4.3 Graphs of Polynomial Functions

Name:_____

Date:_____Hour:_____

Properties of General Polynomial Functions

Continuity

Every graph of a polynomial function is **continuous**, that is, it is an unbroken curve, with no jumps, gaps, or holes. Furthermore, graphs of polynomial functions have no sharp corners. Thus, *neither* of the graphs shown in Figure 4.3-3 represents a polynomial function.

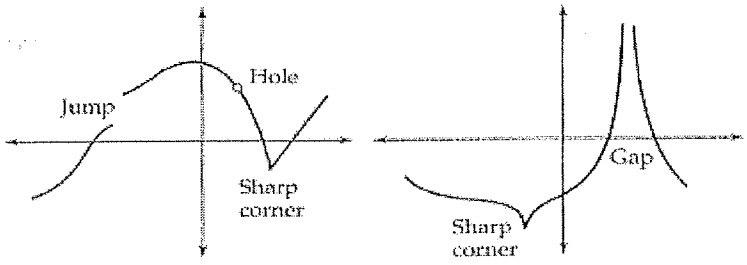
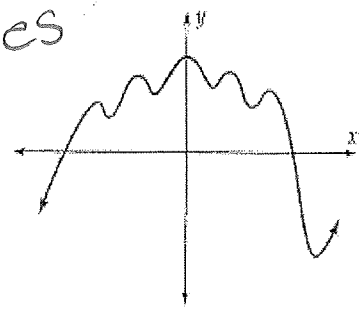


Figure 4.3-3

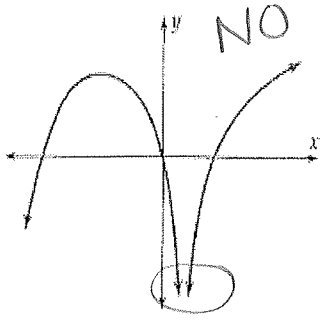
Exercises 4.3

In Exercises 1–6, decide whether the given graph could possibly be the graph of a polynomial function.

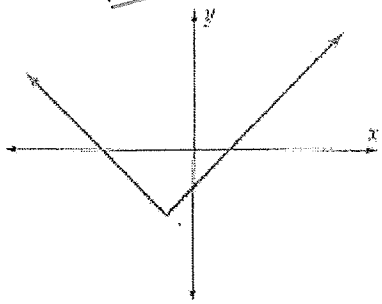
1. YES



2. NO



4. NO

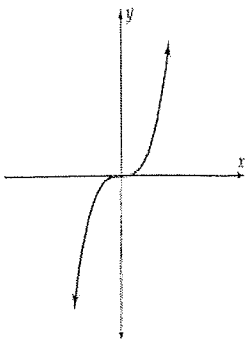


Basic Polynomial Shapes

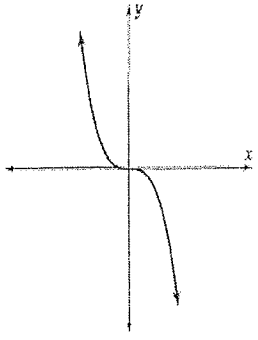
Polynomial Functions of Odd Degree vs Polynomial Functions of Even Degree

$f(x) = ax^n$, n odd

$a > 0$

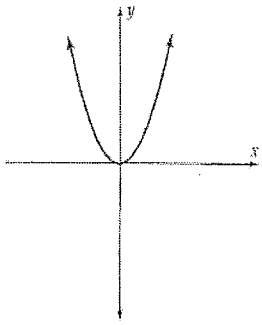


$a < 0$

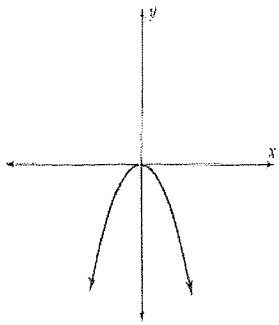


$f(x) = ax^n$, n even

$a > 0$



$a < 0$



End Behavior of Polynomial Functions

When $|x|$ is large, the graph of a polynomial function closely resembles the graph of its highest degree term.

