct choice is A.



$a=2$

3 left
4 up

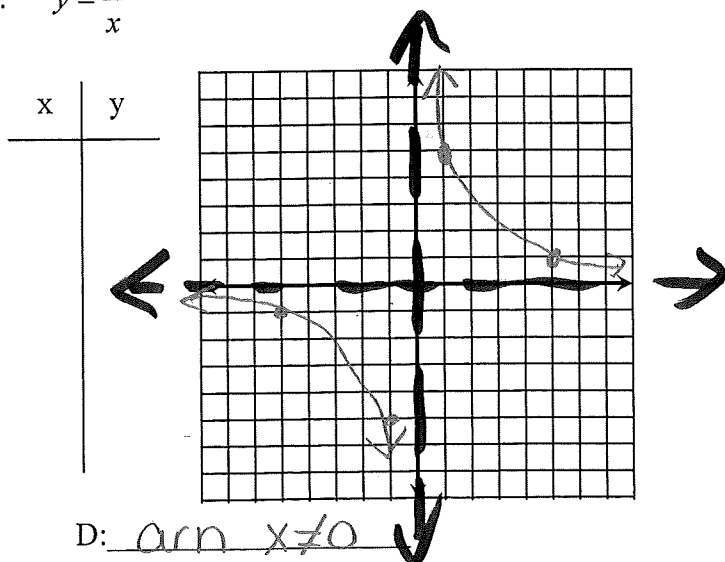
$y = \frac{2}{x+3} + 4$

Graph each function. Identify the x - and y -intercepts and the asymptotes of the graph. Also, state the domain and the range of the function.

See Problem 1.

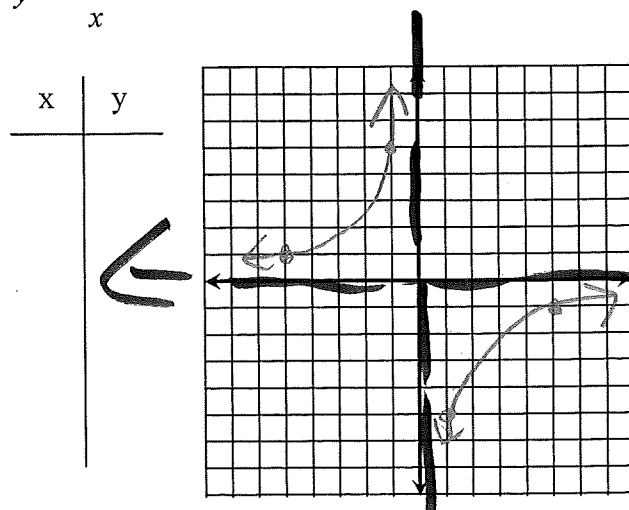
A. $y = \frac{5}{x}$

B. $y = -\frac{5}{x}$



D: $\text{all } x \neq 0$

R: $\text{all } y \neq 0$



D: $\text{all } x \neq 0$

R: $\text{all } y \neq 0$

Graphing Calculator Graph the equations $y = \frac{1}{x}$ and $y = \frac{a}{x}$ using the given value of a . Then identify the effect of a on the graph.

See Problem 2.

13. $a = 2$

Vertical stretch by 2

15. $a = 0.5$

vert. comp by $1/2$

17. $a = 0.75$

vert. comp by .75

Sketch the asymptotes and the graph of each function. Identify the domain and range.

See Problem 3.

