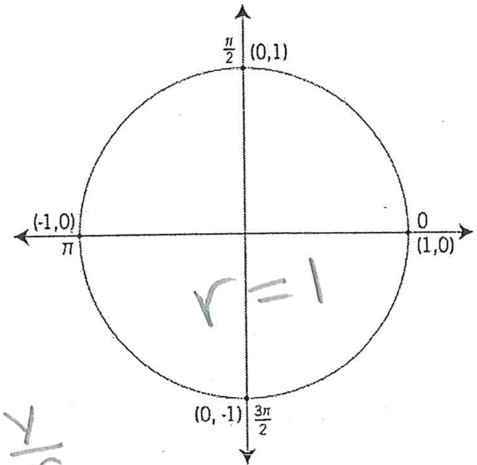
7.1 Sine and Cosine Functions

Day 2

Name: Audia

Date: 2014 Hour: ____________

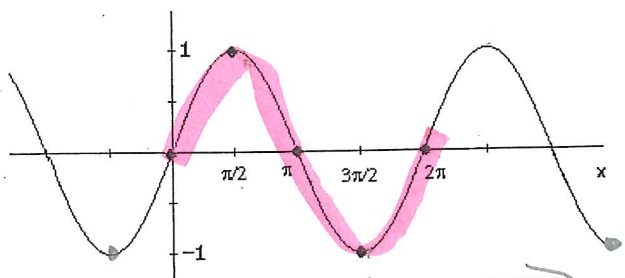
In yesterday's notes, we learned that the period of the sine and cosine functions is 2π. We can use the period of a function to calculate all values of t for which $\sin t$ or $\cos t$ is a given number.



(−∞,∞)

sin= y/r

Example 1: State all values of t for which $\sin t$ is -1.



−2Kπ 2Kπ

x	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	−1
2π	0

$[0, 2\pi]$

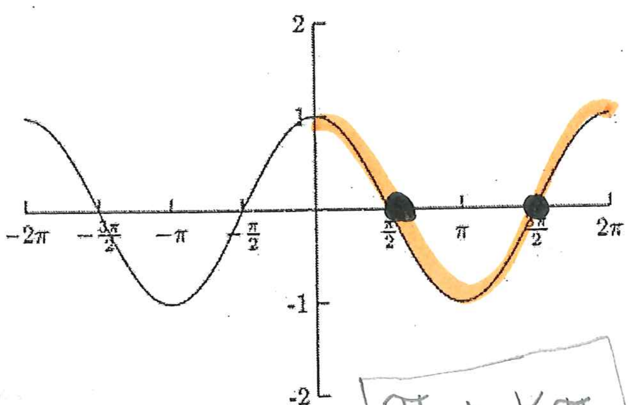
multiples of

$\frac{3\pi}{2} \pm 2K\pi$

Example 2: State all values of t for which $\cos t$ is 0.

$y = \cos x$

cos = x/r



$\frac{\pi}{2} \pm K\pi$

x	y
0	1
$\frac{\pi}{2}$	0 *
π	−1
$\frac{3\pi}{2}$	0 *
2π	1

$\frac{\pi}{2} \pm 2K\pi$

$\frac{3\pi}{2} \pm 2K\pi$

Recall, from yesterday, the domain and range of sine and cosine functions:

