11-2 Probability

Content Standard
Prepares for S.IC.2 Decide if a specified model is consistent with results from a given data generating process, e.g., using simulation.

Objective To find the probability of an event using theoretical, experimental, and simulation methods

Probability measures how likely it is for an event to occur.

Take note

Key Concept Experimental Probability

experimental probability of event: $P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{number of trials}}$

Essential Understanding The probability of an impossible event is 0 (or 0%). The probability of a certain event is 1 (or 100%). Otherwise, the probability of an event is a number between 0 and 1 (or a percent between 0% and 100%).

When you gather data from observations, you can calculate an *experimental probability*. Each observation is an experiment or a trial.

Problem 1 Finding Experimental Probability

Think
What is a trial? What is an event?
A trial is selecting a vehicle parking in the lot. An event is the vehicle you select being a truck.

Gridded Response Of the 60 vehicles in a teachers' parking lot today, 15 are pickup trucks. What is the experimental probability that a vehicle in the lot is a pickup truck?

$$P(\text{pickup truck}) = \frac{\text{number of pickup trucks}}{\text{number of vehicles}} = \frac{15}{60} = 0.25, \text{ or } 25\%$$

The probability that a vehicle in the lot is a pickup truck is 0.25.

Got It? 1. A softball player got a hit in 20 of her last 50 times at bat. What is the experimental probability that she will get a hit in her next at bat?

Dynamic Activity Geometric Probability

Sometimes actual trials are difficult or unreasonable to conduct. In these situations, you can estimate the experimental probability of an event by using a simulation. A **simulation** is a model of the event.

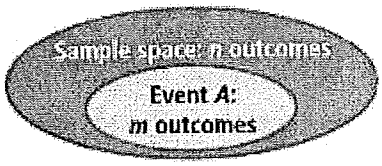
The set of all possible outcomes to an experiment or activity is a **sample space.** When each outcome in a sample space has the same chance of occurring, the outcomes are **equally likely outcomes.**

For one roll of a standard number cube, there are six equally likely outcomes in the sample space. You can calculate *theoretical probability* as a ratio of outcomes.

Take note

Key Concept Theoretical Probability

If a sample space has n equally likely outcomes and an event A occurs in m of these outcomes, then the **theoretical probability** of event A is $P(A) = \frac{m}{n}$.



Problem 3 Finding Theoretical Probability

What is the theoretical probability of each event?

A getting a 5 on one roll of a standard number cube

There are six equally likely outcomes, 1, 2, 3, 4, 5, and 6. The number 5 occurs one way.

$P(5) = \frac{1}{6}$

Plan

How many outcomes are there?
Each cube has six numbers on it, so there are $6 \cdot 6 = 36$ outcomes.