18. $y = \frac{1}{x} - 3$

19. $y = \frac{-2}{x} - 3$

20. $y = \frac{1}{x-2} + 5$

21. $y = \frac{1}{x-3} + 4$

VA = $x = 0$

VA = $x = 0$

VA = $x = 2$

VA = $x = 3$

HA = $y = -3$

HA = $y = -3$

HA = $y = 5$

HA = $y = 4$

D: $\text{all } x \neq 0$

D: $\text{all } x \neq 0$

D: $\text{all } x \neq 2$

D: $\text{all } x \neq 3$

R: $\text{all } y \neq -3$

R: $\text{all } y \neq -3$

R: $\text{all } y \neq 5$

R: $\text{all } y \neq 4$

22. $y = \frac{2}{x+6} - 1$

23. $y = \frac{10}{x+1} - 8$

24. $y = \frac{1}{x} - 2$

25. $y = \frac{-8}{x+5} - 6$

VA = $x = -6$

VA = $x = -1$

VA = $x = 0$

VA = $x = -5$

HA = $y = -1$

HA = $y = -8$

HA = $y = -2$

HA = $y = -6$

D: $\text{all } x \neq -6$

D: $\text{all } x \neq -1$

D: $\text{all } x \neq 0$

D: $\text{all } x \neq -5$

R: $\text{all } y \neq -1$

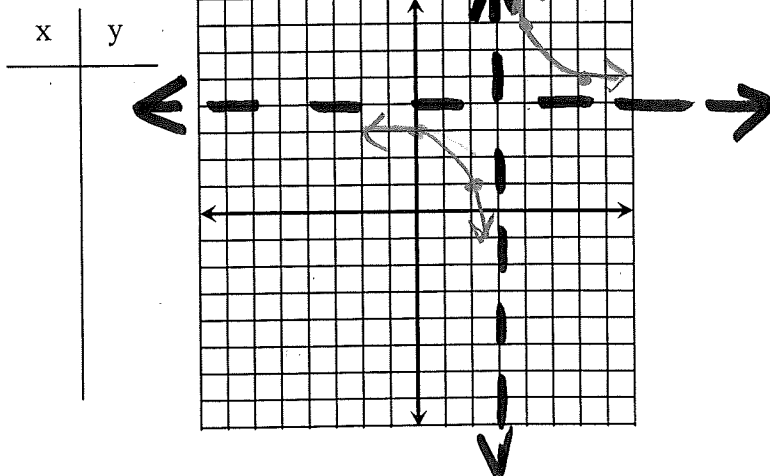
R: $\text{all } y \neq -8$

R: $\text{all } y \neq -2$

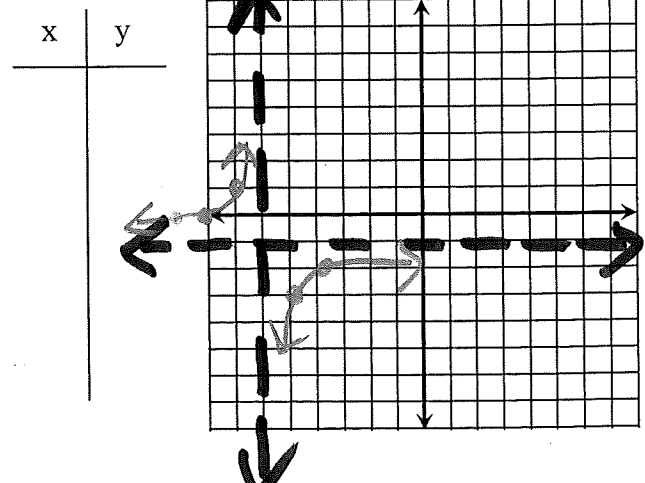
R: $\text{all } y \neq -6$

Graph 21 and 22 above, completely

21.



22.



Write an equation for the translation of $y = \frac{2}{x}$ that has the given asymptotes.

See Problem 4.

