PreCalc

Section 4.2 – Notes and Examples

Name: Key

Date: ______ Hour: ______

$x^2-2=0$
$\sqrt{x^2}=\sqrt{2}$ $x=\pm\sqrt{2}$

$x^2+2=0$
$\sqrt{x^2}=\sqrt{2}$

Theorems About Roots of Polynomial Equations

- A *rational* number is ratio of 2 integers $\pm1, \pm2, \pm3, \frac{1}{2}, \frac{1}{3}$.
- An *irrational* number includes values like: non term. / non ending numbers $\pm\sqrt{2}$ $1\pm\sqrt{2}$

When you solve polynomial equations, some solutions might be rational, while others can be irrational. **If a polynomial is factorable, you will get rational solutions**. If a problem has a square root in its solution, that is considered irrational.

The zeros of a polynomial can be found in a variety of ways:

1. Simply factoring
2. quadratic formula
3. Complete the square
4. Square root both sides
5. graphing calculator
6. RATIONAL ZERO TEST + SYNTHETIC DIVISION

x^3

Not all polynomials are factorable! The ***Rational Root Theorem*** (also known as the Rational Zero Theorem) can help you find all **possible** rational roots of a polynomial equation.

Rational Root Theorem

If the polynomial $P(x)$ has integer coefficients, then every rational root of the polynomial equation $P(x) = 0$ can be written in the form $\frac{p}{q}$, where p is a factor of the constant term of $P(x)$ and q is a factor of the leading coefficient of $P(x)$.

RATIONAL ROOT THEOREM: $\dfrac{\text{factors of constant}}{\text{factors of leading coefficient}}$

Follow these steps to find the list of possible rational zeros:

- Write the equation in standard form, looking for missing terms.
- List all factors of the constant term. This is often called "p."
- List all factors of the leading coefficient. This is often called "q."
- List all combinations of fractions created from $\frac{p}{q}$, including positives and negatives.
- Test each possibility using synthetic divison until you find one with a zero remainder. Always try 1 and –1 first!
- Use the quadratic formula or factor to find the remaining roots.

The Quadratic Formula often is a better choice than factoring, since not all of the polynomials will be factorable!

Use the Rational Root Theorem (RRT) to find all possible roots.
Next, use synthetic division to test and find the first rational root.
Finally, use the quadratic formula to find any other real roots.

$-3, -1, \cancel{1}, \cancel{3}$

Example 1:

Q P

C

$P(x) = x^3 + 5x^2 + 7x + 3$ $p =$ 3 $q =$ 1 $\dfrac{p}{q} = \dfrac{\pm 1, \pm 3}{\pm 1}$

$$\begin{array}{r|rrrr} 1 & 1 & 5 & 7 & 3 \\ & & 1 & 6 & 13 \\ \hline & 1 & 6 & 13 & 16 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 5 & 7 & 3 \\ & & -1 & -4 & -3 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

$(x+1)(x^2+4x+3)$

$(x+1)(x+1)(x+3)$

$(x+1)^2(x+3)$

$-1, -3$

$-3, -1, 1, 3$

Example 2: A. Find all rational zeros **B. Find all real zeros**

$x^3 - x^2 - 3x + 3 = 0$ $p =$ 3 $q =$ 1 $\dfrac{p}{q} = \dfrac{\pm 1, \pm 3}{\pm 1}$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -3 & 3 \\ & & 1 & 0 & -3 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$(x-1)(x^2-3) = 0$

$x^2 - 3 = 0$

$\sqrt{x^2} = \sqrt{3}$

$x = \pm\sqrt{3}$

$= (x-1)(x-(\sqrt{3}))(x-(-\sqrt{3}))$

Example 3: A. Find all rational zeros **B. Find all real zeros**

$f(x) = 2x^6 - 3x^5 - 7x^4 - 6x^3$ $p =$ 6 $q =$ 2 $\dfrac{p}{q} = \dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2}$

