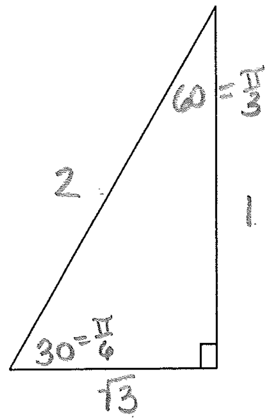
9.2 PreCalculus Notes
Addition and Subtraction Identities

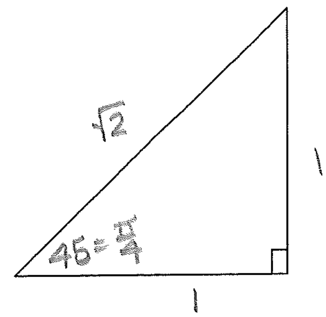
Name Key

This set of identities will help
1. Prove identities
2. Evaluate trig functions of angles w/o a calculator.

S S

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$





$\frac{\pi}{6} = \frac{2\pi}{12}$ $\quad \frac{\pi}{4} = \frac{3\pi}{12}$ $\quad \frac{\pi}{3} = \frac{4\pi}{12}$ $\quad \frac{\pi}{2} = \frac{6\pi}{12}$ $\quad \frac{\pi}{1} = \frac{12\pi}{12}$ $\quad \frac{3\pi}{2} = \frac{18\pi}{12}$ $\quad 2\pi = \frac{24\pi}{12}$

Examples. Evaluate without using a calculator.

1. $\cos\frac{\pi}{12} =$ $\cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$\cos(x+y)$

$\cos x \cdot \cos y - \sin x \cdot \sin y$

$\cos\frac{\pi}{3} \cdot \cos\frac{\pi}{4} - \sin\frac{\pi}{3} \cdot \sin\frac{\pi}{4}$

$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$

Use combinations of the angles above to add or subtract to make equivalent to $\frac{\pi}{12}$.

Then use the addition or subtraction identities to evaluate.

2. $\sin\frac{5\pi}{12} =$ $\sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$\sin\frac{\pi}{6} \cdot \cos\frac{\pi}{4} + \sin\frac{\pi}{4} \cdot \cos\frac{\pi}{6}$

$\left(\frac{1}{2}\right) \cdot \left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$\frac{\sqrt{2} + \sqrt{6}}{4}$

3. $\tan\frac{7\pi}{12} = \tan\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$$\frac{\tan\frac{\pi}{4}+\tan\frac{\pi}{3}}{1-\tan\frac{\pi}{4}\cdot\tan\frac{\pi}{3}}$$

$$\frac{1+\sqrt{3}}{1-1\cdot\sqrt{3}} \to \frac{(1+\sqrt{3})}{(1-\sqrt{3})}$$

$$\frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} \to \frac{1+2\sqrt{3}+3}{1-3} \to \frac{2\sqrt{3}+4}{-2}$$

$$\frac{2\sqrt{3}}{-2}+\frac{4}{-2}$$

$$-\sqrt{3}-2 \qquad -2-\sqrt{3}$$

4. Sin 165º= $(\sin 135 + 30)$ like a 45º

$$\sin 135\cdot\cos 30 + \sin 30\cos 135$$

$$\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{-\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$\frac{\sqrt{6}-\sqrt{2}}{4}$$

5. Simplify: a. $\sin 5\cos 10 + \cos 5 \sin 10$

$\sin(5+10)$

$\sin(15)$

b. $\sin 37 \sin 53 - \cos 37 \cos 53$

$-1(-\sin$

$-\cos 37\cos 53 + \sin 37 \sin 53$

$-1\cos(37+53)$

$-\cos 90$

$-1\cdot 0$

0

6. Rewrite $\csc(x+\frac{\pi}{2})$ in terms of sinx and cosx:

$$\frac{1}{\sin\left(x+\frac{\pi}{2}\right)} \Rightarrow \frac{1}{\sin x\cdot\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\cos x}$$