B getting a sum of 5 on one roll of two standard number cubes

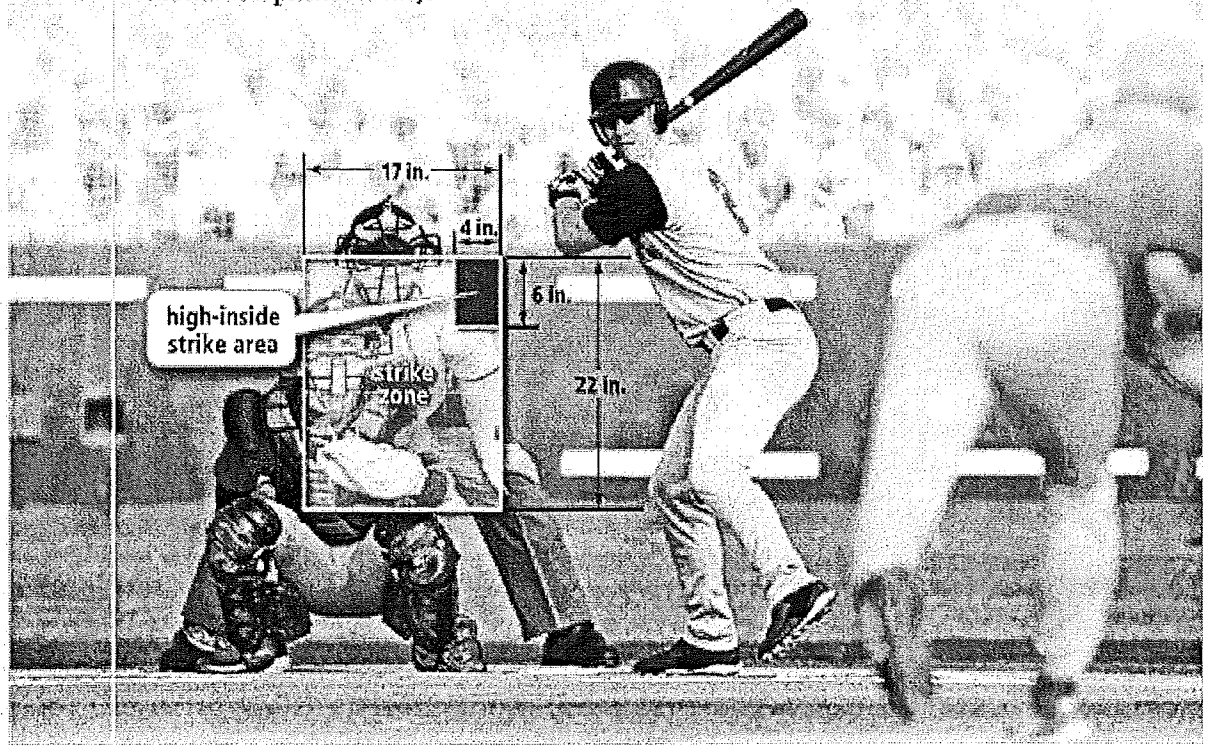
There are 36 possible equally likely outcomes. The favorable outcomes are those with a sum of 5.

$P(\text{sum } 5) = \frac{4}{36} = \frac{1}{9}$

Got It? 3. a. What is the theoretical probability of getting a sum that is an odd number on one roll of two standard number cubes?

Problem 5 Finding Geometric Probability

Geometry A batter's strike zone depends on the height and stance of the batter. What is the geometric probability that a baseball thrown at random within the batter's strike zone, as shown in the figure below, will be a high-inside strike (one of the hardest pitches to hit)?



Think

What are the favorable outcomes? All outcomes?
Favorable outcomes are points in the high-inside region. All outcomes are points in the strike zone.

$$P(\text{high-inside strike}) = \frac{\text{area of high-inside strike zone}}{\text{area of total strike zone}}$$

$$= \frac{4 \cdot 6}{17 \cdot 22}$$

$$\approx 0.064$$

For a baseball thrown at random in the batter's strike zone, the probability that it will be a high-inside strike is about 6.4%.

Got It? 5. Suppose a batter's strike zone is 15 in.-by-20 in. and the high-inside strike zone is 3 in.-by-5 in. What is the probability that a baseball thrown at random within the strike zone will be a high-inside strike?

8. A class tossed coins and recorded 161 heads and 179 tails. What is the experimental probability of heads? Of tails?

See Problem 1.

9. Another class rolled number cubes. Their results are shown in the table. What is the experimental probability of rolling each number?

Number	1	2	3	4	5	6
Occurrences	42	44	45	44	47	46

A jar contains 30 red marbles, 50 blue marbles, and 20 white marbles. You pick one marble from the jar at random. Find each theoretical probability.

See Problem 3.

