4.1 Dividing Polynomials

Problem 1 Polynomial Long Division

Got It? Use polynomial long division to divide $3x^2 - 29x + 56$ by $(x - 7)$. What are the quotient and remainder?

dividend

divisor

$$
\begin{array}{r}
3x \quad -8 \\
x - 7 \overline{)\, 3x^2 - 29x + 56} \\
-(3x^2 - 21x) \downarrow \\
\hline
-8x + 56 \\
-(-8x + 56) \\
\hline
0
\end{array}
$$

$0 \leftarrow$ remainder

$(x-7)(3x-8)$

Problem 2 Checking Factors

Got It? Is $x^4 - 1$ a factor of $P(x) = x^5 + 5x^4 - x - 5$? If it is, write $P(x)$ as a product of two factors.

8. Divide.

$$
\begin{array}{r}
x + 5 \\
x^4 - 1 \overline{)\, x^5 + 5x^4 + 0x^3 + 0x^2 - x - 5} \\
-(x^5 + 0x^4 + 0x^3 + 0x^2 - x) \downarrow \\
\hline
5x^4 + 0x^3 + 0x^2 + 0x - 5 \\
-(5x^4 + 0x^3 + 0x^2 + 0x - 5) \\
\hline
0
\end{array}
$$

9. Write $P(x)$ as a product of two factors.

$P(x) = x^5 + 5x^4 - x - 5$

$= (x^4 - 1)(x + 5)$

Underline the correct word(s), number, or expression to complete each sentence.

10. The remainder of the quotient is 0 / $x + 5$ / $x - 5$.

11. The expression $x^4 - 1$ is / is not a factor of $P(x) = x^5 + 5x^4 - x - 5$.

$1x^1$

$\cancel{\ominus x - 7}$

$\cancel{2x}$

Problem 3 Using Synthetic Division

Got It? Use synthetic division to divide $x^3 - 57x + 56$ by $x - 7$. What are the quotient and remainder?

12. Do the synthetic division. Remember that the sign of the number in the divisor is reversed.

7	1	0	−57	56	Write the coefficients of the polynomial.
		7	49	−56	Bring down the first coefficient. Multiply the coefficient by the divisor.
	1	7	−8	0	Add to the next coefficient. Continue multiplying and adding through the last coefficient.

13. The quotient is $1x^2 + 7x - 8$, and the remainder is 0

Take note

Theorem The Remainder Theorem

If you divide a polynomial $P(x)$ of degree $n \geq 1$ by $x - a$, then the remainder is $P(a)$.

17. If you divide $3x^2 + x - 5$ by $x - 1$, the remainder is $P(1)$. -1

$3(1)^2 + 1 - 5$

18. If you divide $2x^2 + x + 6$ by $x + 1$, the remainder is $P(-1)$.

$2(-1)^2 - 1 + 6$

$2 - 1 + 6 = 7$

Problem 5 Evaluating a Polynomial

Got It? What is $P(-4)$, given $P(x) = x^5 - 3x^4 - 28x^3 + 5x + 20$?

19. $P(-4)$ is the remainder when you divide

$x^5 - 3x^4 - 28x^3 + 5x + 20$ by $x - 4$ / $4 - x$ / $x + 4$

$(-4)^5 - 3(-4)^4 - 28(-4)^3 + 5(-4) + 20$

20. Use synthetic division. Circle the remainder.

$(x+4)$

−4	1	−3	−28	0	5	20
		−4	28	0	0	−20
	1	−7	0	0	5	0

21. $P(-4) = $ 0

Divide using long division. Check your answers.

3. $(x^3 + 5x^2 - 3x - 1) \div (x - 1)$

$$x^2 + 6x + 3 + \frac{2}{x-1}$$

$$x-1 \overline{)\, \cancel{x^3} + 5x^2 - 3x - 1}$$
$$-(\cancel{x^3} - x^2)$$
$$6\cancel{x^2} - 3x$$
$$-(\cancel{6}x^2 - 6x)$$
$$3x - 1$$
$$-(3x - 3)$$
$$2$$

4. $(3x^3 - x^2 - 7x + 6) \div (x + 2)$

Determine whether each binomial is a factor of $x^3 + 3x^2 - 10x - 24$.

7. $x + 4$

8. $x - 3$

9. $x + 6$

-4	1	3	-10	-24
		-4	4	24
	1	-1	-6	0

Yes

Divide using synthetic division.

17. $(x^4 - x^3 + x^2 - x + 1) \div (x - 1)$

18. $(2x^4 + 7x^3 - 11x^2 + 21x + 5) \div (x + 5)$

1	1	-1	1	-1	1
		1	0	1	0
	1	0	1	0	1

$$1x^3 + 1x + \frac{1}{x-1}$$

Use synthetic division and the Remainder Theorem to find $P(a)$.

