

# 8-4

# Rational Expressions

Key

### © Content Standards

- A.SSE.2 Use the structure of an expression to identify ways to rewrite it.
- A.SSE.1.b Interpret complicated expressions by viewing one or more of their parts as a single entity.
- Also A.SSE.1.a

**Objectives** To simplify rational expressions  
To multiply and divide rational expressions

A Rational expression is an expression that is a quotient of two polynomials.

To simplify fractions we divide both the numerator and denominator by a common factor.

Anytime we have variables in the denominator of a fraction, we have to recognize that the variables cannot have a value that would make the denominator equal to zero.

### © Problem 1 Simplifying a Rational Expression

What is  $\frac{x^2 + 7x + 10}{x^2 - 3x - 10}$  in simplest form? State any restrictions on the variable.

$$\frac{x^2 + 7x + 10}{x^2 - 3x - 10} = \frac{(x + 2)(x + 5)}{(x + 2)(x - 5)} \quad \text{Factor the numerator and denominator.}$$

$$= \frac{\cancel{(x + 2)}(x + 5)}{\cancel{(x + 2)}(x - 5)} \quad \text{Divide out common factors.}$$

$$= \frac{x + 5}{x - 5} \quad \text{Simplify.}$$

The simplified form is  $\frac{x + 5}{x - 5}$  for  $x \neq 5$  and  $x \neq -2$ . The restriction  $x \neq -2$  is not evident from the simplified form, but is needed to prevent the denominator of the original expression from being zero.

### Think

Is there more than one restriction?

Yes, before you divided the common factors out,  $(x + 2)$  was one of the factors of the denominator so  $x \neq -2$ .



**Got It?** 1. What is the rational expression in simplest form? State any restrictions on the variables.

a.  $\frac{24x^3y^2}{-6x^2y^3}$

$x \neq 0$   
 $y \neq 0$

$$\frac{-4x}{y}$$

b.  $\frac{x^2 + 2x - 8}{x^2 - 5x + 6}$

$$\frac{(x+4)\cancel{(x-2)}}{(x-3)\cancel{(x-2)}}$$

$x \neq 3, 2$   
 $\frac{x+4}{x-3}$

c.  $\frac{12 - 4x}{x^2 - 9}$

$$= \frac{-4(3-x)}{(x-3)(x+3)}$$

$x \neq \pm 3$  ← restriction  
 $\frac{-4}{x+3}$  ← simplified

## Problem 2 Multiplying Rational Expressions

What is the product  $\frac{x^2 + x - 6}{x - 5} \cdot \frac{x^2 - 25}{x^2 + 4x + 3}$  in simplest form? State any restrictions on the variable.

$$\begin{aligned} & \frac{x^2 + x - 6}{x - 5} \cdot \frac{x^2 - 25}{x^2 + 4x + 3} \\ &= \frac{(x + 3)(x - 2)}{x - 5} \cdot \frac{(x + 5)(x - 5)}{(x + 3)(x + 1)} && \text{Factor all polynomials.} \\ &= \frac{\cancel{(x + 3)}(x - 2)}{\cancel{x - 5}} \cdot \frac{(x + 5)\cancel{(x - 5)}}{\cancel{(x + 3)}(x + 1)} && \text{Divide out common factors.} \\ &= \frac{(x - 2)(x + 5)}{x + 1} && \text{Simplify.} \end{aligned}$$

The product is  $\frac{(x - 2)(x + 5)}{x + 1}$  for  $x \neq -3$ ,  $x \neq 5$ , and  $x \neq -1$ . The restrictions  $x \neq -3$  and  $x \neq 5$  are not evident from the simplified form, but are needed to prevent the denominators in the original product from being zero.

### Plan

How is multiplying rational expressions like multiplying fractions?

To multiply rational expressions, you multiply the numerators and multiply the denominators.

