

### 7.3 Periodic Graphs and Amplitude

We have looked at graphs of the form

$$f(t) = a \sin t + d$$

Where "a" measures the steepness of the graph  
 if  $a > 1$  it's a vertical stretch  
 if  $a < 1$  it's a vertical compression  
 by a factor of "a"

Where "d" shifts the graph up or down vertically by "d" units.

Now let's consider  $f(t) = a \sin bt + d$  where "b" will effect the period of the graph. Before "b" was 1 resulting in a period of  $2\pi$ .

By definition the period for sine and cosine is  $= \frac{2\pi}{|b|}$

The increments (sine: on, above, on, below, on OR cosine: above, on, below, on, above) on the x axis are found by taking the Period and dividing it into 4 increments. So now what you will do, is take  $\frac{2\pi}{|b|}$  and that number is going to be divided into 4 to find the increments..... for example:

Graph 2 full periods:

$$f(t) = \sin 2t$$

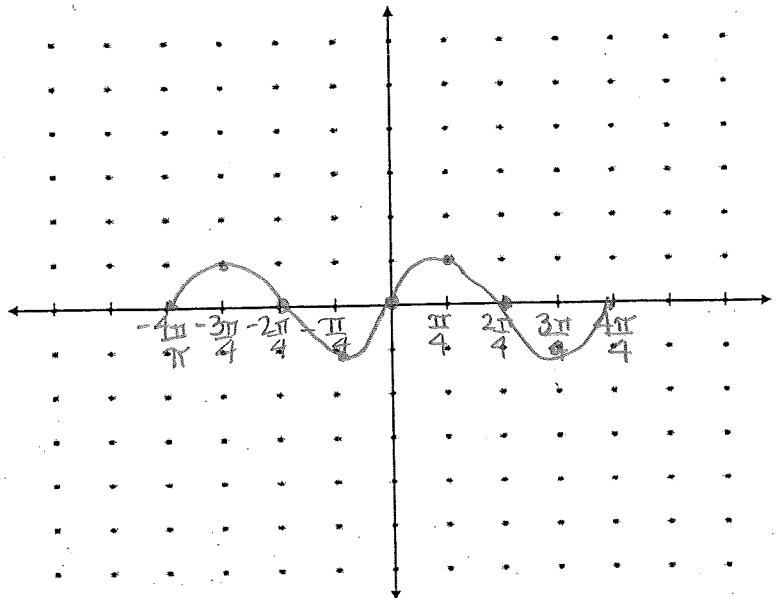
$$y = 1 \sin (2)x + 0$$

Amplitude: 1

Period:  $\frac{2\pi}{2} = \pi$

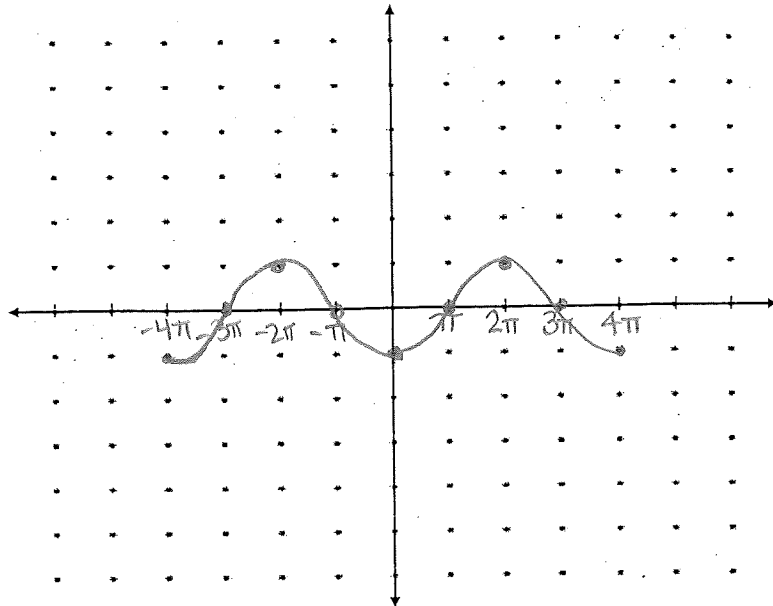
Increments:  $\frac{\pi}{4}$

V. Shift: 0



Graph  $f(t) = -\cos\left(\frac{1}{2}t\right) + 0$

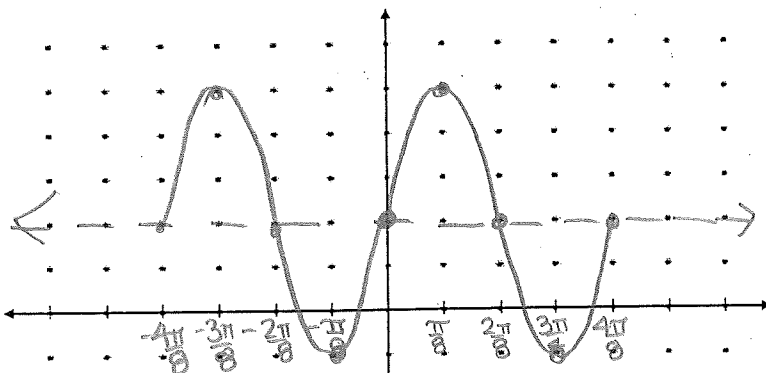
Amplitude:  $\frac{1}{1}$   
 Period:  $\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} = 4\pi$   
 Increments:  $\frac{4\pi}{4} = \pi$   
 V. Shift:  $0$



When  $b > 1$ , you will have a horizontal compression by a factor of  $1/b$ .  
 When  $b < 1$ , you will have a horizontal stretch by a factor of  $1/b$ .

Graph  $f(t) = 3 \sin 4t + 2$   $b=4$

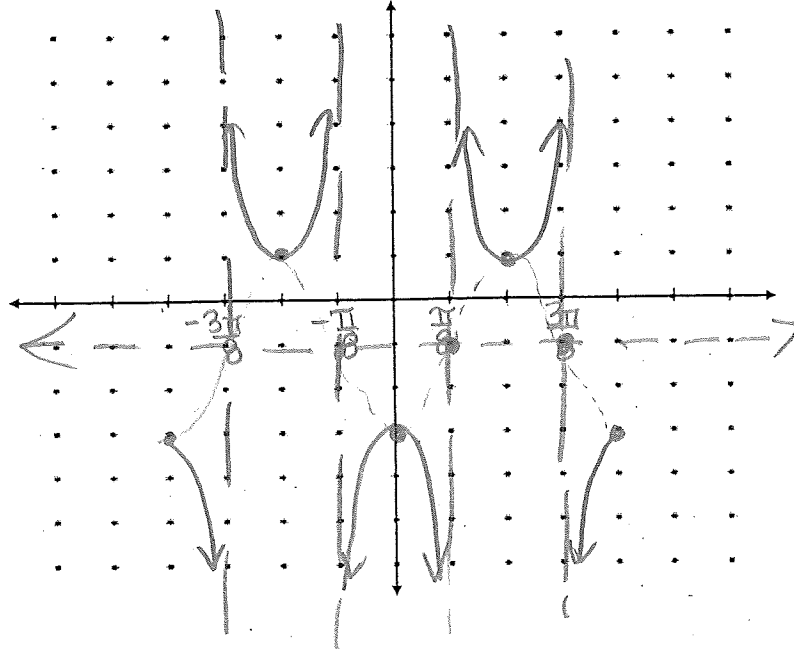
Amplitude:  $3$   
 Period:  $\frac{2\pi}{4} = \frac{\pi}{2}$   
 Increments:  $\frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$   
 V. Shift:  $2$