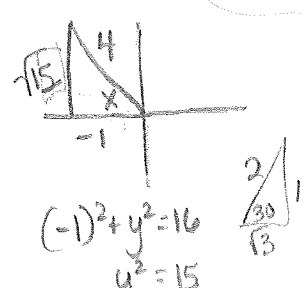
$(0,1)$

$\sin x \cdot 0 \qquad 1\cdot\cos x$

$$\frac{1}{\cos x} \to \sec x$$

7. If $\cos x = -\frac{1}{4}$ and $\frac{\pi}{2} \le x < \pi$ then find $\cos(\frac{\pi}{6} - x)$.

Quadrant II



$(-1)^2+y^2=16$

$y^2=15$

$$\cos\frac{\pi}{6}\cdot\cos x + \sin\frac{\pi}{6}\cdot\sin x$$

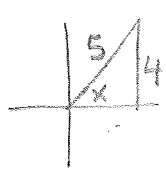
$$\left(\frac{\sqrt{3}}{2}\right)\cdot\left(-\frac{1}{4}\right) + \frac{1}{2}\cdot\frac{\sqrt{15}}{4}$$

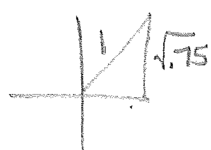
$$-\frac{\sqrt{3}}{8}+\frac{\sqrt{15}}{8}$$

$$\frac{\sqrt{15}-\sqrt{3}}{8}$$

8. Given $\sin x = .8$ and $\sin y = \sqrt{.75}$ and x and y are in Quadrant I, evaluate tan(x – y).

$\frac{8}{10}$ $\quad \sin x = \frac{4}{5}$ $\quad \frac{\sqrt{.75}}{1}$





$$\frac{\tan x - \tan y}{1+\tan x\tan y}$$

9. Prove $\sin x\cos y = \frac{1}{2}[\sin(x+y)+\sin(x-y)]$

$$\frac{1}{2}[(\sin x\cos y + \sin y\cos x) + (\sin x\cos y - \sin y\cos x]$$

$$\frac{1}{2}(2\sin x\cos y)$$

Exercises 9.2

In Exercises 1–12, find the exact value.

1. $\sin\frac{\pi}{12}$

3. $\tan\frac{\pi}{12}$

5. $\cot\frac{5\pi}{12}$

7. $\tan\frac{7\pi}{12}$

9. $\cot\frac{11\pi}{12}$

11. $\sin 105°$

In Exercises 13–18, rewrite the given expression in terms of $\sin x$ and $\cos x$.

13. $\sin\left(\frac{\pi}{2} + x\right)$

15. $\cos\left(x - \frac{3\pi}{2}\right)$

In Exercises 19–24, simplify the given expression.

19. $\sin 3 \cos 5 - \cos 3 \sin 5$

21. $\cos(x + y)\cos y + \sin(x + y)\sin y$

23. $\cos(x + y) - \cos(x - y)$

25. If $\sin x = \frac{1}{3}$ and $0 < x < \frac{\pi}{2}$, then $\sin\left(\frac{\pi}{4} + x\right) = ?$

27. If $\cos x = -\frac{1}{5}$ and $\pi < x < \frac{3\pi}{2}$, then $\sin\left(\frac{\pi}{3} - x\right) = ?$

In Exercises 29–34, assume that $\sin x = 0.8$ and $\sin y = \sqrt{0.75}$ and that x and y lie between 0 and $\frac{\pi}{2}$. Evaluate the given expressions.

29. $\cos(x + y)$

31. $\cos(x - y)$

33. $\tan(x + y)$

37. If x is in the first quadrant and y is in the second quadrant, $\sin x = \frac{24}{25}$, and $\sin y = \frac{4}{5}$, find the exact value of $\sin(x + y)$ and $\tan(x + y)$ and the quadrant in which $x + y$ lies.

39. If x is in the first quadrant and y is in the second quadrant, $\sin x = \frac{4}{5}$, and $\cos y = -\frac{12}{13}$, find the exact value of $\cos(x + y)$ and $\tan(x + y)$ and the quadrant in which $x + y$ lies.

In Exercises 45–56, prove the identity.

45. $\sin(x - \pi) = -\sin x$

47. $\cos(\pi - x) = -\cos x$

49. $\sin(x + \pi) = -\sin x$

51. $\tan(x + \pi) = \tan x$

53. $\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$

55. $\cos(x + y)\cos(x - y) = \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$