**Got It?** 2. What is the product  $\frac{2x - 8}{x^2 - 16} \cdot \frac{x^2 + 5x + 4}{x^2 + 8x + 16}$  in simplest form? State any restrictions on the variable.

$$\begin{aligned} & \frac{2(x - 4)}{\cancel{(x - 4)}(x + 4)} \cdot \frac{\cancel{(x + 4)}(x + 1)}{(x + 4)(x + 4)} \\ & \quad x \neq \pm 4 \\ & \quad \frac{2(x + 1)}{(x + 4)^2} \end{aligned}$$

To divide rational expressions, you multiply by the reciprocal of the divisor, just as you do when you divide rational numbers.

## Problem 3 Dividing Rational Expressions

What is the quotient  $\frac{2 - x}{x^2 + 2x + 1} \div \frac{x^2 + 3x - 10}{x^2 - 1}$  in simplest form? State any restrictions on the variable.

$$\begin{aligned} & \frac{(2 - x)}{(x + 1)(x + 1)} \div \frac{(x + 5)(x - 2)}{(x - 1)(x + 1)} \\ & \frac{\cancel{(2 - x)}}{(x + 1)\cancel{(x + 1)}} \cdot \frac{(x - 1)\cancel{(x + 1)}}{(x + 5)\cancel{(x - 2)}} \end{aligned}$$

$$\frac{-(x - 1)}{(x + 1)(x + 5)}$$

simplified answer

restrictions  
 $x \neq \pm 1$   
 $x \neq -5, 2$



**Got It?** 3. a. What is the quotient  $\frac{x^2 + 5x + 4}{x^2 + x - 12} \div \frac{x^2 - 1}{2x^2 - 6x}$  in simplest form? State any restrictions on the variable.

b. **Reasoning** Without doing the calculation, what is greatest number of restrictions the quotient  $\frac{x^2 + 8x + 7}{x^2 - x - 12} \div \frac{x^2 + 2x - 8}{x^2 + 13x + 24}$  could have? Explain.

$$\frac{(x+4)(x+1)}{(x+4)(x-3)} \cdot \frac{(x-1)(x+1)}{2x(x-3)} \quad \left| \quad \frac{\cancel{(x+4)}(\cancel{x+1})}{\cancel{(x+4)}(\cancel{x-3})} \cdot \frac{2x(\cancel{x-3})}{(x-1)(\cancel{x+1})}$$

$$x \neq -4, 3, 0, \pm 1 \quad \quad \quad 2x/x-1$$

Simplify each rational expression. State any restrictions on the variables.

See Problem 1.

8.  $\frac{5x^2y}{15xy^3}$

9.  $\frac{2x}{4x^2 - 2x}$

10.  $\frac{6c^2 + 9c}{3c}$

11.  $\frac{49 - z^2}{z + 7}$

12.  $\frac{x^2 + 8x + 16}{x^2 - 2x - 24}$

13.  $\frac{12 - x - x^2}{x^2 - 8x + 15}$

Multiply. State any restrictions on the variables.

See Problem 2.

14.  $\frac{4x^2}{5y} \cdot \frac{7y}{12x^4}$

15.  $\frac{2x^4}{10y^{-2}} \cdot \frac{5y^3}{4x^3}$

$$16. \frac{8y - 4}{10y - 5} \cdot \frac{5y - 15}{3y - 9}$$

$$17. \frac{2x + 12}{3x - 9} \cdot \frac{6 - 2x}{3x + 8}$$

$$18. \frac{x^2 - 4}{x^2 - 1} \cdot \frac{x + 1}{x^2 + 2x}$$

$$19. \frac{x^2 - 5x + 6}{x^2 - 4} \cdot \frac{x^2 + 3x + 2}{x^2 - 2x - 3}$$

Divide. State any restrictions on the variables.

◆ See Problem 3.

$$20. \frac{7x}{4y^3} \div \frac{21x^3}{8y}$$

$$21. \frac{3x^3}{5y^2} \div \frac{6y^{-3}}{5x^{-5}}$$

$$22. \frac{6x + 6y}{y - x} \div \frac{18}{5x - 5y}$$

$$23. \frac{3y - 12}{2y + 4} \div \frac{6y - 24}{8 + 4y}$$

$$24. \frac{x^2}{x^2 + 2x + 1} \div \frac{3x}{x^2 - 1}$$

$$25. \frac{y^2 - 5y + 6}{y^3} \div \frac{y^2 + 3y - 10}{4y^2}$$