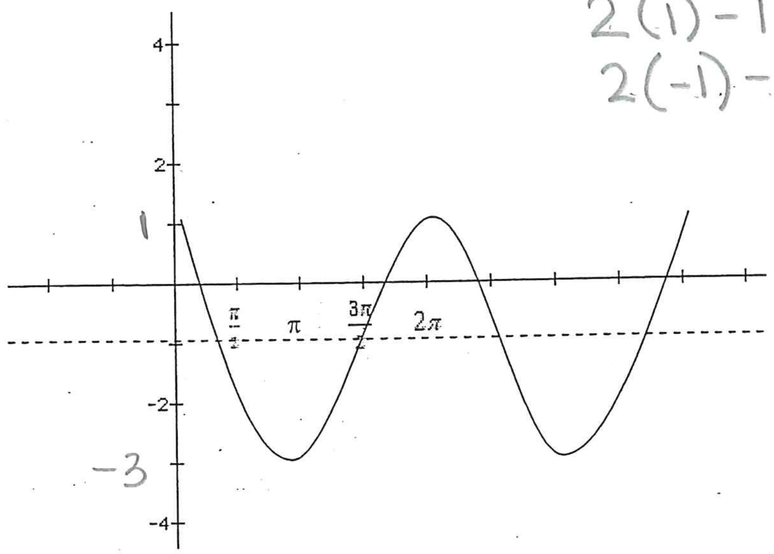
Domain: $(-\infty, \infty)$ arn

Range: $[-1, 1]$

When we alter the amplitude or include a vertical shift, the range of the function will change.

Example 3: Determine the Domain and Range of $f(t) = 2\cos t - 1$

$2(1) - 1 = 1$
$2(-1) - 1 = -3$



Max: 1

Min: -3

Domain: arn $(-\infty, \infty)$

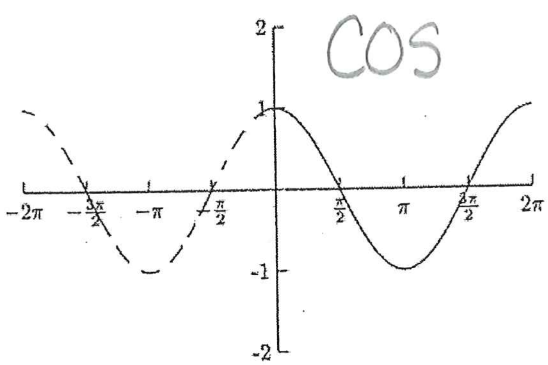
Range: $[-3, 1]$

Symmetry of Sine and Cosine Functions

Even Functions

A function f is even if $f(-x) = f(x)$ for every x in the domain of f.

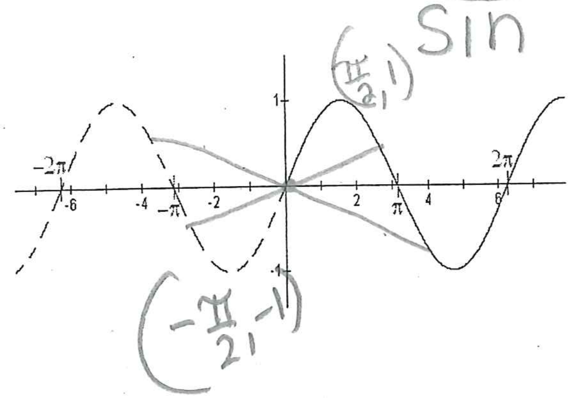
The graph of an even function is symmetric with respect to the y-axis.



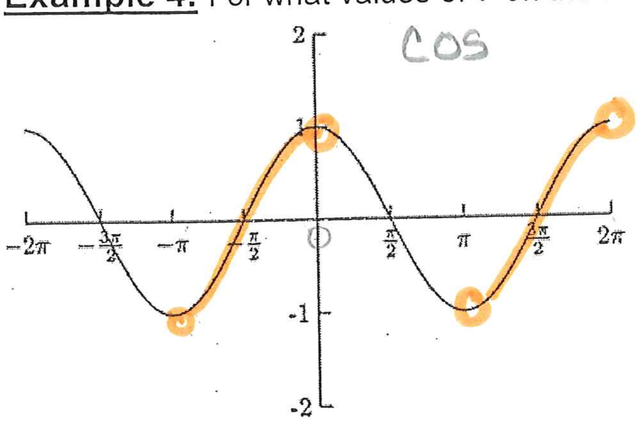
Odd Function

A function f is odd if $f(-x) = -f(x)$ for every x in the domain of f.

The graph of an odd function is symmetric with respect to the origin.



Example 4: For what values of t on the interval $[-2\pi, 2\pi]$ is $g(t) = \cos t$ increasing?