$y = -2 \cos 4t - 1$

Graph  $f(t) = -2 \sec 4t - 1$

Amplitude:  $2$   
 Period:  $\frac{2\pi}{4} = \frac{\pi}{2}$   
 Increments:  $\frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$   
 V. Shift:  $1 \text{ down}$



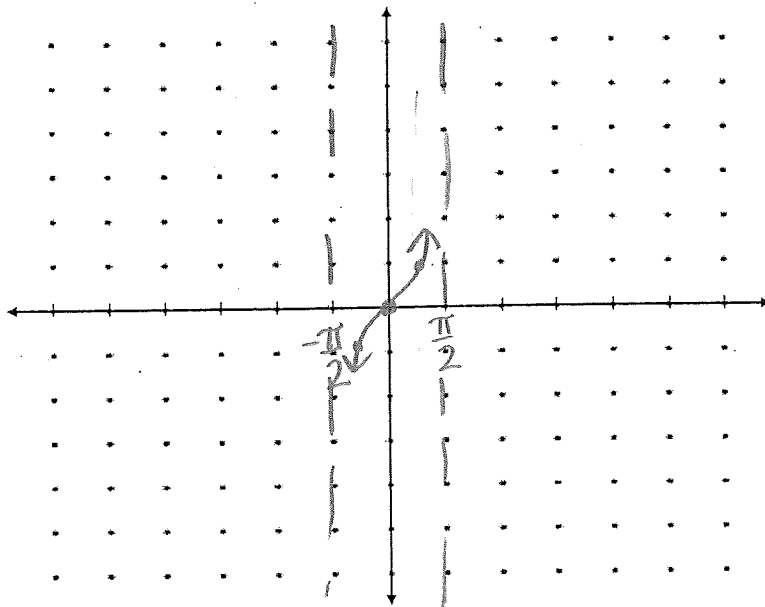
For the tan and cot graphs, the period was every  $\pi$  and not  $2\pi$ .

By definition the period for tan and cot =  $\frac{\pi}{|b|}$ .

The increments are  $\frac{P}{|b|}$

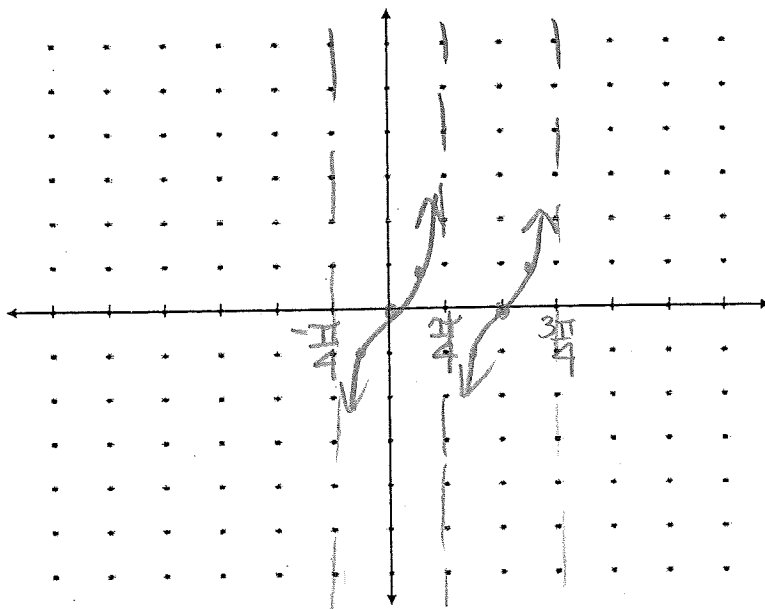
Graph  $f(t) = \tan t$

Steepness : 1  
 Period:  $\pi$   
 Increments:  $\frac{\pi}{2}$   
 V. Shift: ~~0~~



$f(t) = \tan 2t$

Steepness : 1  
 Period:  $\frac{\pi}{b} = \frac{\pi}{2}$   
 Increments:  $\frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$   
 V. Shift: ~~0~~



