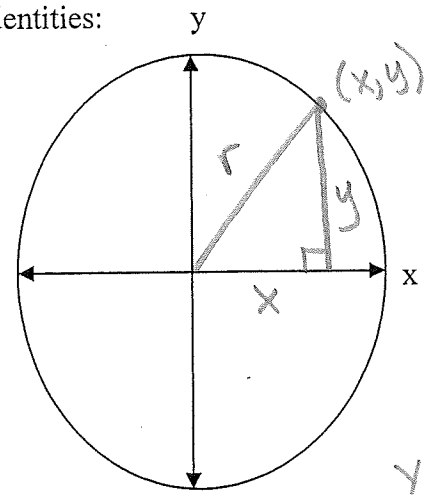
9.1 PreCalculus

Trig Identities and Proofs.

Pythagorean Identities:



$\sin = \frac{y}{r}$

$\cos = \frac{x}{r}$

$\frac{\sin}{\cos} = \tan$

1) $x^2 + y^2 = r^2$

2) divide everything by r^2

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$

3) $\cos^2 x + \sin^2 x = 1$

1. $\cos^2 x + \sin^2 x = 1$

2. divide everything by $\cos^2 x$

$\cos^2 x + \sin^2 x = 1$

$\frac{\cos^2 x}{\cos^2 x} \quad \frac{\sin^2 x}{\cos^2 x} \quad \frac{\cos^2 x}{\cos^2 x}$

3. $1 + \tan^2 x = \sec^2 x$

1. $\cos^2 x + \sin^2 x = 1$

$\frac{\sin^2 x}{\sin^2 x} \quad \frac{\sin^2 x}{\sin^2 x} \quad \frac{\sin^2 x}{\sin^2 x}$

2. divide everything by $\sin^2 x$

$\cos^2 x + \sin^2 x = 1$

$\frac{\sin^2 x}{\sin^2 x} \quad \frac{\sin^2 x}{\sin^2 x} \quad \frac{\sin^2 x}{\sin^2 x}$

3. $\cot^2 x + 1 = \csc^2 x$

Now you can take these 3 Identities and transform them into other equations:

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\sin^2 x = 1 - \cos^2 x$

$-\sin^2 x = \cos^2 x - 1$

$1 + \tan^2 x = \sec^2 x$

$1 = \sec^2 x - \tan^2 x$

$\tan^2 x = \sec^2 x - 1$

$\cot^2 x + 1 = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

$1 = \csc^2 x - \cot^2 x$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \csc x = \frac{1}{\sin x}$$

$$\cos x = \frac{1}{\sec x} \qquad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x} \qquad \cot x = \frac{1}{\tan x}$$

Negative Angle Identities

$$\sin(-x) = -\sin x$$

$$\tan(-x) = -\tan x$$

$$\cos(-x) = \cos x$$

Pythagorean Identites

$$\cos^2 x + \sin^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x \qquad \cot^2 x + 1 = \csc^2 x$$

To Prove Identities, we will use the following stategies.....

1. Transform one side of the equation into the other. Choose only one side to work with... NOT BOTH!
2. Use substitution to help simplify
3. Write the given in terms of sine and cosine.
4. Use all methods of factoring to break down the problem into factors (ie. Difference of 2 perfect squares)
5. Know all of the identities above. The better you know them, the easier this will be.
6. Always work DOWN (VERTICALLY NOT HORIZONTALLY) when proving the identity.

9.1 Trig identities. Prove each! Examples!

1. $\cot x \sin x = \cos x$

$\frac{\cos x}{\sin x} \cdot \frac{\sin x}{1} =$

$\cos x = \cos x$

2. $\frac{\cot x}{\csc x} = \cos x$

$\frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} =$

$\frac{\cos x}{\sin x} \cdot \frac{\sin x}{1}$

$\cos x = \cos x$

factoring

3. $(\csc x - 1)(\csc x + 1) = \cot^2 x$

$\csc^2 x - 1 =$

$\cot^2 x = \cot^2 x$

4. $\sec^4 x - \tan^4 x = 1 + 2\tan^2 x$

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$(1)(\sec^2 x + \tan^2 x)$

$(1 + \tan^2 x) + \tan^2 x$

$1 + 2\tan^2 x =$

5. $\sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$

$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$

$\frac{\sin^2 x}{\cos^2 x \sin^2 x} + \frac{\cos^2 x}{\cos^2 x \sin^2 x}$

$\frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$

$\frac{1}{\cos^2 x \sin^2 x}$

$\sec^2 x \cdot \csc^2 x$

6. $\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x} = 2\csc x$

$\frac{(1+\cos x)}{(1+\cos x)}$

$\frac{1 + 2\cos x + \cos^2 x}{(\sin x)(1+\cos x)} + \frac{\sin^2 x}{(\sin x)(1+\cos x)}$

$\frac{1 + 2\cos x + \cos^2 x + \sin^2 x}{(\sin x)(1+\cos x)}$

$\frac{2 + 2\cos x}{(\sin x)(1+\cos x)}$

$\frac{2(1+\cos x)}{(\sin x)(1+\cos x)}$

$\frac{2}{\sin x}$

$\frac{2}{1} \cdot \frac{1}{\sin x} = 2\csc x$

7. $\frac{1}{\csc x - \sin x} = \sec x \tan x$

$\frac{1}{\frac{1}{\sin x} - \frac{\sin x \cdot \sin x}{1 \cdot \sin x}}$

$\frac{1}{\frac{1 - \sin^2 x}{\sin x}}$

$\frac{1}{\frac{\cos^2 x}{\sin x}}$

$\frac{1}{1} \cdot \frac{\sin x}{\cos^2 x}$

$\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$

$\tan x \cdot \sec x$

$\sec x \cdot \tan x$

9.1 Homework

Name_____

These problems come from your book. You can refer to 9.1 in your book, page 580. The answers to the odd are in the back of your book.

In Exercises 9–18, prove the identity.

9. $\tan x \cos x = \sin x$

11. $\cos x \sec x = 1$

13. $\tan x \csc x = \sec x$

15. $\dfrac{\tan x}{\sec x} = \sin x$

17. $(1 + \cos x)(1 - \cos x) = \sin^2 x$

27. $\sec^2 x - \csc^2 x = \tan^2 x - \cot^2 x$

31. $\sin^2 x - \tan^2 x = -\sin^2 x \tan^2 x$

33. $(\cos^2 x - 1)(\tan^2 x + 1) = -\tan^2 x$

35. $\tan x = \dfrac{\sec x}{\csc x}$

37. $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$

41. $\dfrac{\sec x}{\csc x} + \dfrac{\sin x}{\cos x} = 2 \tan x$

43. $\dfrac{\sec x + \csc x}{1 + \tan x} = \csc x$