$(-\pi, 0) \cup (\pi, 2\pi)$

decreasing $(-2\pi, -\pi) \cup (0, \pi)$

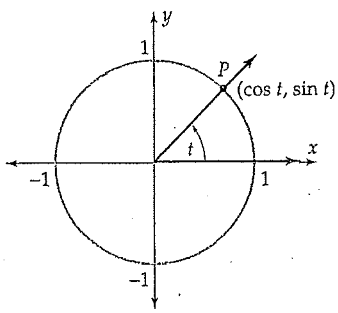
7.1 Tangent Functions

Day 3

Name:______________________________

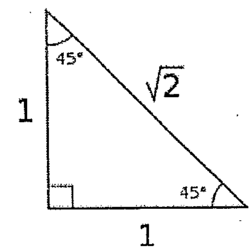
Date:________________ Hour:__________

To learn about tangent functions, we will be using the same approach as we did with sine and cosine functions. First start out with a t-table, then make some generalizations, and last graph "by heart".



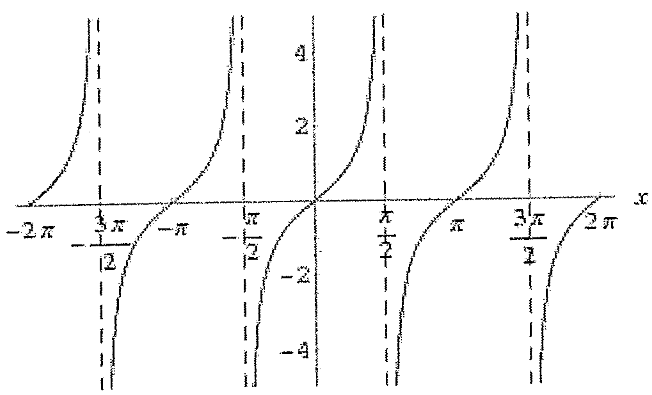
We can use point P and the origin in the figure to the left to make a connection between the tangent function and the slope:

$$slope = \frac{\sin t - 0}{\cos t - 0} = \frac{\sin t}{\cos t} = \tan t \qquad \tan t = \frac{y}{x}$$



Example 1: $f(t) = \tan t$

x	y



Period:____________

Max:____________

Min:____________

Domain:____________

Range:____________

The **Period** is measured from one asymptote to another or $\frac{3\pi}{2} - \frac{\pi}{2} = \frac{2\pi}{2} = \pi$

Notice that $\tan t$ goes through the origin and looks like a positive slope. Also, the first asymptote is at $\frac{\pi}{2}$

Vertical shifts of tangent functions work the same way as sine and cosine functions. However, the amplitude works a little bit differently.

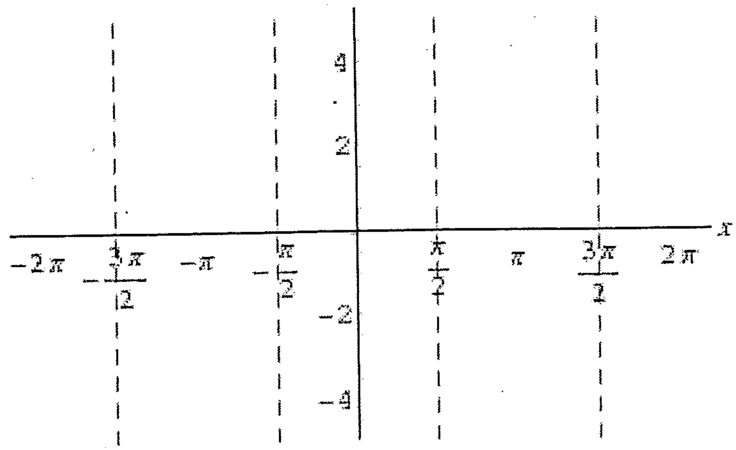
$$f(t) = a \tan t + d$$

Steepness ↗ (a)   Up or Down ↖ (d)

Example 2: Graph $f(t) = -3\tan t$

Reflection over the x-axis

The 3 makes it steep



Because the period of the tangent function is π, we can calculate all values of $\tan t$ for a given number.