When a polynomial function has *odd* degree, one end of its graph shoots upward and the other end downward.

When a polynomial function has *even* degree, both ends of its graph shoot upward or both ends shoot downward.

Multiplicity and Graphs

Let c be a zero of multiplicity k of a polynomial f.

- If k is odd, the graph of f crosses the x-axis at c.
- If k is even, the graph of f touches, but does not cross, the x-axis at c.

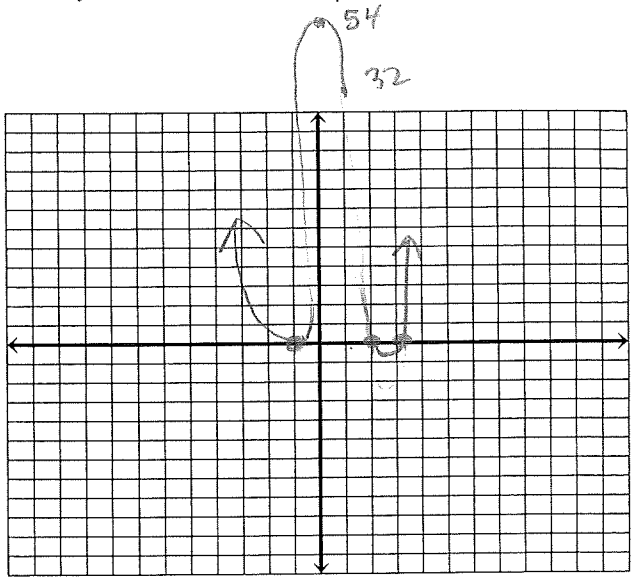
Example 1 Multiplicity of Zeros $y = +x^6$

Find all zeros of $f(x) = (x + 1)^2(x - 2)(x - 3)^3$.

$x=-1 \quad x=2 \quad x=3$

Zero	Multiplicity	x-Intercept
−1	2 even	touches
2	1 odd	crosses
3	3 odd	crosses

x	plug in x	y
0	$1 \cdot -2 \cdot -27$	54
1	$4 \cdot -1 \cdot -8$	32
2.5		−.6



Number of Local Extrema

A polynomial function of degree n has at most $n - 1$ local extrema.

Number of Points of Inflection

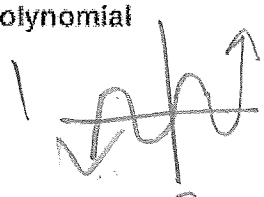
- The graph of a polynomial function of degree n, with $n \geq 2$, has at most $n - 2$ points of inflection.
- The graph of a polynomial function of odd degree has at least one point of inflection.

Example 2 A Complete Graph of a Polynomial

$f(x) = x^5 - x^4 - 8x^3 + 12x^2$

$= x^2(x^3 - x^2 - 8x + 12)$

$= x^2$

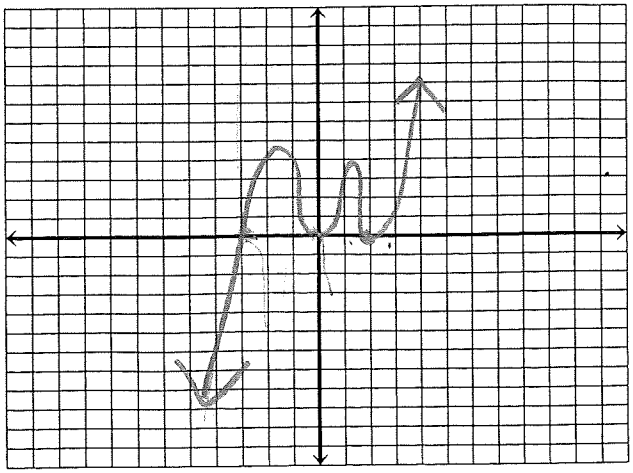


Degree 5 Leading coefficient: + # of Inflection points: 3

Y intercept: (0,0) Zeros 0 2 −3

Multiplicity of each zero: even even odd

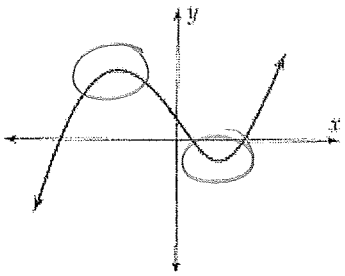
Write in factored form: $y = x^2(x-2)^2(x+3)$



