8-6 Solving Rational Equations

Content Standards
A.APR.7 . . . Add, subtract, multiply, and divide rational expressions.
Also A.APR.6, A.CED.1, A.REI.11

Objectives To solve rational equations
To use rational equations to solve problems

A **rational equation** contains at least one rational expression. You can simplify solving a rational equation if you first clear the equation of denominators. You can do this by multiplying by the LCD of the rational expressions in the equation.

Rational Equation	Not a Rational Equation
$\frac{x}{x+1} + \frac{x}{x-1} = \frac{2}{x^2-1}$	$x + \frac{1}{2} = \frac{2}{3}$

Any time you multiply each side of an equation by an algebraic expression, it is possible to introduce an extraneous solution. Recall that an extraneous solution is a solution of the derived equation, but not a solution of the original equation. You must check all solutions in the original equation to confirm that they are indeed solutions.

Problem 1 Solving a Rational Equation

What are the solutions of the rational equation?

A $\frac{x}{x-3} + \frac{x}{x+3} = \frac{2}{x^2-9}$

$\times (x+3)(x-3)$

$\frac{x}{(x-3)} + \frac{x}{(x+3)} = \frac{2}{(x+3)(x-3)}$

$x(x+3) + x(x-3) = 2$

$x^2 + 3x + x^2 - 3x = 2$

$2x^2 = 2$

$x^2 = 1$

$x = \pm 1$ ← good

B $\frac{x-1}{x^2+3x+2} + \frac{2x}{x+2} = \frac{x-1}{x+1}$

$\times (x+2)(x+1)$

$\frac{(x-1)}{(x+2)(x+1)} + \frac{2x(x+2)(x+1)}{(x+2)} = \frac{(x-1)(x+2)(x+1)}{(x+1)}$

$(x-1) + 2x(x+1) = (x-1)(x+2)$

$x - 1 + 2x^2 + 2x = x^2 + x - 2$

$x^2 + 2x + 1 = 0$

$(x+1)(x+1) = 0$

$x = -1$

No Sol.

a. $\frac{x-1}{x+2} = \frac{x^2+2x-3}{x+2}$ $(x+2)$

$x - 1 = x^2 + 2x - 3$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x = -2 \quad x = 1$

$x = 1$

Answer

b. $\frac{x}{(x+1)} + \frac{3}{x+4} = \frac{x+3}{x+4}$ $(x+4)(x+1)$

$x(x+4) + 3(x+1) = (x+3)(x+1)$

$x^2 + 4x + 3x + 3 = x^2 + 4x + 3$

$3x = 0$

$x = 0$

Solve each equation. Check each solution.

See Problem 1.

8. $\frac{1}{4} - x = \frac{x}{8}$

9. $\frac{y}{5} + \frac{y}{2} = 7$

10. $\frac{2x}{3} - \frac{1}{2} = \frac{2x + 5}{6}$

11. $\frac{3x - 2}{12} - \frac{1}{6} = \frac{1}{6}$

12. $\frac{1}{x} + \frac{x}{2} = \frac{x + 4}{2x}$

13. $\frac{11}{3x} - \frac{1}{3} = \frac{-4}{x^2}$

14. $\frac{3}{2x} - \frac{5}{3x} = 2$

15. $\frac{5x}{4} - \frac{3}{x} = \frac{1}{4}$

16. $\frac{2}{y} + \frac{1}{2} = \frac{5}{2y}$

17. $x + \frac{6}{x} = -5$

18. $\frac{1}{4x} - \frac{3}{4} = \frac{7}{x}$

19. $\frac{5}{2x} - \frac{2}{3} = \frac{1}{x} + \frac{5}{6}$