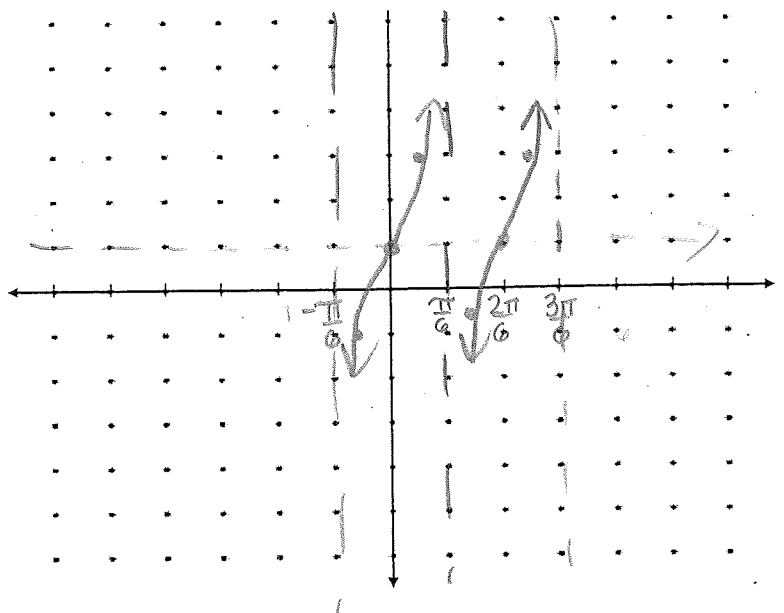
Graph  $f(t) = 2 \tan 3t + 1$

Steepness: 2

Period:  $\frac{\pi}{3}$

Increments:  $\frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

V. Shift: up 1



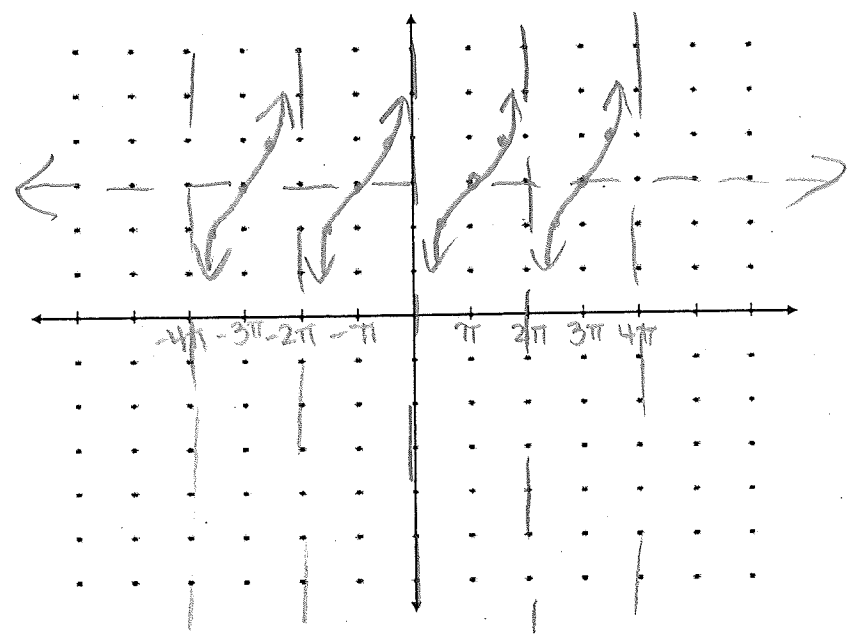
Graph  $f(t) = -\cot \frac{1}{2}t + 3$

Steepness: 1

Period:  $\frac{\pi}{\frac{1}{2}} = 2\pi$

Increments:  $\frac{2\pi}{2} = \pi$

V. Shift: up 3



Example: Write an equation for a sine function with the given information:

Amplitude of  $\frac{1}{2}$ , and period is  $\frac{\pi}{2}$  and shifted 3 units down.