26. $P(x) = 3x^3 - 4x^2 - 5x + 1;\ a = 2$

27. $P(x) = x^3 + 7x^2 + 12x - 3;\ a = -5$

2 |

$$3(2)^3 - 4(2)^2 - 5(2) + 1 = 0$$

$$(x-2)$$

Divide.

30. $(6x^3 + 2x^2 - 11x + 12) \div (3x + 4)$

31. $(x^4 + 2x^3 + x - 3) \div (x - 1)$

32. $(2x^4 + 3x^3 - 4x^2 + x + 1) \div (2x - 1)$

33. $(x^5 - 1) \div (x - 1)$

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Remainder Theorem

If a polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

Simply evaluate the function for f(1). Whatever answer you get, is the remainder!!! So a 0 would tell you there is no remainder!

Example 4 The Remainder When Dividing by $x - c$

Find the remainder when $x^{79} + 3x^{24} + 5$ is divided by $x - 1$.

~~1 | 1 0 0 0 0 0 0 0 0 0 0 0~~

$1^{79} + 3(1)^{24} + 5$

$1 + 3 + 5 = 9$

Example 5 The Remainder When Dividing by $x + k$

Find the remainder when $3x^4 - 8x^2 + 11x + 1$ is divided by $x + 2$.

$3(-2)^4 - 8(-2)^2 + 11(-2) + 1$

$3(16) - 32 - 22 + 1$

$48 - 32 - 22 + 1$

$16 - 22 + 1 = -5$

$-6 + 1 = -5$

In Exercises 31–40, find the remainder when $f(x)$ is divided by $g(x)$, without using division.

31. $f(x) = x^{10} + x^8;\quad g(x) = x - 1$

$(1)^{10} + (1)^8$

2

33. $f(x) = 3x^4 - 6x^3 + 2x - 1;\quad g(x) = x + 1$

35. $f(x) = x^3 - 2x^2 + 5x - 4;\quad g(x) = x + 2$

37. $f(x) = 2x^5 - 3x^4 + x^3 - 2x^2 + x - 8;\ g(x) = x - 10$

In Exercises 41–46, use the Factor Theorem to determine whether $h(x)$ is a factor of $f(x)$.

41. $h(x) = x - 1;\quad f(x) = x^5 + 1$

43. $h(x) = x + 2;\quad f(x) = x^3 - 3x^2 - 4x - 12$

45. $h(x) = x - 1;\quad f(x) = 14x^{99} - 65x^{56} + 51$

Number of Zeros | A polynomial of degree n has at most n distinct real zeros.

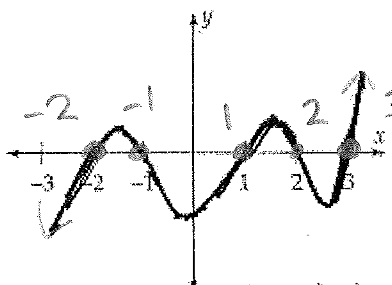
Example 8 A Polynomial with Specific Zeros

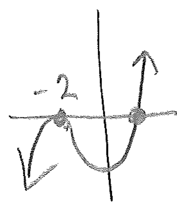
Find three polynomials of different degrees that have 1, 2, 3, and −5 as zeros.

$$f(x) = (x-1)(x-2)(x-3)(x+5)$$
$$= (x-1)^2(x-2)(x-3)(x+5)$$
$$= (x-1)^2(x-2)^3(x-3)(x+5)$$

In Exercises 51–54, each graph is of a polynomial function $f(x)$ of degree 5 whose leading coefficient is 1, but the graph is not drawn to scale. Use the Factor Theorem to find the polynomial. *Hint:* What are the zeros of $f(x)$? What does the Factor Theorem tell you?

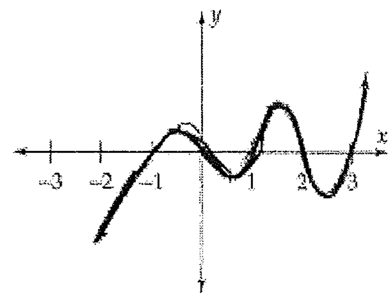
51.





$$f(x) = (x+2)(x+1)(x-1)(x-2)(x-3)$$

53.



In Exercises 55–58, find a polynomial with the given degree n, the given zeros, and no other zeros.

55. $n = 3$; zeros, 1, 7, −4

57. $n = 6$; zeros 1, 2, π

59. Find a polynomial function f of degree 3 such that $f(10) = 17$ and the zeros of $f(x)$ are 0, 5, and 8.

In Exercises 61–64, find a number k satisfying the given condition.

61. $x + 2$ is a factor of $x^3 + 3x^2 + kx - 2$.

63. $x - 1$ is a factor of $k^2x^4 - 2kx^2 + 1$.