26. $x = 0$ and $y = 4$

$y = \frac{2}{x} + 4$

27. $x = -2$ and $y = 3$

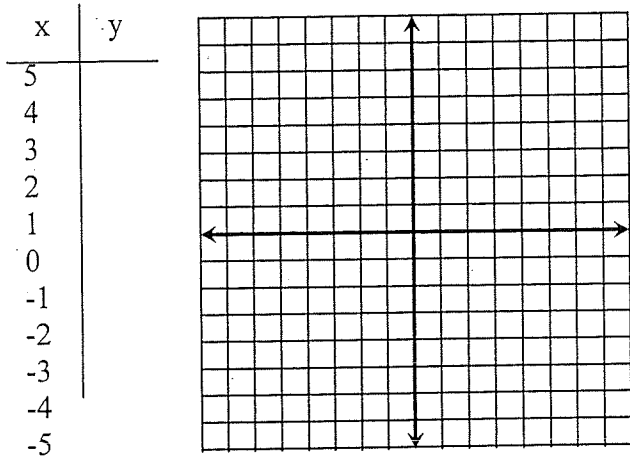
$y = \frac{2}{x+2} + 3$

28. $x = 4$ and $y = -8$

$y = \frac{2}{x-4} - 8$

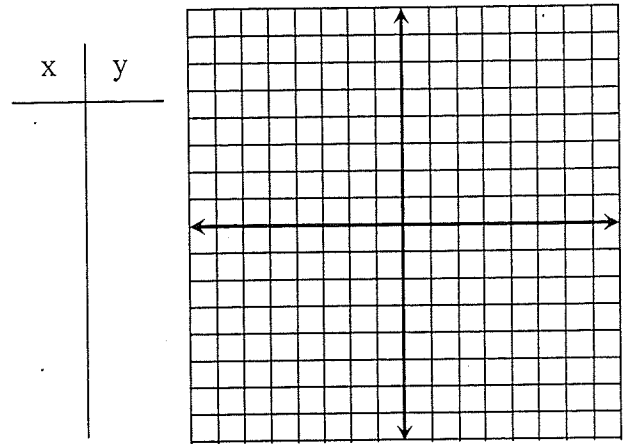
8.2 Algebra 2
Reciprocal Functions

The function $f(x) = \frac{1}{x}$

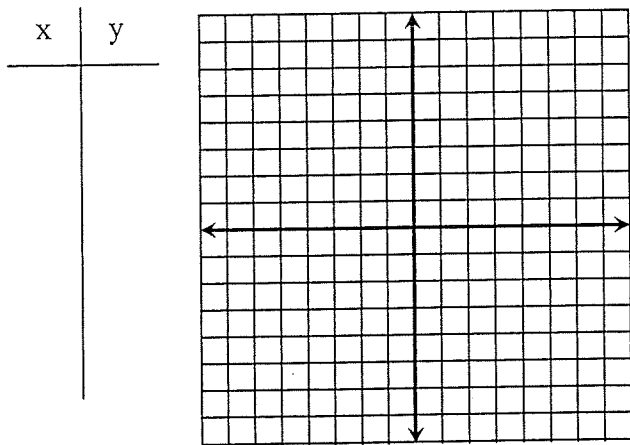


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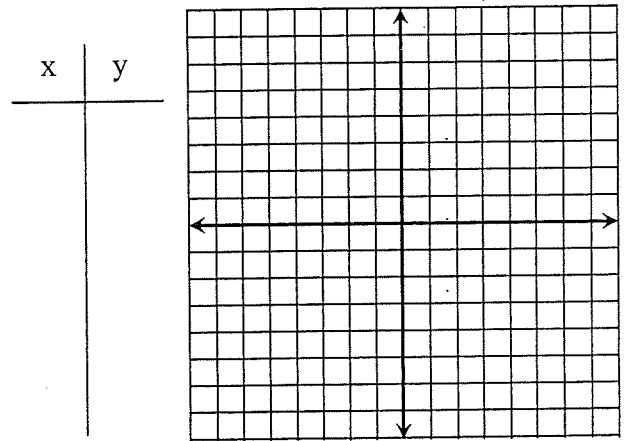
The function $f(x) = \frac{2}{x}$