$f(t) = a \sin bt + d$

$f(t) = \frac{1}{2} \sin 4t - 3$

~~$\frac{2\pi}{b} = \frac{\pi}{2}$~~   
 $\pi b = 4\pi$   
 $b = 4$

$\tan \frac{\pi}{b} = \cot \frac{\pi}{b}$

For #'s 1-8, find all values for which the given statement is true.

1.  $\cos t = -\frac{\sqrt{2}}{2}$

1. \_\_\_\_\_

2.  $\tan t = -\sqrt{3}$

2. \_\_\_\_\_

3.  $\cos t = -\frac{1}{2}$

3. \_\_\_\_\_

4.  $\sin t = -\frac{\sqrt{3}}{2}$

4. \_\_\_\_\_

5.  $\cos t = -1$

5. \_\_\_\_\_

6.  $\tan t = -1$

6. \_\_\_\_\_

7.  $\sin t = 0$

7. \_\_\_\_\_

8.  $\tan t = \frac{\sqrt{3}}{3}$

8. \_\_\_\_\_

In 9-17, graph two complete periods!!!  
LABEL YOUR GRAPHS! BE NEAT!!

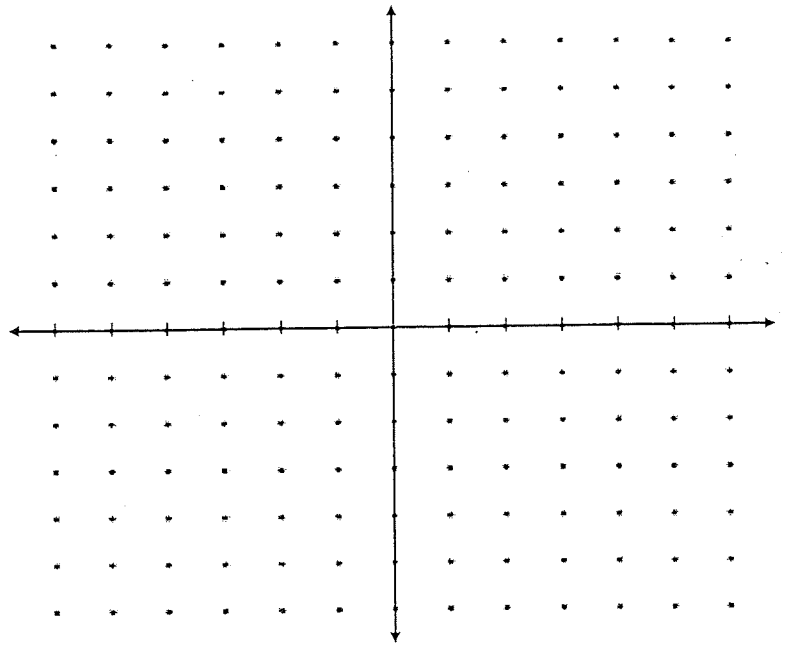
9. Graph  $f(t) = -3\cos t + 2$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