$= x^3(2x^3 - 3x^2 - 7x - 6)$

$$\begin{array}{r|rrrr} 1 & 2 & -3 & -7 & -6 \\ & & 2 & -1 & -8 \\ \hline & 2 & -1 & -8 & -14 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 2 & -3 & -7 & -6 \\ & & 4 & 2 & -10 \\ \hline & 2 & 1 & -5 & -16 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -7 & -6 \\ & & 6 & 9 & 6 \\ \hline & 2 & 3 & 2 & 0 \end{array}$$

$x^3(x-3)(2x^2+3x+2)$

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\dfrac{-3 \pm \sqrt{9 - 4(2)(2)}}{4}$

$\dfrac{-3 \pm \sqrt{9 - 16}}{4}$

$\dfrac{-3 \pm \sqrt{-7}}{4}$

There is another HINT to use when testing seems to be getting you nowhere. This hint is useful when there are a lot of possibilities listed. When using the Rational Zero Test + synthetic division does not give a 0 remainder, use the UPPER AND LOWER BOUND RULE:

C= number used in synthetic division

Upper Bound: when c>0 and all the signs in the last row are nonnegative (+), "C" is an upper bound....this means it's too big...try something smaller:

$$\begin{array}{r|l} 3 & \\ \hline & + \ + \ + \ + \ + \end{array}$$

Lower Bound: when c<0 and all the signs in the last row alternate (+, -, +, -, +, -) then "c" is a lower bound...it's too small... try something larger:

$$\begin{array}{r|l} -3 & \\ \hline & + \ - \ + \ - \ + \ - \end{array}$$

$- + - + - +$

Example 4: A. Find all rational zeros B. Find all real zeros C. Factor Completely

$f(x) = x^5 - 2x^4 + 2x^3 - 3x + 2$ $p =$ 2 $q =$ 1 $\frac{p}{q} = \frac{\pm 1, \pm 2}{\pm 1}$

$$\begin{array}{r|rrrrrr} 1 & 1 & -2 & 2 & 0 & -3 & 2 \\ & & 1 & -1 & 1 & 1 & -2 \\ \hline & 1 & -1 & 1 & 1 & -2 & 0 \end{array}$$

$(x-1)(x^4 - x^3 + 1x^2 + x - 2)$

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & 1 & 1 & -2 \\ & & 1 & 0 & 1 & 2 \\ \hline & 1 & 0 & 1 & 2 & 0 \end{array}$$

$(x-1)(x-1)(x^3 + x + 2)$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & 2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 4 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 1 & 2 \\ & & -1 & 1 & -2 \\ \hline & 1 & -1 & 2 & 0 \end{array}$$

$(x-1)^2(x+1)(x^2 - 1x + 2)$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{1 \pm \sqrt{1 - 4(1)(2)}}{2}$

In Exercises 1-11, find all the rational zeros of the polynomial

1. $x^3 + 3x^2 - x - 3$

3. $x^3 + 5x^2 - x - 5$

7. $12\left(\frac{1}{12}x^3 - \frac{1}{12}x^2 - \frac{2}{3}x + 1\right)$ *Hint:* The Rational Zero Test can only be used on polynomials with integer coefficients. Note that $f(x)$ and $12f(x)$ have the same zeros. (Why?)

$\boxed{x^3 - x^2 - 8x + 12}$

$x^3 - 19x + 30$

11. $0.1x^3 - 1.9x + 3$

In Exercises 13-17, factor the polynomial as a product of linear factors and a factor g(x) such that g(x) is either a constant or a polynomial that has no rational zeros.

$(\quad)(\quad)(\quad)$

13. $2x^3 - 4x^2 + x - 2$

15. $x^6 + 2x^5 + 3x^4 + 6x^3$

In Exercises 23-31, find all real zeros of the polynomial.

23. $2x^3 - 5x^2 + x + 2$

27. $x^4 + x^3 - 19x^2 + 32x - 12$