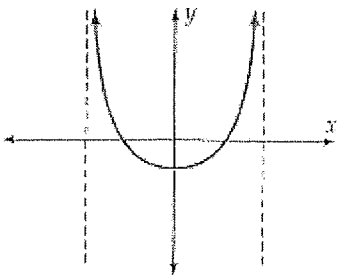
In Exercises 7–12, determine whether the given graph could possibly be the graph of a polynomial function of degree 3, degree 4, or degree 5.

4 5

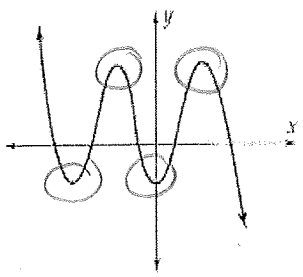
7. 3



9.

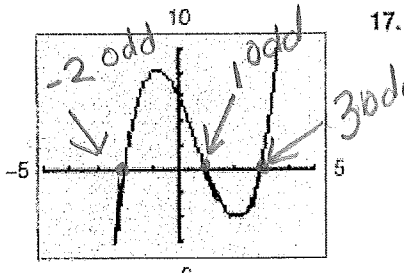


11.



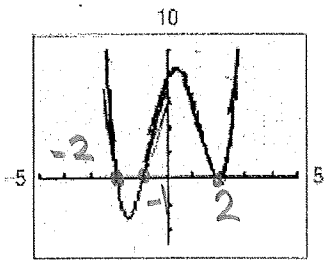
In Exercises 15–18, the graph of a polynomial function is shown. List each zero of the polynomial and state whether its multiplicity is even or odd.

15.



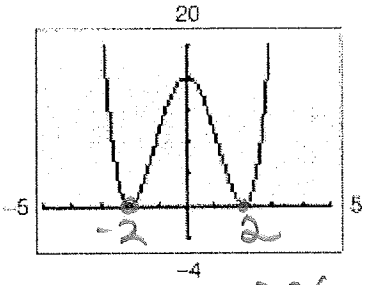
$y = (x+2)(x-1)(x-3)$

17.



$y = (x+2)(x+1)(x-2)^2$

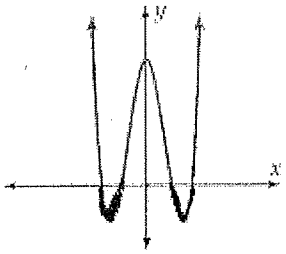
18.



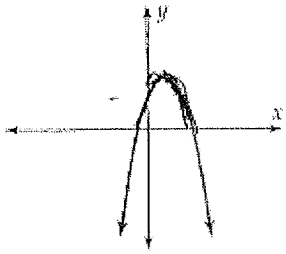
$y = (x+2)^2(x-2)^2$

In Exercises 19–24, use your knowledge of polynomial graphs, *not* a calculator, to match the given function with one of graphs a–f.

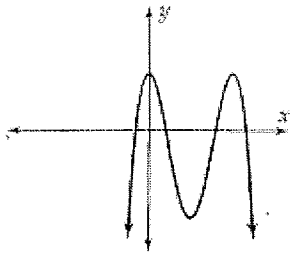
a.



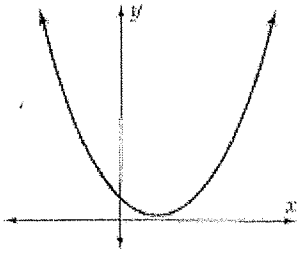
b.



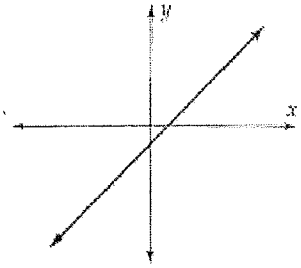
c.