Increments: \_\_\_\_\_

V. Shift: \_\_\_\_\_



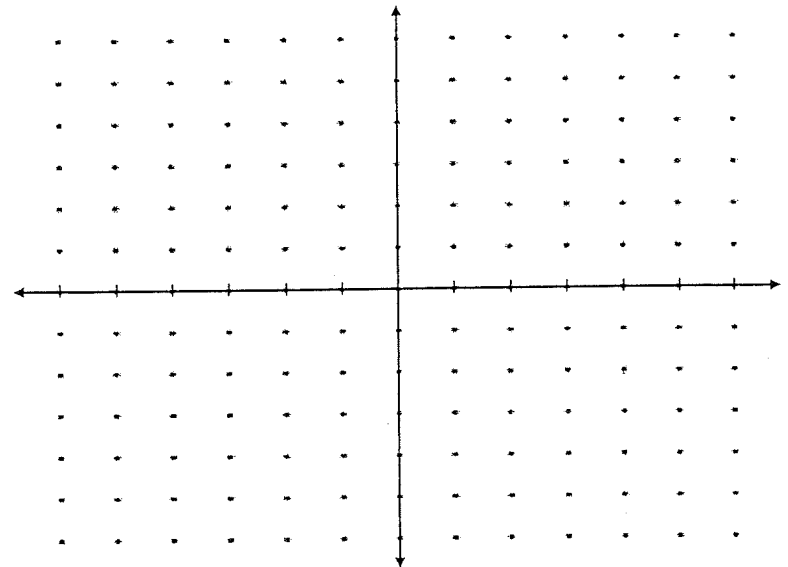
10. Graph  $f(t) = 4\cos\frac{t}{2} + 1$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

Increments: \_\_\_\_\_

V. Shift: \_\_\_\_\_



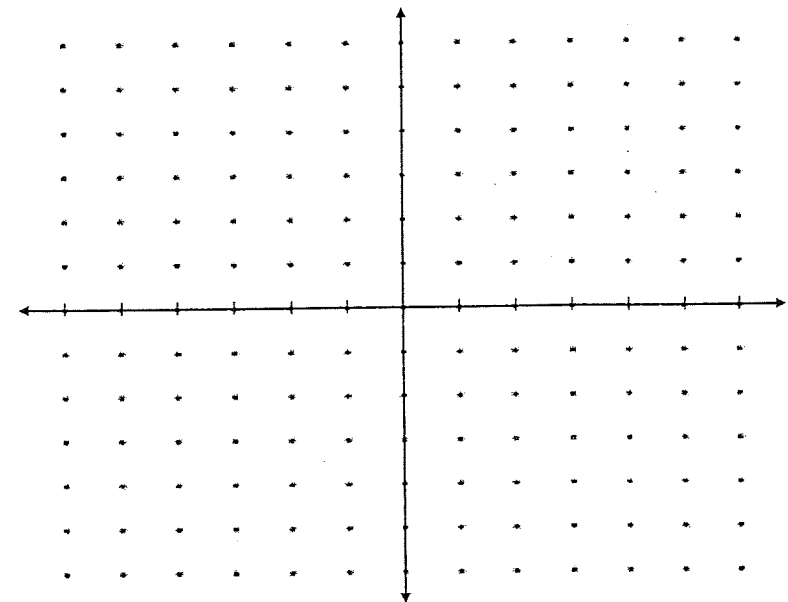
11. Graph  $f(t) = 2\sin \pi t$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

Increments: \_\_\_\_\_

V. Shift: \_\_\_\_\_



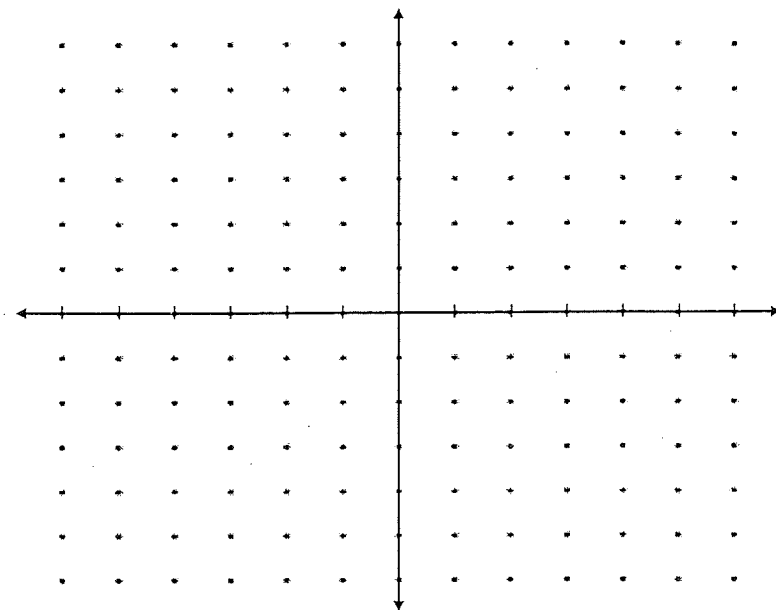
12. Graph  $f(t) = -3 \cos 2t + 2$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