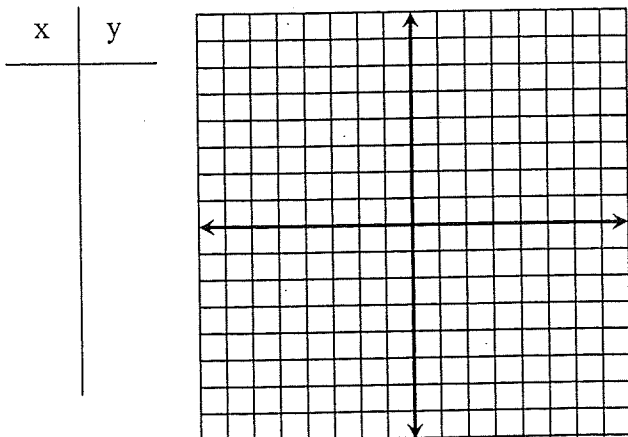
The function $f(x) = \frac{3}{x}$



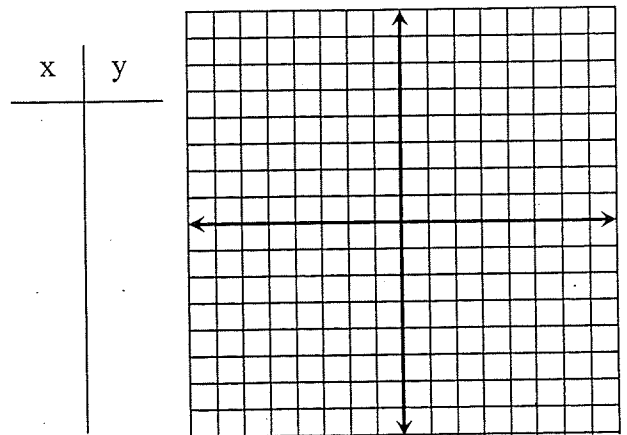
The function $f(x) = \frac{4}{x}$



The function $f(x) = \frac{5}{x}$



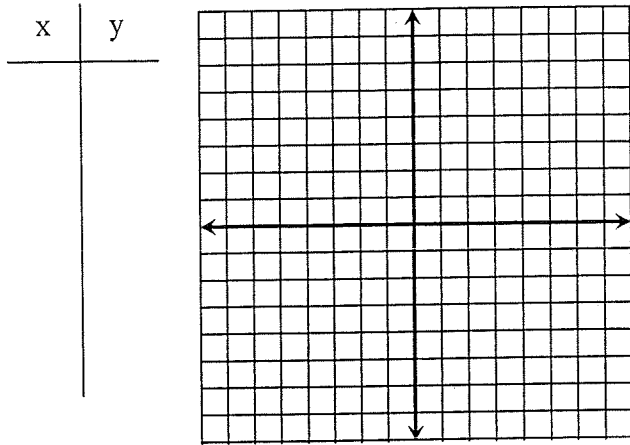
The function $f(x) = -\frac{5}{x}$



$$f(x) = \frac{3}{x-2} + 1$$

Vertical Asymptote: _____ Domain: _____

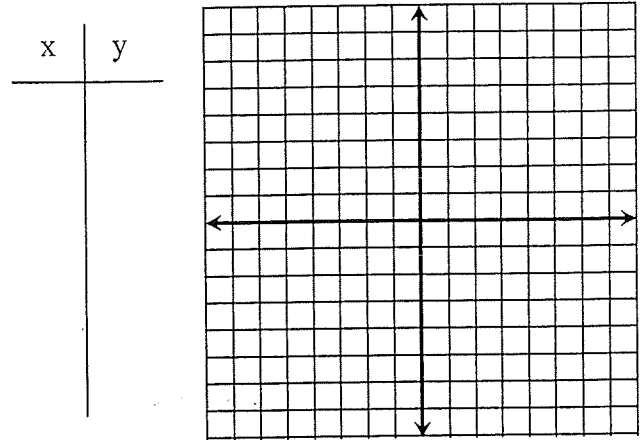
Horizontal Asymptote: _____ Range: _____



$$f(x) = \frac{2}{x+2} - 4$$

Vertical Asymptote: _____ Domain: _____

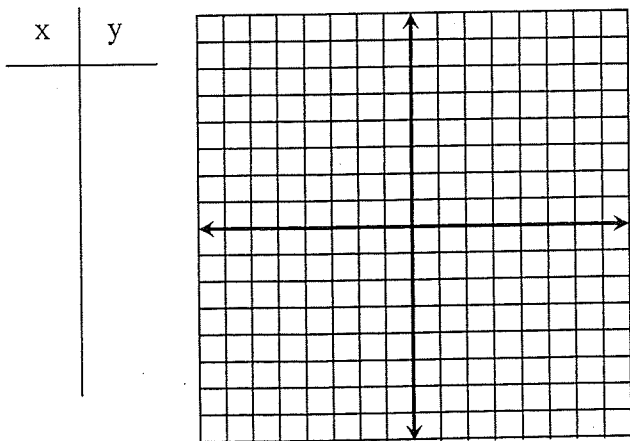
Horizontal Asymptote: _____ Range: _____



The function $f(x) = \frac{3}{x+2} - 4$

Vertical Asymptote: _____ Domain: _____

Horizontal Asymptote: _____ Range: _____



The function $f(x) = \frac{4}{x} - 3$

Vertical Asymptote: _____ Domain: _____

Horizontal Asymptote: _____ Range: _____

