



Algebra 2

Name: _____

Date: _____ Hour: _____

Section 5.4- Notes and Examples

Dividing Polynomials

When you try to factor a polynomial, you are trying to find a divisor of the polynomial that gives a quotient (the other factor) and a remainder of 0. This suggests that being able to divide one polynomial by another could help you factor polynomials.

Divide using long division.

$$1. \quad (x^3 - 2x^2 - 5x + 6) \div (x + 2)$$

$$\begin{array}{r} x^2 - 4x + 3 \\ x+2 \overline{) x^3 - 2x^2 - 5x + 6} \\ \underline{-(x^3 + 2x^2)} \\ -4x^2 - 5x \\ \underline{-(-4x^2 - 8x)} \\ 3x + 6 \\ \underline{-(3x + 6)} \\ 0 \end{array}$$

$x^2 - 4x + 3$
 $(x-3)(x-1)$

$$2. \quad (6x^3 + x^2 - 2x + 18) \div (2x + 3)$$

$$\begin{array}{r} 3x^2 - 4x + 5 \\ 2x+3 \overline{) 6x^3 + x^2 - 2x + 18} \\ \underline{-(6x^3 + 9x^2)} \\ -8x^2 - 2x \\ \underline{-(-8x^2 - 12x)} \\ 10x + 18 \\ \underline{-(10x + 15)} \\ 3 \end{array}$$

$3x^2 - 4x + 5 + \frac{3}{2x+3}$

$$3. \quad (x^3 + 48) \div (x + 3)$$

$$\begin{array}{r} x^2 - 3x + 9 \\ x+3 \overline{) x^3 + 0x^2 + 0x + 48} \\ \underline{-(x^3 + 3x^2)} \\ -3x^2 + 0x \\ \underline{-(-3x^2 - 9x)} \\ 9x + 48 \\ \underline{-(9x + 27)} \\ 21 \end{array}$$

$x^2 - 3x + 9 + \frac{21}{x+3}$

Synthetic division simplifies the long-division process for dividing by a **linear** expression $x - a$.

To use synthetic division:

- 1) Write the polynomial in standard form, including zeros for missing terms.
- 2) Write the coefficients (including zeros) of the polynomial. Omit all variables and exponents
- 3) For the divisor, reverse the sign (use a , this allows you to add instead of subtract).

Divide using synthetic division.

$$4. \quad (x^3 - 2x^2 - 5x + 6) \div (x + 2)$$

$$\begin{array}{r|rrrr} -2 & 1 & -2 & -5 & 6 \\ & & -2 & 8 & -6 \\ \hline & 1 & -4 & 3 & 0 \end{array} \leftarrow \text{remainder}$$

$1x^2 - 4x + 3$

$$5. \quad (x^3 + 3x + 4) \div (x + 1)$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 3 & 4 \\ & & -1 & 1 & -4 \\ \hline & 1 & -1 & 4 & 0 \end{array}$$

$x^2 - 1x + 4$

Can only use synthetic division to divide a polynomial by a polynomial that is a

L. Coefficient is +1 : degree of divisor is 1

Remember!!! Always look for _____.

Take note

Theorem The Remainder Theorem

If you divide a polynomial $P(x)$ of degree $n \geq 1$ by $x - a$, then the remainder is $P(a)$.

Examples:

$$(2x^3 + 7x^2 + 2x + 1) \div (x - 5)$$

For each function below, use synthetic division **and** substitution to find the indicated value.

6. $P(x) = 2x^3 + 7x^2 + 2x + 1$ find $P(5)$

Method one: synthetic division

$$\begin{array}{r|rrrr}
 5 & 2 & 7 & 2 & 1 \\
 & & 10 & 85 & 435 \\
 \hline
 & 2 & 17 & 87 & 436
 \end{array}$$

remainder

Method two: Substitution

$$\begin{aligned}
 P(5) &= 2(5)^3 + 7(5)^2 + 2(5) + 1 \\
 &= 436
 \end{aligned}$$

$$(3x^3 - 4x + 3) \div (x + 2)$$

7. $P(x) = 3x^3 - 4x + 3$ find $P(-2)$

Method one: synthetic division

$$\begin{array}{r|rrrr}
 -2 & 3 & 0 & -4 & 3 \\
 & & -6 & 12 & -16 \\
 \hline
 & 3 & -6 & 8 & -13
 \end{array}$$

$$3x^2 - 6x + 8 - \frac{13}{x+2}$$

Method two: Substitution

$$\begin{aligned}
 3(-2)^3 - 4(-2) + 3 \\
 -24 + 8 + 3 \\
 -24 + 11 \\
 -13
 \end{aligned}$$

Determine whether the binomial is a factor of $3x^3 + 10x^2 - x - 12$. If so, completely factor the polynomial.

8. $x + 3$

$$\begin{array}{r|rrrr}
 -3 & 3 & 10 & -1 & -12 \\
 & & -9 & -3 & 12 \\
 \hline
 & 3 & 1 & -4 & 0
 \end{array}$$

9. $x + 2$

$$\begin{array}{r|rrrr}
 -2 & 3 & 10 & -1 & -12 \\
 & & -6 & -8 & 18 \\
 \hline
 & 3 & 4 & -9 & 6
 \end{array}$$

or

$$3(-3)^3 + 10(-3)^2 - (-3) - 12$$

5-4

Practice

Form G

Dividing Polynomials

Divide using long division. Check your answers.

1. $(x^2 - 13x - 48) \div (x + 3)$

2. $(2x^2 + x - 7) \div (x - 5)$

Determine whether each binomial is a factor of $x^3 + 3x^2 - 10x - 24$.

7. $x + 4$

8. $x - 3$

9. $x + 6$

10. $x + 2$

Divide using synthetic division.

11. $(x^3 - 8x^2 + 17x - 10) \div (x - 5)$

12. $(x^3 + 5x^2 - x - 9) \div (x + 2)$

17. $(x^4 - x^3 + x^2 - x + 1) \div (x - 1)$

18. $(2x^4 + 7x^3 - 11x^2 + 21x + 5) \div (x + 5)$

Use synthetic division and the given factor to completely factor each polynomial function.

21. $y = x^3 + 3x^2 - 13x - 15; (x + 5)$

22. $y = x^3 - 3x^2 - 10x + 24; (x - 2)$

Use synthetic division and the Remainder Theorem to find $P(a)$.

26. $P(x) = 3x^3 - 4x^2 - 5x + 1; a = 2$ 27. $P(x) = x^3 + 7x^2 + 12x - 3; a = -5$

Divide.

30. $(6x^3 + 2x^2 - 11x + 12) \div (3x + 4)$ 31. $(x^4 + 2x^3 + x - 3) \div (x - 1)$

32. $(2x^4 + 3x^3 - 4x^2 + x + 1) \div (2x - 1)$ 33. $(x^5 - 1) \div (x - 1)$

37. Writing What are the divisor, quotient, and remainder represented by the synthetic division below?

$$\begin{array}{r|rrrr} -5 & 1 & 0 & -19 & 30 \\ & & -5 & 25 & -30 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$
