

Prove the following identities. Show all solution steps for full credit!

1. $\sec^4 x - \tan^4 x = 2 \tan^2 x + 1$

2. $\csc x(\csc x + \cot x) = \frac{1}{1 - \cos x}$

3. $\frac{\cot x}{\csc x - \sin x} = \sec x$

For #4-8, simplify. Show all work for full credit!

4. $[\sin(x - y) + \sin(x + y)]$

4. _____

5. $\tan(\pi + x)$

5. _____

6. $\cos\left(\frac{\pi}{2} - x\right)$

6. _____

7. Simplify: $\cos(x-y)\cos y - \sin(x-y)\sin y$ 7. _____

8. Simplify: $\sin 7 \cos 3 + \cos 7 \sin 3$ 8. _____

For #9-11, use Addition and Subtraction Identities to find the **exact** values.

9. $\cos \frac{5\pi}{12}$ 9. _____

10. $\sin 105^\circ$ 10. _____

11. $\tan 75^\circ$ 11. _____
Unrationalized

_____ Rationalized

12. Given: $\cos x = -\frac{4}{5}$, where $\frac{\pi}{2} < x < \pi$, 12. _____
evaluate and simplify $\cos\left(\frac{\pi}{4} + x\right)$. Find the **exact** value.

13. Given: $\sin x = \frac{4}{5}$, where $0 < x < \frac{\pi}{2}$ and $\csc y = \frac{13}{5}$, where $\frac{\pi}{2} < y < \pi$
evaluate and simplify $\sin(x+y)$. Find the **exact** value.

13. _____

Prove the following identities. Show all solution steps for full credit!

1. $\sec^4 x - \tan^4 x = 2 \tan^2 x + 1$

$$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$$

$$1(\sec^2 x + \tan^2 x)$$

$$\sec^2 x + \tan^2 x$$

$$\downarrow$$

$$1 + \tan^2 x + \tan^2 x$$

$$\underline{\underline{2 \tan^2 x + 1}}$$

2. $\csc x(\csc x + \cot x) = \frac{1}{1 - \cos x}$

$$\frac{1}{\sin x} \left(\frac{1}{\sin x} + \frac{\cos x}{\sin x} \right)$$

$$\frac{1}{\sin^2 x} + \frac{\cos x}{\sin^2 x}$$

$$\frac{1 + \cos x}{\sin^2 x}$$

$$\frac{1 + \cos x}{1 - \cos^2 x}$$

$$\frac{(1 + \cos x) \cdot 1}{(1 + \cos x)(1 + \cos x)}$$

$$\frac{1}{1 - \cos x}$$

3. $\frac{\cot x}{\csc x - \sin x} = \sec x$

$$\frac{\frac{\cos}{\sin}}{\frac{1}{\sin} - \frac{\sin x \cdot \sin x}{1 \cdot \sin x}}$$

$$\frac{\frac{\cos}{\sin}}{\frac{1 - \sin^2 x}{\sin x}}$$

$$\frac{\cos x}{1 - \sin^2 x}$$

$$\frac{\cos x}{\cos^2 x}$$

$$\frac{\cancel{\cos x}}{\cancel{\cos x}}$$

$$\frac{1}{\cos x} = \sec x$$

For #4-8, simplify. Show all work for full credit!

4. $[\sin(x-y) + \sin(x+y)]$ 4. $2 \sin x \cos y$

$$\sin x \cos y - \sin y \cos x + \sin x \cos y + \sin y \cos x$$

5. $\tan(\pi + x)$ 5. $\tan x$

$$\frac{\cancel{\tan \pi} + \tan x}{1 - \cancel{\tan \pi} \cdot \tan x}$$

6. $\cos\left(\frac{\pi}{2} - x\right)$ 6. $\sin x$

$$\cancel{\cos \frac{\pi}{2}} \cos x + \sin \frac{\pi}{2} \sin x$$

$$0 \cos x + 1 \sin x$$

$$0 + \sin x$$

$$\sin x$$

7. Simplify: $\cos(x-y)\cos y - \sin(x-y)\sin y$

$\cos(a+b) \cos(x-y+y)$

7. $\cos x$

8. Simplify: $\sin 7 \cos 3 + \cos 7 \sin 3$

$\sin(7+3)$

8. $\sin 10$

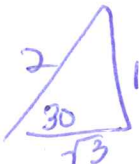
For #9-11, use Addition and Subtraction Identities to find the exact values.

9. $\cos \frac{5\pi}{12}$

$\cos(\frac{3\pi}{12} + \frac{2\pi}{12})$

$\cos(\frac{\pi}{4} + \frac{\pi}{6})$

$\cos \frac{\pi}{4} \cdot \cos \frac{\pi}{6} - \sin \frac{\pi}{4} \cdot \sin \frac{\pi}{6}$



9. $\frac{\sqrt{6}-\sqrt{2}}{4}$

10. $\sin 105^\circ$

$\sin(45+60)$

$\sin 45 \cdot \cos 60 + \sin 60 \cdot \cos 45$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

10. $\frac{\sqrt{2}+\sqrt{6}}{4}$

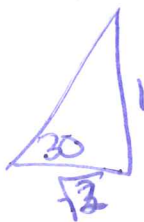
11. $\tan 75^\circ$

$\tan(30^\circ + 45^\circ)$

$\tan 30 + \tan 45$

$\frac{1 + \tan 30 \cdot \tan 45}{1 - \tan 30 \cdot \tan 45}$

$\frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}}$

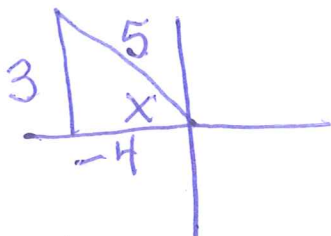


11. $\frac{3+\sqrt{3}}{3-\sqrt{3}}$
Unrationalized

$2+\sqrt{3}$
Rationalized

12. Given: $\cos x = \frac{3}{5}$, where $\frac{\pi}{2} < x < \pi$

evaluate and simplify $\cos(\frac{\pi}{4} + x)$. Find the exact value.



$\cos \frac{\pi}{4} \cdot \cos x - \sin \frac{\pi}{4} \sin x$

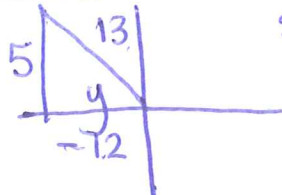
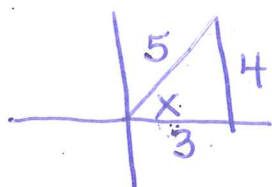
$\frac{\sqrt{2}}{2} \cdot \frac{-4}{5} - \frac{\sqrt{2}}{2} \cdot \frac{3}{5}$

$\frac{-4\sqrt{2}}{10} - \frac{3\sqrt{2}}{10} = \frac{-7\sqrt{2}}{10}$

12. $\frac{-7\sqrt{2}}{10}$

13. Given: $\sin x = \frac{4}{5}$, where $0 < x < \frac{\pi}{2}$ and $\csc y = \frac{13}{5}$, where $\frac{\pi}{2} < y < \pi$

evaluate and simplify $\sin(x+y)$. Find the exact value.



$\sin x \cdot \cos y + \sin y \cdot \cos x$

$\frac{4}{5} \cdot \frac{-12}{13} + \frac{5}{13} \cdot \frac{3}{5}$

$\frac{-48}{65} + \frac{15}{65}$

13. $\frac{-33}{65}$