13. P(red) **14.** P(blue) **15.** P(not white) **16.** P(red or blue)

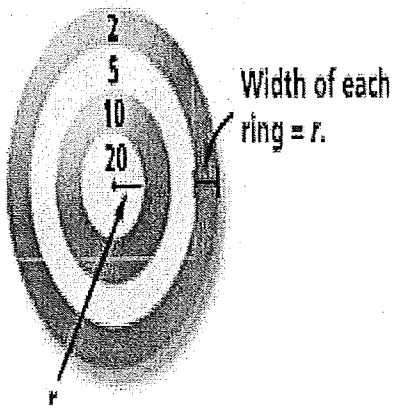
A bag contains 36 red blocks, 48 green blocks, 22 yellow blocks, and 19 purple blocks. You pick one block from the bag at random. Find each theoretical probability.

17. P(green) **18.** P(purple) **19.** P(not yellow)

20. P(green or yellow) **21.** P(yellow or not green) **22.** P(purple or not red)

Geometry Suppose that a dart lands at random on the dartboard shown at the right. Find each theoretical probability.

See Problem 5.



24. The dart lands in the bull's-eye.

25. The dart lands in a green region.

26. The dart scores at least 10 points.

27. The dart scores less than 10 points.

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The probability that a vehicle in the lot is a pickup truck is 0.25.

Got It? 1. A softball player got a hit in 20 of her last 50 times at bat. What is the experimental probability that she will get a hit in her next at bat?

$\frac{20}{50}$ 40%

Dynamic Activity Geometric Probability

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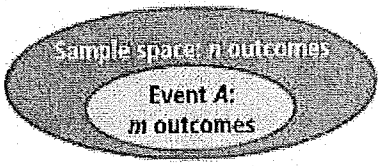

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There are 36 possible equally likely outcomes. The favorable outcomes are those with a sum of 5.

$P(\text{sum } 5) = \frac{4}{36} = \frac{1}{9}$

Got It? 3. a. What is the theoretical probability of getting a sum that is an odd number on one roll of two standard number cubes?

18/36

1,2
1 4
1,6
2,1
2,3
2,5
3,2
3,4
3,6
4,1
4,3
4,5
5,2
5,4
5,6
6,1
6,3
6,5

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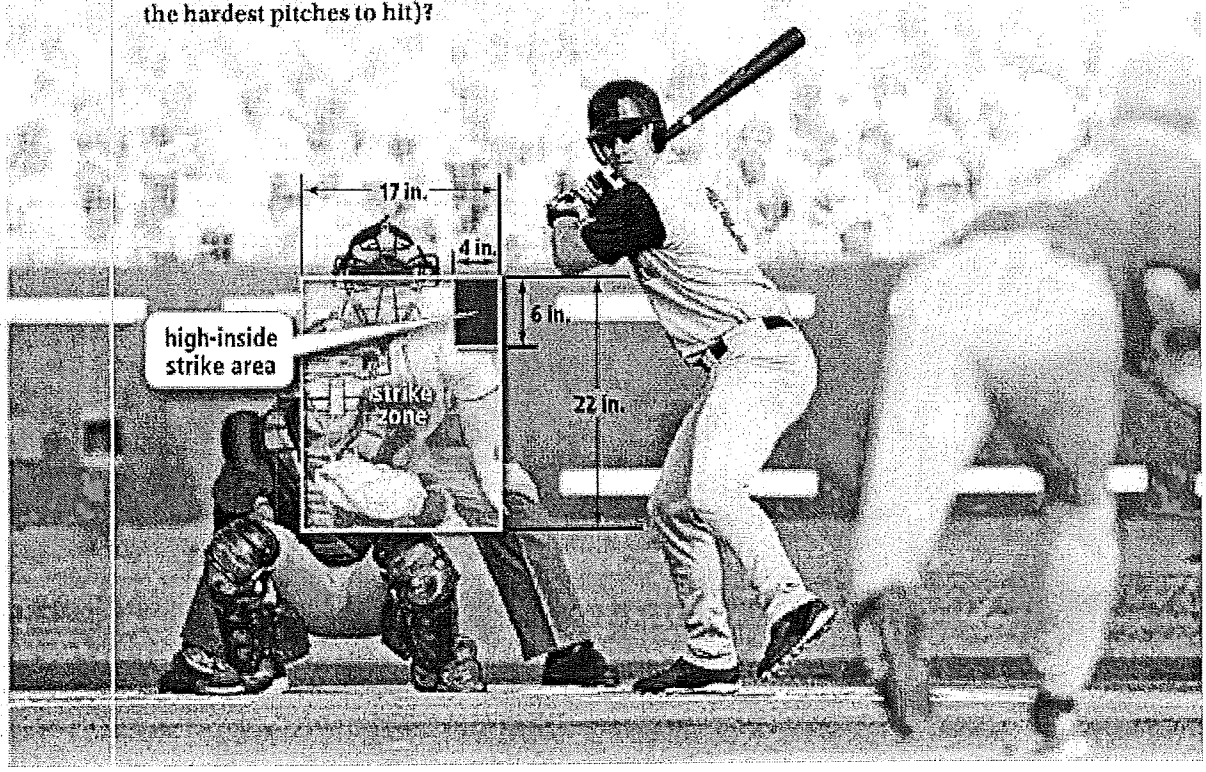