Example 3: Find all values for which $\tan t = 0$.

Domain and Range of Tangent Functions

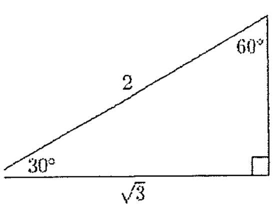
Domain: $(-\infty, \infty)$ ← except odd multiples of $\frac{\pi}{2}$

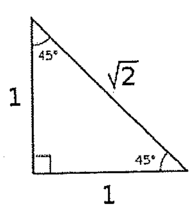
Range: $(-\infty, \infty)$

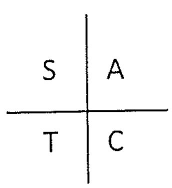
Even or Odd?

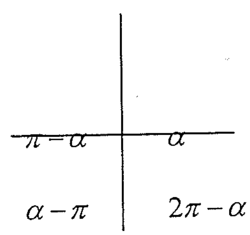
Because tangent is symmetric about the origin, it is an **odd** function.

Review:





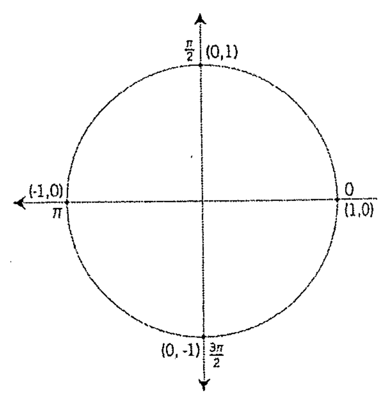




$30° = \frac{\pi}{6}$

$45° = \frac{\pi}{4}$

$60° = \frac{\pi}{3}$



1. $\cos x = -\frac{\sqrt{3}}{2}$

2. $\sin x = \frac{1}{2}$

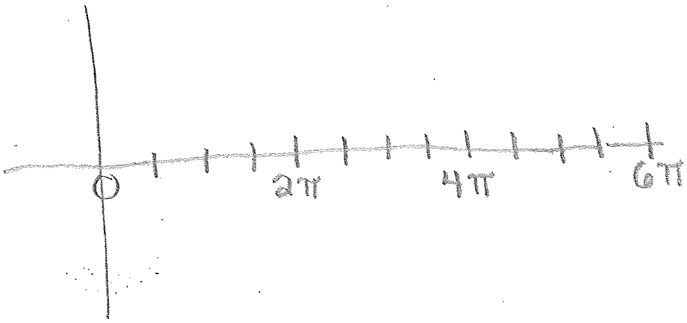
3. $\tan x = -1$

4. $\csc x = -2$

Exercises 7.1

Graph each function on the given interval.

1. $f(t) = \sin t$; $[2\pi, 6\pi]$



2. $g(t) = \cos t$; $[\pi, 3\pi]$

3. $h(t) = \tan t$; $[\pi, 2\pi]$

7. For what values of t on the interval $[-2\pi, 2\pi]$ is $\sin t = 1$?

8. For what values of t on the interval $[-2\pi, 2\pi]$ is $\cos t = 0$?

9. What is the maximum value of $g(t) = \cos t$?

10. What is the minimum value of $f(t) = \sin t$?

11. For what values of t on the interval $[-2\pi, 2\pi]$ does the graph of $h(t) = \tan t$ have vertical asymptotes?

12. What is the y-intercept of the graph of $f(t) = \sin t$?

13. What is the y-intercept of the graph of $g(t) = \cos t$?

14. What is the y-intercept of the graph of $h(t) = \tan t$?

15. For what values of t on the interval $[-\pi, \pi]$ is $f(t) = \sin t$ increasing?

16. For what values of t on the interval $[-3\pi, -\pi]$ is $g(t) = \cos t$ decreasing?

In Exercises 20–33, find all the exact t-values for which the given statement is true.

