11-3 Probability of Multiple Events

Content Standards

S.CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

Also S.CP.2, S.CP.5

Objectives To find the probability of the event A and B
To find the probability of the event A or B

Essential Understanding To find the probability of two events occurring together, you have to decide whether one event occurring affects the other event.

When the occurrence of one event affects how a second event can occur, the events are **dependent events**. Otherwise, the events are **independent events**.

Problem 1 Classifying Events

Are the outcomes of each trial dependent or independent events?

A Roll a number cube. Then spin a spinner.

The two events do not affect each other. They are independent.

B Pick one flash card, then another from a stack of 30 flash cards.

Picking the first card affects the possible outcomes of picking the second card. The events are dependent.

Got It? 1. You select a coin at random from your pocket. You replace the coin and select again. Are your selections independent events? Explain.

Think

What must you ask yourself?
Does the first event have any effect on the outcome of the second event?

Multiply to find the probability that two independent events will both occur.

Take Note

Key Concept Probability of A and B

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

A Practice Tell whether the outcomes of each trial are dependent events or independent events. See Problem 1.

9. A month is selected at random; a number from 1 to 30 is selected at random.
10. A month is selected at random; a day of that month is selected at random.
11. A letter of the alphabet is selected at random; one of the remaining letters is selected at random.
12. The color of a car is selected at random; the type of transmission is selected at random.

Problem 2 Finding the Probability of Independent Events

Dynamic Activity Independent and Dependent Events

Picnic At a picnic there are 10 diet drinks and 5 regular drinks. There are also 8 bags of fat-free chips and 12 bags of regular chips. If you grab a drink and a bag of chips without looking, what is the probability that you get a diet drink and fat-free chips?

Think

Is it important that you don't look?
Yes; probability is based on random events. It is not random if you look.

Event A = picking a diet drink Event B = picking fat-free chips

A and B are independent. Picking a drink has no effect on picking the chips.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$= \frac{\text{number of diet drinks}}{\text{total number of drinks}} \cdot \frac{\text{number of bags of fat-free chips}}{\text{total number of bags of chips}}$$

$$= \frac{10}{15} \cdot \frac{8}{20} = \frac{4}{15} \approx 0.267, \text{ or } 26.7\%$$

The probability that you get a diet drink and fat-free chips is about 26.7%.

Q and R are independent events. Find $P(Q \text{ and } R)$.

See Problem 2.

13. $P(Q) = \frac{1}{4}, P(R) = \frac{2}{3}$

14. $P(Q) = \frac{12}{17}, P(R) = \frac{3}{8}$

15. $P(Q) = 0.6, P(R) = 0.9$

16. $P(Q) = \frac{1}{3}, P(R) = \frac{6}{7}$

17. Reading Suppose you have five books in your book bag. Three are novels, one is a biography, and one is a poetry book. Today you grab one book out of your bag without looking, and return it later. Tomorrow you do the same thing. What is the probability that you grab a novel both days?

Two events that cannot happen at the same time are **mutually exclusive events.** If A and B are mutually exclusive events, then $P(A \text{ and } B) = 0$.

Problem 3 Mutually Exclusive Events

You roll a standard number cube. Are the events mutually exclusive? Explain.

Think
Can you roll a 2 and a 3 at the same time?
No; just one number comes up on one roll of one number cube.

A rolling a 2 and a 3

You cannot roll a 2 and 3 at the same time. The events are mutually exclusive.

B rolling an even number and a multiple of 3

You can roll a 6—an even number and a multiple of 3—at the same time. The events are not mutually exclusive.

Got It? 3. You roll a standard number cube. Are the events mutually exclusive? Explain.
- **a.** rolling an even number and rolling a prime number
- **b.** rolling an even number and rolling a number less than 2

To find the probability of either event A or event B occuring, you need to determine whether events A and B are mutually exclusive.

Take Note

Key Concept Probability of A or B

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

If A and B are mutually exclusive events, then $P(A \text{ or } B) = P(A) + P(B)$.

Two fair number cubes are rolled. State whether the events are mutually exclusive. Explain your reasoning. See Problem 3.

18. The sum is a prime number; the sum is less than 4.

19. The numbers are equal; the sum is odd.

20. The product is greater than 20; the product is a multiple of 3.

Problem 4 Finding Probability for Mutually Exclusive Events

Languages At your high school, a student can take one foreign language each term. About 37% of the students take Spanish. About 15% of the students take French. What is the probability that a student chosen at random is taking Spanish or French?

Know
- The percentages of students taking Spanish or French.
- Students can take one foreign language at a time.

Need
The probability that a student is taking Spanish or French.

Plan
Use the correct formula for $P(A \text{ or } B)$.

"One foreign language each term" means the events are mutually exclusive.

$$P(\text{Spanish or French}) = P(\text{Spanish}) + P(\text{French}) \quad P(A \text{ or } B) = P(A) + P(B) \text{ for mutually exclusive events.}$$
$$\approx 0.37 + 0.15$$
$$= 0.52$$

The probability that a student chosen at random is taking Spanish or French is about 0.52, or about 52%.