Increments: \_\_\_\_\_

V. Shift: \_\_\_\_\_



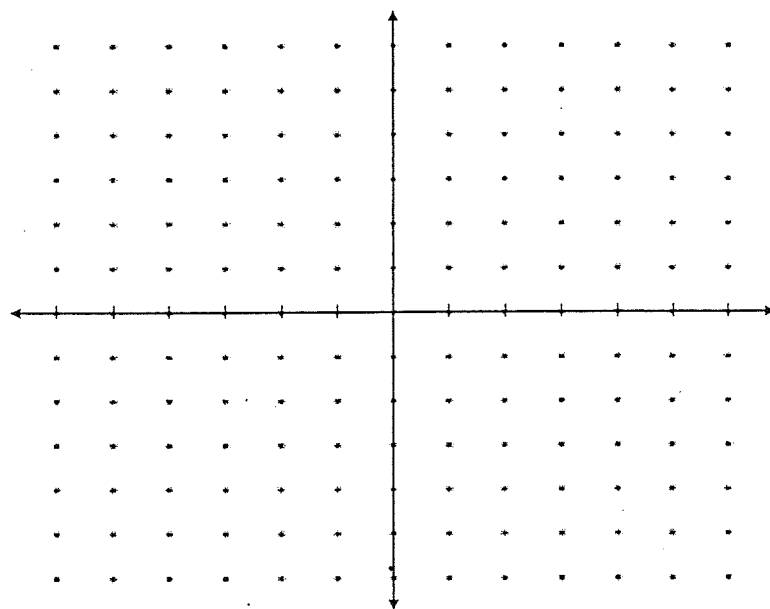
13. Graph  $f(t) = 2 \tan 3t$

Steepness: \_\_\_\_\_

Period: \_\_\_\_\_

Increments: \_\_\_\_\_

V. Shift: \_\_\_\_\_



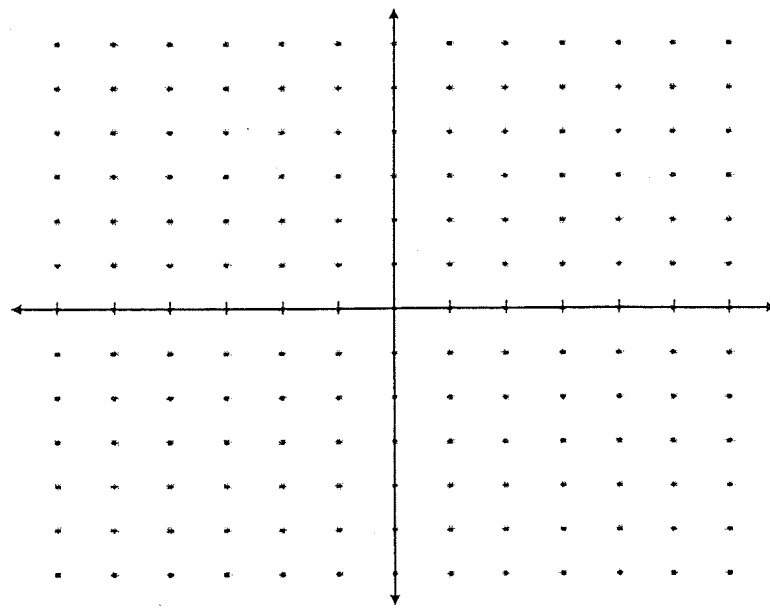
14. Graph  $f(t) = \cot \frac{\pi}{2} t$