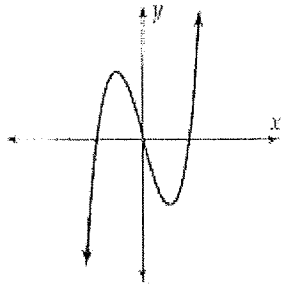
d.



e.



f.



e d c

19. $f(x) = 2x - 3$

20. $g(x) = x^2 - 4x + 7$

23. $f(x) = -x^4 + 6x^3 - 9x^2 + 2$

21. $g(x) = x^3 - 4x$

22. $f(x) = x^4 - 5x^2 + 4$

24. $g(x) = -2x^2 + 3x + 1$

f a b

43. A complete graph of a polynomial function g is shown below.

a. Is the degree of $g(x)$ even or odd?

b. Is the leading coefficient of $g(x)$ positive or negative?

c. What are the real zeros of $g(x)$?

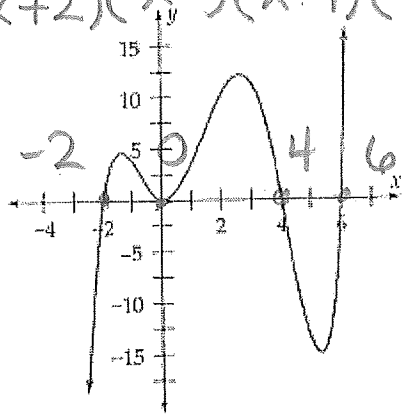
d. What is the smallest possible degree of $g(x)$?

a. even b. neg.

c. $-2, 0, 2$

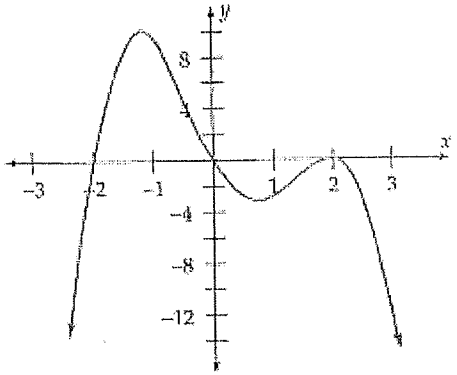
$y = (x+2)(x)(x-2)^2$

$y = (x+2)(x^2)(x-4)(x-6)$



44. Do Exercise 43 for the polynomial function g whose complete graph is shown here.

4 degree



Find the zeros and graph the polynomial. SHOW ALL WORK. BE NEAT!

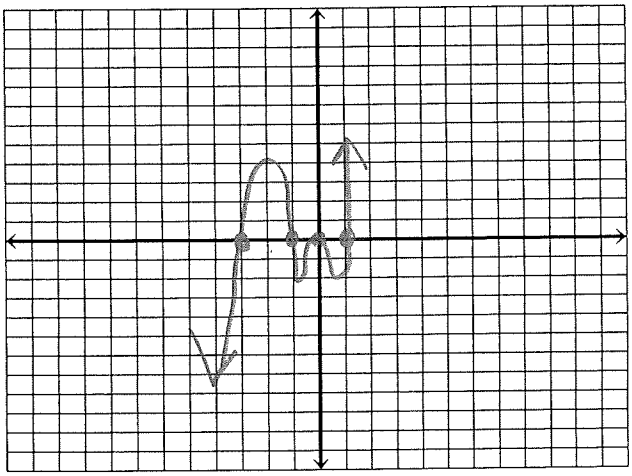
A. $f(x) = x^5 + 3x^4 - x^3 - 3x^2$

$x^2(x^3 + 3x^2 - x - 3)$

$x^2(x-1)(x^2+4x+3)$

$x^2(x-1)(x+3)(x+1)$

x	plug in x	y



Find the zeros and graph the polynomial. SHOW ALL WORK. BE NEAT!!!

$y = x^3 + 2x^2 - 8x$

$x(x^2 + 2x - 8)$

$x(x+4)(x-2)$

x	plug in x	y
-2	$-2 \cdot 2 \cdot -4$	16
1	$1 \cdot 5 \cdot -1$	-10