Got It? 4. a. In Problem 4, about 9% of the students take Mandarin Chinese. What is the probability that a student chosen at random is taking Spanish, French, or Mandarin Chinese?

S and T are mutually exclusive events. Find $P(S \text{ or } T)$. See Problem 4.

21. $P(S) = \frac{5}{8}, P(T) = \frac{1}{8}$

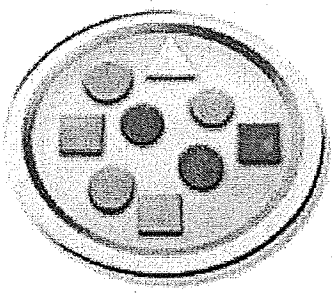
22. $P(S) = \frac{3}{5}, P(T) = \frac{1}{3}$

23. $P(S) = 12\%, P(T) = 27\%$

24. **Population** About 30% of the U.S. population is under 20 years old. About 17% of the population is over 60. What is the probability that a person chosen at random is under 20 or over 60?

Problem 5 Finding Probability

Multiple Choice Suppose you reach into the dish and select a token at random. What is the probability that the token is round or green?



(A) $\frac{2}{9}$ (B) $\frac{3}{9}$ (C) $\frac{6}{9}$ (D) $\frac{8}{9}$

Think
Are the events mutually exclusive?
No; it is possible to have a round *and* green token.

$P(\text{round or green})$

$= P(\text{round}) + P(\text{green}) - P(\text{round and green})$

$= \frac{5}{9} + \frac{3}{9} - \frac{2}{9} = \frac{6}{9}$

The probability of selecting a round or green token is $\frac{6}{9}$, or $\frac{2}{3}$. The correct answer is C.

Got It? 5. Suppose you select a token at random from the dish above. What is each probability?

a. the token is square or red

b. the token is green or square

A standard number cube is tossed. Find each probability. See Problem 5.

25. $P(3 \text{ or odd})$

26. $P(4 \text{ or even})$

27. $P(\text{even or less than } 4)$

28. $P(\text{odd or greater than } 2)$

29. $P(\text{odd or prime})$

30. $P(4 \text{ or less than } 6)$

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What must you ask yourself?
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Got It? 1. You select a coin at random from your pocket. You replace the coin and select again. Are your selections independent events? Explain.

Independent events.

Multiply to find the probability that two independent events will both occur.

Take Note

Key Concept Probability of *A* and *B*

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A Practice

Tell whether the outcomes of each trial are dependent events or independent events.

See Problem 1.

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10. A month is selected at random; a day of that month is selected at random. dependent
11. A letter of the alphabet is selected at random; one of the remaining letters is selected at random. dependent
12. The color of a car is selected at random; the type of transmission is selected at random. indep.

10/15 15 drinks 8/20

Problem 2 Finding the Probability of Independent Events

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$\frac{1}{4} \cdot \frac{2}{3} = \frac{1}{6}$

14. $P(Q) = \frac{12}{17}, P(R) = \frac{3}{8}$

$\frac{12}{17} \cdot \frac{3}{8} = \frac{9}{34}$

15. $P(Q) = 0.6, P(R) = 0.9$

$(.6)(.9) = .54$

16. $P(Q) = \frac{1}{3}, P(R) = \frac{6}{7}$

$\frac{1}{3} \cdot \frac{6}{7} = \frac{2}{7}$

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19. The numbers are equal; the sum is odd. 1 1 22 33 44 55 66

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not mutually exclusive (can do it)

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.09 + .37 + .15

.61

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$\frac{5}{8} + \frac{1}{8} = \frac{6}{8}$

3/4

22. $P(S) = \frac{3}{5}$, $P(T) = \frac{1}{3}$

$\frac{3}{5} + \frac{1}{3}$

14/15

23. $P(S) = 12\%$, $P(T) = 27\%$

.12 + .27 = .39

39%

24. **Population** About 30% of the U.S. population is under 20 years old. About 17% of the population is over 60. What is the probability that a person chosen at random is under 20 or over 60?

.30 + .17

47

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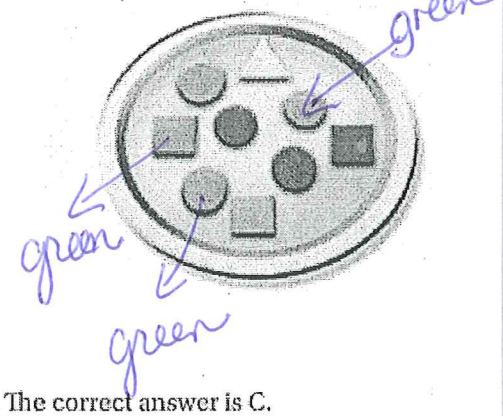
9 total



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A standard number cube is tossed. Find each probability. See Problem 5.

25. $P(3 \text{ or odd})$

$P(3) + P(\text{odd}) - P(3 \text{ and odd})$

$\frac{1}{6} + \frac{3}{6} - \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

26. $P(4 \text{ or even})$

27. $P(\text{even or less than } 4)$

$P(\text{even}) + P(\text{less than } 4)$

$\frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6}$

28. $P(\text{odd or greater than } 2)$

29. $P(\text{odd or prime})$

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