20. $\tan t = 0$

21. $\sin t = \frac{\sqrt{2}}{2}$

22. $\sin t = 0$

23. $\cos t = -\frac{1}{2}$

24. $\tan t = 1$

25. $\sin t = -\frac{\sqrt{3}}{2}$

26. $\cos t = 0$

27. $\cos t = \frac{\sqrt{3}}{2}$

28. $\sin t = 1$

29. $\sin t = \frac{1}{2}$

30. $\tan t = -\frac{\sqrt{3}}{3}$

31. $\cos t = -\frac{\sqrt{2}}{2}$

In Exercises 34–43, list the transformations that change the graph of f into the graph of g. State the domain and range of g.

34. $f(t) = \cos t$; $g(t) = \cos t - 2$

35. $f(t) = \cos t$; $g(t) = -\cos t$

41. $f(t) = \sin t$; $g(t) = 3 \sin t + 2$

42. $f(t) = \cos t$; $g(t) = 5 \cos t + 3$

43. $f(t) = \sin t$; $g(t) = \sin t + 3$

In Exercises 44–48, sketch the graph of each function.

44. $f(t) = -2 \cos t$

45. $f(t) = 5 \sin t + 1$

46. $f(t) = 4 \tan t$

47. $f(t) = -\frac{1}{4} \cos t$

48. $f(t) = 3 \sin t - \frac{1}{2}$

In Exercises 49–54, match a graph to a function. Only one graph is possible for each function.

49. $h(t) = -2\tan t$

50. $g(t) = 2.5\cos t$

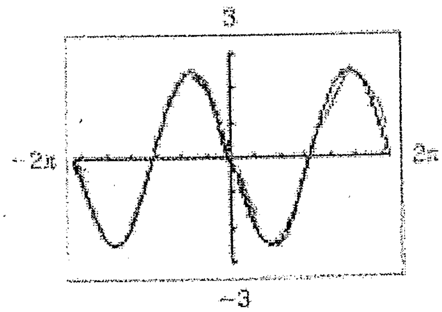
51. $h(t) = -\sin t + 1$

52. $f(t) = -2.5\sin t$

53. $g(t) = 3\tan t - 1$

54. $f(t) = -\cos t + 1$

a.



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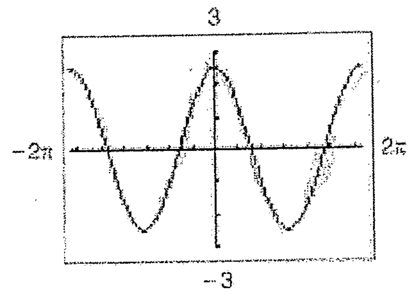

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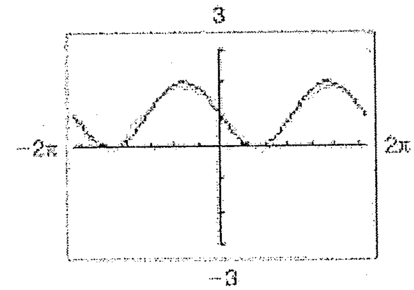

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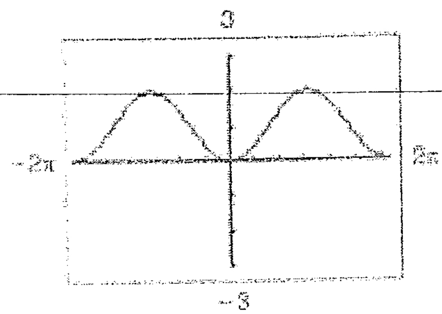

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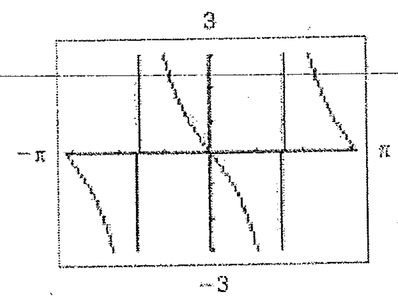

f.