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Got It? 5. Suppose a batter's strike zone is 15 in.-by-20 in. and the high-inside strike zone is 3 in.-by-5 in. What is the probability that a baseball thrown at random within the strike zone will be a high-inside strike?

$$\frac{3 \times 5}{15 \times 20} = \frac{15}{300} = .05$$

5%

161/340 = 47%

8. A class tossed coins and recorded 161 heads and 179 tails. What is the experimental probability of heads? Of tails? ◀ See Problem 1.

179/340 = 53%

9. Another class rolled number cubes. Their results are shown in the table. What is the experimental probability of rolling each number?

Number	1	2	3	4	5	6
Occurrences	42	44	45	44	47	46

268

A jar contains 30 red marbles, 50 blue marbles, and 20 white marbles. You pick one marble from the jar at random. Find each theoretical probability. ◀ See Problem 3.

13. P(red) $\frac{30}{100} = 30\%$

14. P(blue) $\frac{50}{100} = 50\%$

15. P(not white) $\frac{80}{100} = 80\%$

16. P(red or blue) $\frac{80}{100} = 80\%$

A bag contains 36 red blocks, 48 green blocks, 22 yellow blocks, and 19 purple blocks. You pick one block from the bag at random. Find each theoretical probability.

17. P(green) $\frac{48}{125} = 38.4\%$

18. P(purple) $\frac{19}{125} = 15.2\%$

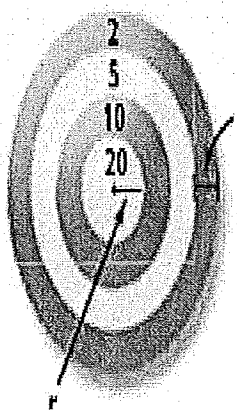
19. P(not yellow) $36+48+19$ $\frac{103}{125} = 82.4\%$

20. P(green or yellow) $48+22$ $\frac{70}{125} = 56\%$

21. P(yellow or not green) $36+22+19$ $\frac{77}{125} = 61.6\%$

22. P(purple or not red) $19+48+22$ $\frac{89}{125} = 71.2\%$

Geometry Suppose that a dart lands at random on the dartboard shown at the right. Find each theoretical probability. ◀ See Problem 5.



24. The dart lands in the bull's-eye. 1/16

25. The dart lands in a green region.

26. The dart scores at least 10 points. $\frac{4\pi}{16\pi} = \frac{1}{4} = 25\%$

27. The dart scores less than 10 points. $16\pi - 4\pi = \frac{12\pi}{16\pi} =$ $\frac{1}{16}$

area $= \pi r^2$

$\pi \cdot 1^2$

3.14

$\pi \cdot 4^2$

16π