Steepness: \_\_\_\_\_

Period: \_\_\_\_\_

Increments: \_\_\_\_\_

V. Shift: \_\_\_\_\_



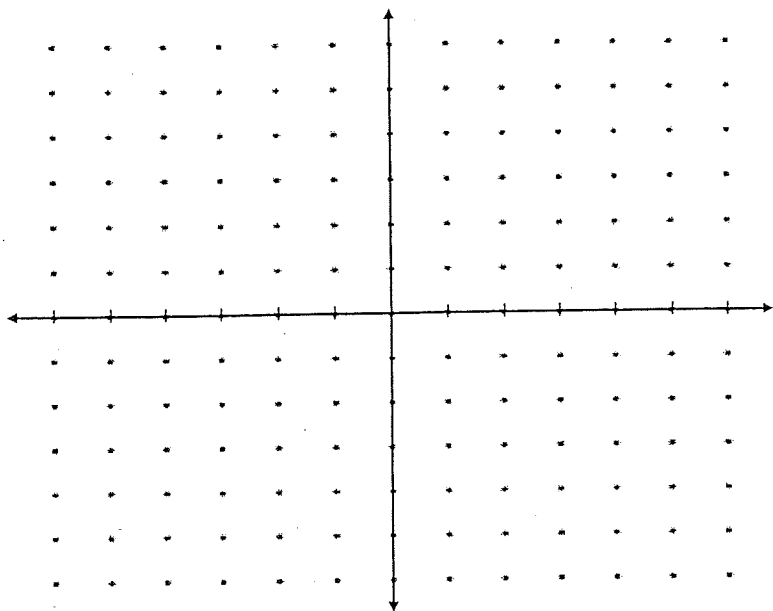
15. Graph  $f(t) = \frac{1}{2} \tan \pi t + 2$

Steepness: \_\_\_\_\_

Period: \_\_\_\_\_

Increments: \_\_\_\_\_

V. Shift: \_\_\_\_\_



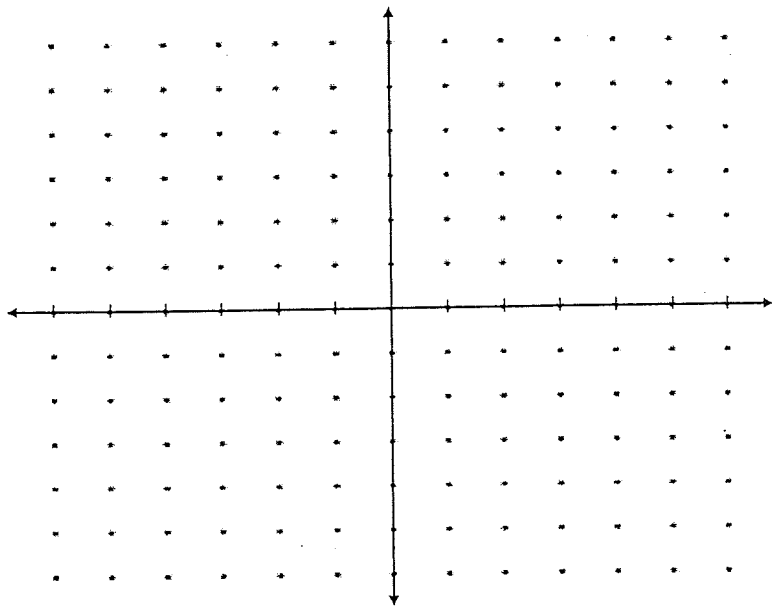
16. Graph  $f(t) = 2 \cos \pi t$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

Increments: \_\_\_\_\_

V. Shift: \_\_\_\_\_



17. Graph  $f(t) = 3 \sin 2\pi t$

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

Increments: \_\_\_\_\_

V. Shift: \_\_\_\_\_

